

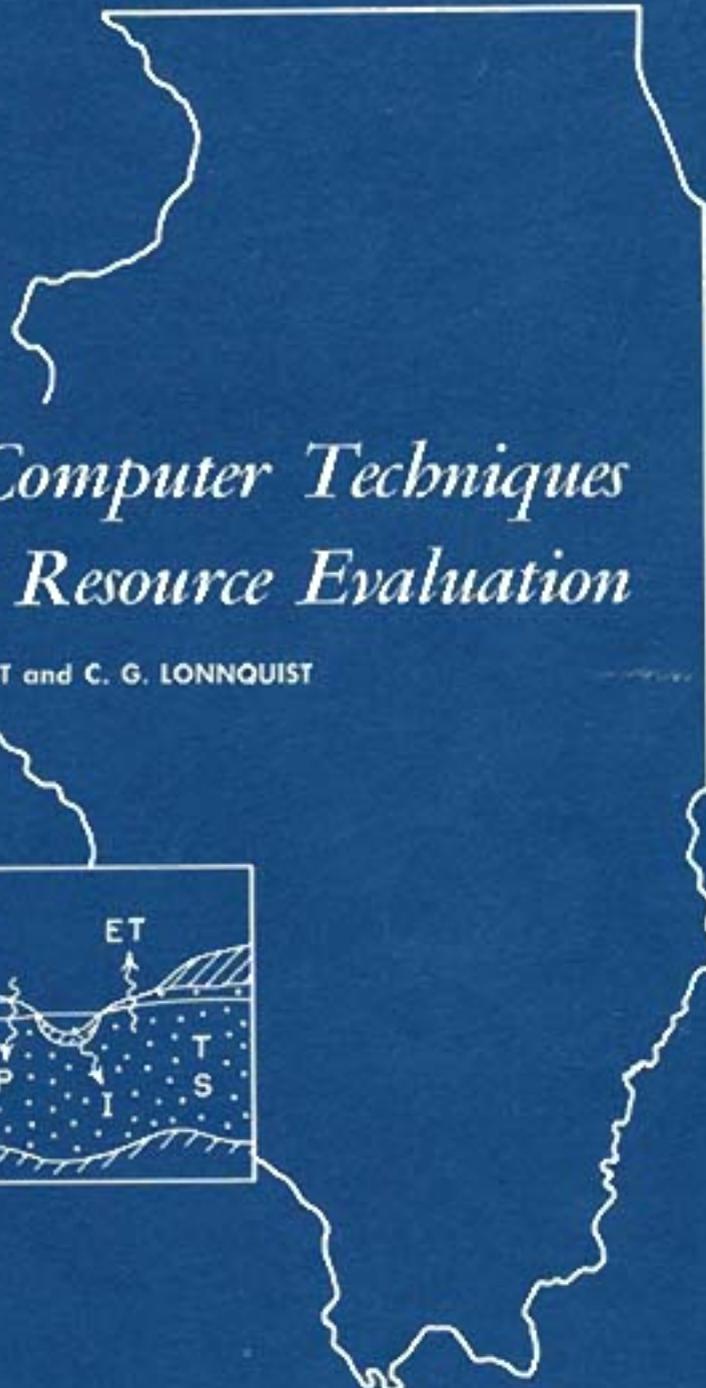
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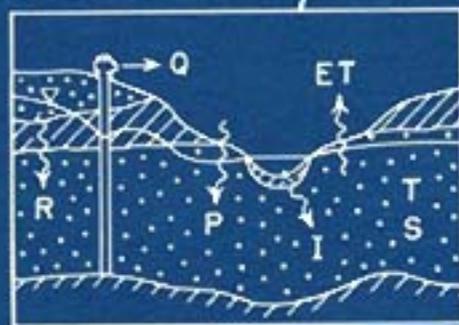
STATE OF ILLINOIS

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# *Selected Digital Computer Techniques for Groundwater Resource Evaluation*

by T. A. PRICKETT and C. G. LONNQUIST



ILLINOIS STATE WATER SURVEY

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# *Selected Digital Computer Techniques for Groundwater Resource Evaluation*

by T. A. PRICKETT and C. G. LONNQUIST

**Title:** Selected Digital Computer Techniques for Groundwater Resource Evaluation.

**Abstract:** Generalized digital computer program listings are given that can simulate one-, two-, or three-dimensional nonsteady flow of groundwater in heterogeneous aquifers under water table, nonleaky, and leaky artesian conditions. Included are listings for time varying pumpage from wells, natural or artificial recharge rates, the relationships of water exchange between surface waters and the groundwater reservoir, the process of groundwater evapotranspiration, and the mechanism of converting from artesian to water table conditions. Discussion includes necessary mathematical background, documented program listings, sample input data, job setup procedures, theoretical versus computer comparisons, and field examples. A finite difference approach is used to formulate the equations of groundwater flow; a modified alternating direction implicit method solves the set of resulting finite difference equations. Programs are written in FORTRAN IV and will operate with any consistent set of units. The digital computers can deal with much more complex problems than is practical with electric analog or analytical methods.

**Reference:** Prickett, T. A., and C. G. Lonquist. Selected Digital Computer Techniques for Groundwater Resource Evaluation. Illinois State Water Survey, Urbana, Bulletin 55, 1971.

**Indexing Terms:** Case histories, digital computers, finite difference theory, groundwater, groundwater evapotranspiration, induced infiltration, leakage, models, numerical analysis, program listings, recharge, three-dimensional flow, variable pumpage.

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*Printed by authority of the State of Illinois-Ch. 127, IRS, Par. 58.29*  
*(12-71-2000)*

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# *Selected Digital Computer Techniques for Groundwater Resource Evaluation*

by T. A. Prickett and C. G. Lonquist

## ABSTRACT

Generalized digital computer program listings are given that can simulate one-, two-, and three-dimensional nonsteady flow of groundwater in heterogeneous aquifers under water table, nonleaky, and leaky artesian conditions. Programming techniques involving time varying pumpage from wells, natural or artificial recharge rates, the relationships of water exchange between surface waters and the groundwater reservoir, the process of groundwater evapotranspiration, and the mechanism of converting from artesian to water table conditions are also included.

The discussion of the digital techniques includes the necessary mathematical background, documented program listings, theoretical versus computer comparisons, and field examples. Also presented are sample computer input data and explanations of job setup procedures.

A finite difference approach is used to formulate the equations of groundwater flow. A modified alternating direction implicit method is used to solve the set of resulting finite difference equations.

The programs included are written in FORTRAN IV and will operate with any consistent set of units.

## INTRODUCTION

The use of digital computers in groundwater resource evaluation has grown rapidly within the past few years. Computers are now widely available that allow solution of large sets of simultaneous equations that are involved in studying cause and effect relationships in heterogeneous aquifer systems with a wide variety of boundary conditions. The digital computer can deal with problems of much greater complexity than is practical with electric analog or analytical methods. However, digital computers will not cause analytical methods or electric analog simulators to become obsolete; used in conjunction with these other tools available to the hydrologist, digital computers can greatly improve the analysis of groundwater problems.

This report, which describes the digital computer techniques used at this time by the Illinois State Water Survey, is aimed primarily to the engineer and geologist involved in evaluating groundwater resources. The discussion of the digital techniques includes the necessary mathematical background, documented program listings, theoretical examples, and field applications.

The main object of this report is to present generalized computer programs that will simulate one-, two-, or three-dimensional nonsteady-state flow problems in heterogeneous aquifers under water table, nonleaky, and leaky artesian conditions. These programs cover time varying pumpage from wells, natural or artificial recharge rates, the relationships of water exchange between surface waters and the groundwater reservoir, the process of groundwater evapo-

transpiration, the mechanism of possible conversion of storage coefficients from artesian to water table conditions, and the three-dimensional problem.

A finite difference approach that is identical to that taken in designing electric analog simulators has been adopted in formulating the equations of groundwater flow. Thus investigators whose experience is solely with designing electric analogs will find many similarities in the digital model designing procedures.

Part 1 of this report provides a description of the mathematical derivation of the finite difference equations governing the flow of groundwater in a form amenable to digital computer solution. Also described is a modified alternating direction implicit procedure by which the computer programs solve the set of finite difference equations.

Part 2 presents a 'basic aquifer simulation program' and describes its use in detail. The basic aquifer simulation program is intended for studying cause and effect relationships in heterogeneous aquifer systems with constant pumpage taking place from wells simulated at any node point of the digital model.

The basic aquifer simulation program forms the nucleus of all remaining programs of the report. Part 3 includes the various modifications made to the basic program that allow incorporating additional features such as specialized types of printout and the differing boundary conditions mentioned above. The program modifications are added and discussed one at a time. The modified program listings

are given, sample data sets are included along with job setup instructions, and comparisons are made between theoretical and field examples where possible.

It is intended that simulation of various conditions, applicable for particular aquifer situations, can be accomplished by combining separate features of several computer programs. Given as an example is a combined aquifer simulation program that can simultaneously handle all of the conditions mentioned earlier.

Also included are a specialized computer model for a field example, an analog-digital computer comparison of solutions of a hypothetical model problem, and an analysis of heads or drawdowns at pumped well nodes of the digital model.

The computer programs included in this report were written in FORTRAN Iv for use on an IBM 360 system model 75 with a G level compiler. However, the programs will operate, with modifications, on other computers. Also, these computer programs are written so that they will operate with any consistent set of units.

The reader should be cautioned that the program listings of this report are not intended to be 'canned programs' but are meant to present the basic concepts of individual boundary condition modeling which, in combination with one

another, will serve as a basis for designing digital models for particular aquifer situations.

### **Acknowledgments**

This report was prepared under the general supervision of Dr. William C. Ackermann, Chief of the Illinois State Water Survey, and H. F. Smith, Head of the Hydrology Section. R. J. Schicht and A. F. Moench of the Hydrology Section, and Jacob Bear of the Israel Institute of Technology, Haifa, reviewed and criticized the material and made useful suggestions.

Many individuals made helpful comments on the computer programming and acknowledgment is due R. A. Sinclair of the Water Survey Statistical Unit; P. Ungemach of the Service Geologique D'Alsace et de Lorraine in Strasbourg, France; and many programming consultants of the Department of Computer Science at the University of Illinois.

Thanks are also due Miss Judy Flynn and M. C. Clevenger of the Water Survey Computer Data Processing Section for keypunching most of the programs and data decks; Mrs. Karen Helm for typing the manuscript; Mrs. J. Loreena Ivens for final editing; and J. W. Brother, Jr., and William Motherway, Jr., for preparing the illustrations.

## Part 1. Mathematical Background

### Mathematical Derivation of Finite Difference Equations

The partial differential equation (Bittinger et al., 1967) governing the nonsteady-state, two-dimensional flow of groundwater in an artesian, nonhomogeneous, and isotropic aquifer is

$$\partial/\partial x (T \partial h/\partial x) + \partial/\partial y (T \partial h/\partial y) = S \partial h/\partial t + Q \quad (1)$$

where

$T$  = aquifer transmissivity

$h$  = head

$t$  = time

$S$  = aquifer storage coefficient

$Q$  = net groundwater withdrawal rate per unit area

$x, y$  = rectangular coordinates

There is no general solution to equation 1; however, a numerical solution of that equation can be obtained through a finite difference approach. The finite difference approach first involves replacing the continuous aquifer system parameters by an equivalent set of discrete elements. In working with digital computers both the space and time variables are treated as discrete parameters. Secondly, the equations governing the flow of groundwater in the discretized model are written in finite difference form. Finally, the resulting set of finite difference equations is solved numerically with the aid of a digital computer.

The finite difference equations can be derived in two ways; i.e., from the physical standpoint involving Darcy's law and the principle of conservation of mass, or by a conventional mathematical treatment, substituting finite difference approximations for the derivatives of equation 1. Both derivational routes lead to the same result. The finite difference equations of this report are derived from the physical standpoint as follows.

A finite difference grid is superposed over a map of an aquifer as illustrated in figure 1. The aquifer is thus subdivided into volumes having dimensions  $m \times y$  where  $m$  is the thickness of the aquifer. The differentials  $x$  and  $y$  are approximated by the finite lengths  $\Delta x$  and  $\Delta y$ , respectively. The area  $\Delta x \Delta y$  should be small compared with the total area of the aquifer to the extent that the discrete model is a reasonable representation of the continuous system. The intersections of grid lines are called nodes and are referenced with a column ( $i$ ) and row ( $j$ ) coordinate system colinear with the  $x$  and  $y$  directions, respectively.

Flow rate terms ( $Q_1, 2, 3, \dots, n$ ) are arbitrarily assigned flow directions as illustrated in figure 1. Flow rate terms  $Q_1, Q_2, Q_3$ , and  $Q_4$  represent node-to-node water transfer rates.  $Q_5$  is the flow rate associated with the amount of water taken into or released from storage per unit time increment  $t$ . Water is being taken into storage since the assigned direction of the flow rate term  $Q_5$  is out of the node  $i, j$  as il-

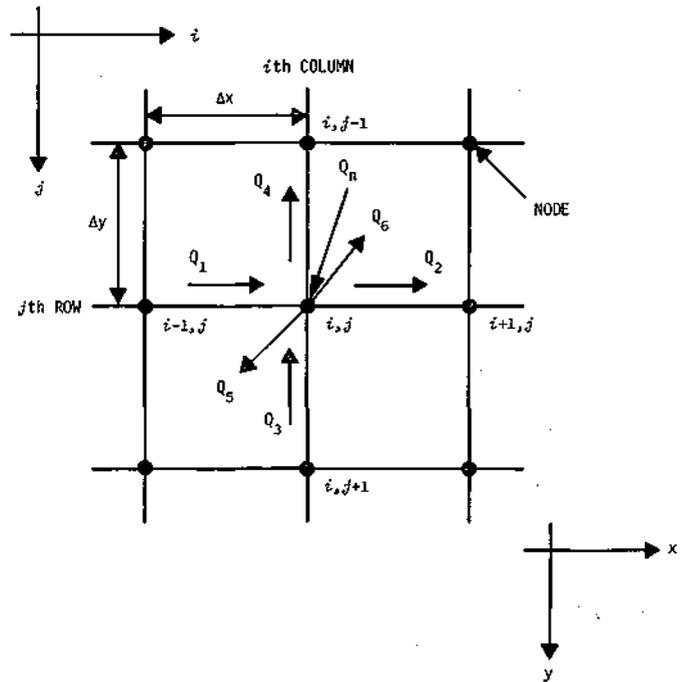


Figure 1. Finite difference grid

lustrated in figure 1.  $Q_6$  is defined as a net constant withdrawal rate. A seventh generalized flow rate term  $Q_n$  is included to account for other special conditions such as leakage, induced infiltration, and effects of evapotranspiration which will be added later in this report.

The continuity condition relating the flow rates entering and leaving the node  $i, j$  of figure 1 requires that the rates be equal as follows

$$Q_4 + Q_1 + Q_3 = Q_2 + Q_5 + Q_6 + Q_n \quad (2)$$

Determining the values of the flow rate terms of equation 2 involves three considerations. First, it is necessary to define what portion of the aquifer is represented by each individual term. Secondly, it must be kept in mind that, although the flow rates may take place in any direction in the aquifer system, they are restricted to the  $x$  and  $y$  directions in the finite difference approach. The portions of the aquifer included in the flow rate terms then may be referred to as 'vector volumes' to emphasize that not only a volume but also the direction of flow is being considered (Karplus, 1958). Finally, since time is discretized, equation 2 represents an instantaneous balance at the end of a time increment.

Horizontal projections of the vector volumes of the node-to-node flow rate terms  $Q_1, Q_2, Q_3$ , and  $Q_4$  are defined as illustrated in figure 2. All vector volumes of figure 2 have a vertical dimension extending the full depth of the aquifer,  $m$ . Furthermore, the portion of aquifer involved with each of these flow rate terms extends in width one half of the grid interval on either side of the line between node points

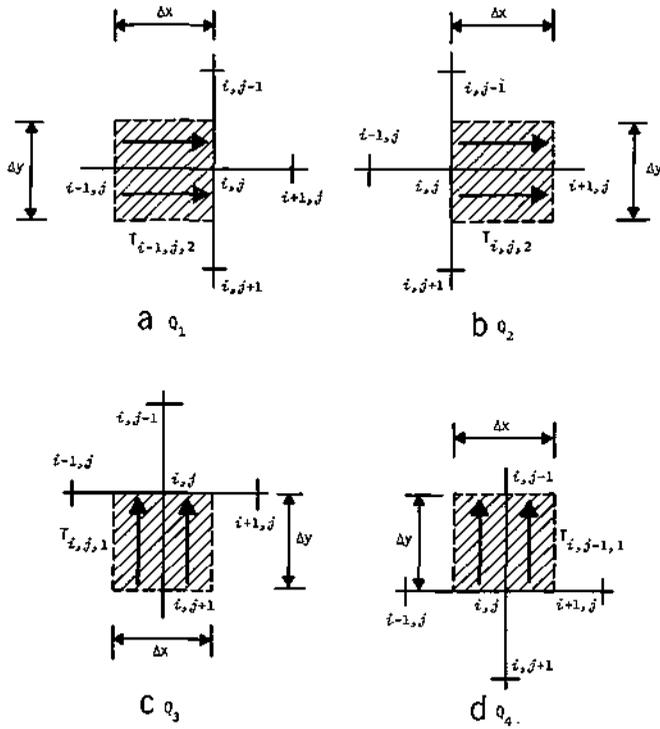


Figure 2. Vector volumes for node-to-node flow rate terms

and is equal in length to the grid interval. Darcy's law is then applied to the flow rate terms  $Q_1$  through  $Q_4$  to give

$$Q_1 = T_{i-1,j,2} (h_{i-1,j} - h_{i,j}) \Delta y / \Delta x \quad (3a)$$

$$Q_2 = T_{i,j,2} (h_{i,j} - h_{i+1,j}) \Delta y / \Delta x \quad (3b)$$

$$Q_3 = T_{i,j,1} (h_{i,j+1} - h_{i,j}) \Delta x / \Delta y \quad (3c)$$

$$Q_4 = T_{i,j-1,1} (h_{i,j} - h_{i,j-1}) \Delta x / \Delta y \quad (3d)$$

where

$T_{i,j,1}$  = aquifer transmissivity within the vector volume between nodes  $i,j$  and  $i,j + 1$  (see figures 2c and d)

$T_{i,j,2}$  = aquifer transmissivity within the vector volume between nodes  $i,j$  and  $i,j + 1$  (see figures 2a and b)

$h_{i,j}$  = calculated heads at the end of a time increment measured from an arbitrary reference level at node  $i,j$

Horizontal projections of the vector volumes of the flow terms  $Q_5$ ,  $Q_6$ , and  $Q_n$  are identical (figure 3). These vector volumes, except that related to  $Q_n$ , extend the full depth of the aquifer and have horizontal dimensions of  $x \times y$ , the volumes being centered around the node point  $i,j$ .

The flow rate term  $Q_5$ , representing the rate at which water is taken into storage, is given by

$$Q_5 = S \Delta x \Delta y (h_{i,j} - h_{\phi i,j}) / \Delta t \quad (4)$$

where

$h_{\phi i,j}$  = calculated head at node  $i,j$  at the end of the previous time increment  $t$

$t$  = time increment elapsed since last calculation of heads

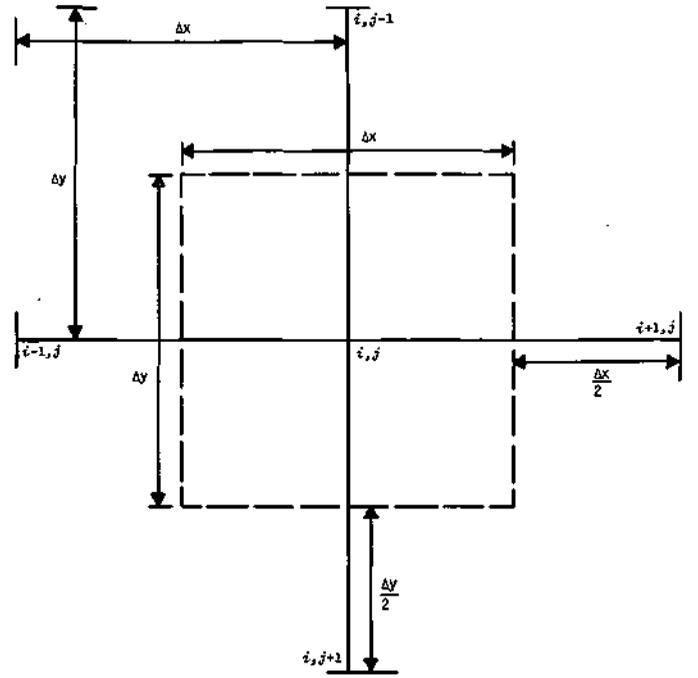


Figure 3. Vector volumes for flow rate terms  $Q_5$ ,  $Q_6$ , and  $Q_n$

The flow rate term  $Q_6$  is made equal to a constant net withdrawal rate from the vector volume of node  $i,j$  of figure 3 as follows

$$Q_6 = Q_{i,j} \quad (5)$$

Since no special source or sink functions are included at this time, the flow rate term  $Q_n$  is set equal to zero. However, the term  $Q_n$  will be carried along in the derivations that follow so that when special functions are included their relationships may be substituted without re-derivation. Therefore

$$Q_n = Q_n \quad (6)$$

Substitution of equations 3, 4, 5, and 6 into equation 2 results in

$$Q_n + T_{i-1,j,2} (h_{i-1,j} - h_{i,j}) \Delta y / \Delta x + T_{i,j,1} (h_{i,j+1} - h_{i,j}) \Delta x / \Delta y = T_{i,j,2} (h_{i,j} - h_{i+1,j}) \Delta y / \Delta x + T_{i,j-1,1} (h_{i,j} - h_{i,j-1}) \Delta x / \Delta y + S \Delta x \Delta y (h_{i,j} - h_{\phi i,j}) / \Delta t + Q_{i,j} \quad (7)$$

Dividing both sides of equation 7 by the product of  $x \times y$  and rearranging terms yields

$$T_{i-1,j,2} (h_{i-1,j} - h_{i,j}) / \Delta x^2 + T_{i,j,2} (h_{i+1,j} - h_{i,j}) / \Delta x^2 + T_{i,j,1} (h_{i,j+1} - h_{i,j}) / \Delta y^2 + T_{i,j-1,1} (h_{i,j-1} - h_{i,j}) / \Delta y^2 = S (h_{i,j} - h_{\phi i,j}) / \Delta t + Q_{i,j} / \Delta x \Delta y - Q_n / \Delta x \Delta y \quad (8)$$

With  $Q_n$  equal to zero, equation 8 is the finite difference form of the partial differential equation (see equation 1) governing the nonsteady-state, two-dimensional flow of groundwater in an artesian, nonhomogeneous aquifer. A similar equation was derived in a more formal manner by Pinder and Bredehoeft (1968, equation 20).

Since there is an equation of the same form as equation 8 for every node of the digital model, a large set of simultaneous equations must be solved for the principal unknowns  $h_{i,j}$ . A modified form of the alternating direction implicit

method of Peaceman and Rachford (1955), as described below, is used to solve the set of simultaneous equations.

### Iterative Alternating Direction Implicit Method

Briefly, the iterative alternating direction implicit method involves first, for a given time increment, reducing a large set of simultaneous equations down to a number of small sets. This is done by solving the node equations by Gauss elimination of an individual column of the model while all terms related to the nodes in adjacent columns are held constant. According to Peaceman and Rachford (1955), the set of column equations is then implicit in the direction along the column and explicit in the direction orthogonal to the column alignment. The solution of the set of column equations is then a straightforward process which will be explained in the next section.

After all column equations have been processed column by column, attention is focused on solving the node equations again by Gauss elimination of an individual row while all terms related to adjacent rows are held constant. Finally, after all equations have been solved row by row, an 'iteration' has been completed. The above process is repeated a sufficient number of times to achieve convergence, and this completes the calculations for the given time increment. The calculated heads are then used as initial conditions for the next time increment. This total process is repeated for successive time increments. Peaceman and Rachford (1955) point out that this technique is unconditionally stable regardless of the size of the time increment.

Equation 8 is modified and rearranged to facilitate node equation solving by columns and rows. First it is assumed that the finite difference grid is made up of squares such that  $y = x$ . (The case where  $y$  does not equal  $x$  will be treated later in this report.) Equation 8 can then be rewritten by multiplying both sides by  $x^2$  as follows:

$$\begin{aligned} & T_{i-1,j,2} (h_{i-1,j} - h_{i,j}) + T_{i,j,2} (h_{i+1,j} - h_{i,j}) \\ & + T_{i,j,1} (h_{i,j+1} - h_{i,j}) + T_{i,j-1,1} (h_{i,j-1,1} - h_{i,j}) \\ & = (S\Delta x^2/\Delta t) (h_{i,j} - h\phi_{i,j}) + Q_{i,j} - Q_n \end{aligned} \quad (9)$$

Equation 9 is expanded, the signs are reversed, and terms of  $h_{i,j}$  are grouped together to yield

$$\begin{aligned} & h_{i,j} (T_{i-1,j,2} + T_{i,j,2} + T_{i,j,1} + T_{i,j-1,1} + S\Delta x^2/\Delta t) \\ & - T_{i-1,j,2} h_{i-1,j} - T_{i,j,2} h_{i+1,j} - T_{i,j,1} h_{i,j+1} \\ & - T_{i,j-1,1} h_{i,j-1} = (S\Delta x^2/\Delta t) h\phi_{i,j} - Q_{i,j} + Q_n \end{aligned} \quad (10)$$

Equation 10 is then written in two forms, one for solving the node equations by columns and the other for solving the node equations by rows.

For calculations by *columns*, equation 10 is rearranged as follows

$$\begin{aligned} & -T_{i,j-1,1} h_{i,j-1} + h_{i,j} (T_{i-1,j,2} + T_{i,j,2} + T_{i,j,1} + T_{i,j-1,1} \\ & + S\Delta x^2/\Delta t) - T_{i,j,1} h_{i,j+1} = (S\Delta x^2/\Delta t) h\phi_{i,j} - Q_{i,j} \\ & + T_{i-1,j,2} h_{i-1,j} + T_{i,j,2} h_{i+1,j} + Q_n \end{aligned} \quad (11)$$

Equation 11 is of the form

$$AA_j h_{i,j-1} + BB_j h_{i,j} + CC_j h_{i,j+1} = DD_j \quad (11')$$

where the constant terms are

$$AA_j = -T_{i,j-1,1} \quad (11a)$$

$$BB_j = T_{i-1,j,2} + T_{i,j,2} + T_{i,j,1} + T_{i,j-1,1} + S\Delta x^2/\Delta t \quad (11b)$$

$$CC_j = -T_{i,j,1} \quad (11c)$$

$$\begin{aligned} DD_j = & (S\Delta x^2/\Delta t) h\phi_{i,j} - Q_{i,j} + T_{i-1,j,2} h_{i-1,j} \\ & + T_{i,j,2} h_{i+1,j} + Q_n \end{aligned} \quad (11d)$$

For calculations by *rows*, equation 10 is rearranged as follows

$$\begin{aligned} & -T_{i-1,j,2} h_{i-1,j} + h_{i,j} (T_{i-1,j,2} + T_{i,j,2} + T_{i,j,1} + T_{i,j-1,1} \\ & + S\Delta x^2/\Delta t) - T_{i,j,2} h_{i+1,j} = (S\Delta x^2/\Delta t) h\phi_{i,j} - Q_{i,j} \\ & + T_{i,j-1,1} h_{i,j-1} + T_{i,j,1} h_{i,j+1} + Q_n \end{aligned} \quad (12)$$

Equation 12 is of the form

$$AA_i h_{i-1,j} + BB_i h_{i,j} + CC_i h_{i+1,j} = DD_i \quad (12')$$

where the constant terms are

$$AA_i = -T_{i-1,j,2} \quad (12a)$$

$$BB_i = T_{i-1,j,2} + T_{i,j,2} + T_{i,j,1} + T_{i,j-1,1} + S\Delta x^2/\Delta t \quad (12b)$$

$$CC_i = -T_{i,j,2} \quad (12c)$$

$$\begin{aligned} DD_i = & (S\Delta x^2/\Delta t) h\phi_{i,j} - Q_{i,j} + T_{i,j-1,1} h_{i,j-1} \\ & + T_{i,j,1} h_{i,j+1} + Q_n \end{aligned} \quad (12d)$$

There are three head unknowns in each equation written for each node along a column (equation 11') or row (equation 12'). (In standard matrix notation a set of equations defined by 11' or 12' forms what is termed a tri-diagonal matrix.) The solution of a set of column or row head equations is accomplished by Gauss elimination incorporating what Peaceman and Rachford term G and B arrays applied to tri-diagonal matrices. An explanation of what is included in the G and B arrays follows.

### Calculation of Heads with G and B Arrays

An example can be used to illustrate how heads are calculated with the use of the G and B arrays. Although the method outlined can be applied to any number of nodes in a row or column, a four-node row as shown in figure 4 will be considered here, for completeness and simplicity. The heads at the nodes 1, 2, 3, and 4 along the  $j$ th row can be calculated by first writing the flow equations (of the form given by equation 12') for each node going in order of increasing column number ( $i$ ). Secondly, the resulting equations are arranged in such a manner that the head at the node of interest,  $h_{i,j}$ , is a function of known parameters and the head at only the node  $h_{i+1,j}$ . When this is done, the head at the last node of the row,  $h_{4,j}$ , will be a function of known parameters. Finally, all other heads can then be calculated in order of decreasing column number.

The equations for the four nodes shown in figure 4 can be written as follows.

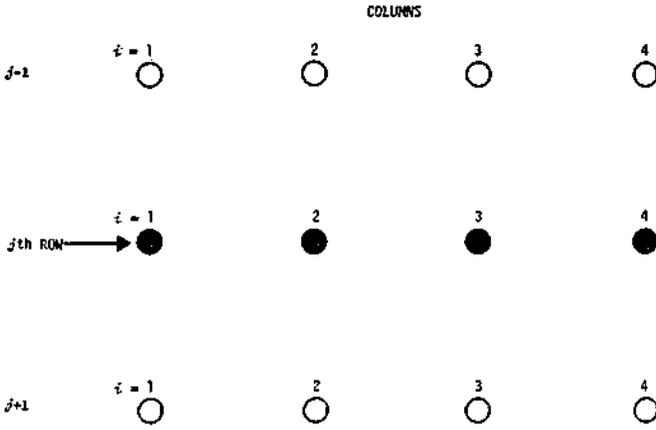


Figure 4. Example four-node row

At the first  $j$ th row node,  $i = 1$  and the flow equation is (from equation 12')

$$AA_1 h_{0,j} + BB_1 h_{1,j} + CC_1 h_{2,j} = DD_1 \quad (13)$$

Since no node with coordinates  $0,j$  exists,  $AA_1$  is set equal to zero and equation 13 becomes

$$BB_1 h_{1,j} + CC_1 h_{2,j} = DD_1 \quad (14)$$

Equation 14 is rearranged into

$$h_{1,j} = DD_1/BB_1 - (CC_1/BB_1) h_{2,j} \quad (15)$$

At this point it is defined that

$$G_1 = DD_1/BB_1 \quad (16)$$

and

$$B_1 = CC_1/BB_1 \quad (17)$$

Substituting equations 16 and 17 into 15 gives

$$h_{1,j} = G_1 - B_1 h_{2,j} \quad (18)$$

Now the head at the node of interest  $h_{1,j}$  is a function of the known parameters  $G_1$  and  $B_1$  and the head at only the node  $h_{i+1,j}$  or  $h_{2,j}$ .

Proceeding to the next  $j$ th row node of figure 4 where  $i = 2$ , the equation of flow is written as

$$AA_2 h_{1,j} + BB_2 h_{2,j} + CC_2 h_{3,j} = DD_2 \quad (19)$$

Solving for  $BB_2 h_{2,j}$  from equation 19 yields

$$BB_2 h_{2,j} = DD_2 - CC_2 h_{3,j} - AA_2 h_{1,j} \quad (20)$$

Substitution of equation 18 into 20 gives

$$BB_2 h_{2,j} = DD_2 - CC_2 h_{3,j} - AA_2 (G_1 - B_1 h_{2,j}) \quad (21)$$

Carrying out the multiplication, rearranging terms, and solving for  $h_{2,j}$  result in

$$h_{2,j} = (DD_2 - AA_2 G_1)/(BB_2 - AA_2 B_1) - [CC_2/(BB_2 - AA_2 B_1)] h_{3,j} \quad (22)$$

The known parameters of equation 22 are defined as

$$G_2 = (DD_2 - AA_2 G_1)/(BB_2 - AA_2 B_1) \quad (23)$$

and

$$B_2 = CC_2/(BB_2 - AA_2 B_1) \quad (24)$$

Substitution of equations 23 and 24 into 22 gives

$$h_{2,j} = G_2 - B_2 h_{3,j} \quad (25)$$

Again the head of interest  $h_{2,j}$  is a function of the known parameters  $G_2$  and  $B_2$  and the head at only the node  $h_{i+1,j}$  or  $h_{3,j}$ .

Proceeding to the next  $j$ th row node where  $i = 3$ , the flow equation can be written as

$$AA_3 h_{2,j} + BB_3 h_{3,j} + CC_3 h_{4,j} = DD_3 \quad (26)$$

Solving for the terms  $BB_3 h_{3,j}$  yields

$$BB_3 h_{3,j} = DD_3 - CC_3 h_{4,j} - AA_3 h_{2,j} \quad (27)$$

Substitution of equation 25 into equation 27 gives

$$BB h_{3,j} = DD_3 - CC_3 h_{4,j} - AA_3(G_2 - B_2 h_{3,j}) \quad (28)$$

Carrying out the multiplication, rearranging terms, and solving for  $h_{3,j}$  result in

$$h_{3,j} = (DD_3 - AA_3 G_2)/(BB_3 - AA_3 B_2) - [CC_3/(BB_3 - AA_3 B_2)] h_{4,j} \quad (29)$$

The known parameters of equation 29 are defined as

$$G_3 = (DD_3 - AA_3 G_2)/(BB_3 - AA_3 B_2) \quad (30)$$

$$B_3 = CC_3/(BB_3 - AA_3 B_2) \quad (31)$$

Substitution of equations 30 and 31 into equation 29 yields

$$h_{3,j} = G_3 - B_3 h_{4,j} \quad (32)$$

Finally, the flow equation is written for the last  $j$ th row node as

$$AA_4 h_{3,j} + BB_4 h_{4,j} + CC_4 h_{5,j} = DD_4 \quad (33)$$

Since there is no node at  $h_{5,j}$  the term  $CC_4$  is set equal to zero and equation 33 becomes

$$AA_4 h_{3,j} + BB_4 h_{4,j} = DD_4 \quad (34)$$

Rearranging terms of equation 34 gives

$$BB_4 h_{4,j} = DD_4 - AA_4 h_{3,j} \quad (35)$$

Substitution of equation 32 into equation 35 gives

$$BB_4 h_{4,j} = DD_4 - AA_4 (G_3 - B_3 h_{4,j}) \quad (36)$$

Carrying out the multiplication, rearranging terms, and solving for  $h_{4,j}$  result in

$$h_{4,j} = (DD_4 - AA_4 G_3)/(BB_4 - AA_4 B_3) \quad (37)$$

The head  $h_{4,j}$  is a function of only known terms, defined as

$$G_4 = (DD_4 - AA_4 G_3)/(BB_4 - AA_4 B_3) \quad (38)$$

Substitution of equation 38 into equation 37 gives

$$h_{4,j} = G_4 \quad (39)$$

Since the head  $h_{4,j}$  is now known, substitution of its value back into equation 32 allows calculation of the head at node  $h_{3,j}$ . The head  $h_{3,j}$  is then substituted into equation 25 allowing calculation of  $h_{2,j}$ . Finally  $h_{2,j}$  is substituted into equation 18 giving the value  $h_{1,j}$ . Thus all heads in the  $j$ th row have been determined.

A study of the G and B terms given in equations 16, 17, 23, 24, 30, 31, and 38 reveals their general form as

$$\mathbf{G}_N = (\mathbf{DD}_N - \mathbf{AA}_N \mathbf{G}_{N-1}) / (\mathbf{BB}_N - \mathbf{AA}_N \mathbf{B}_{N-1}) \quad (40)$$

and

$$\mathbf{B}_N = \mathbf{CC}_N / (\mathbf{BB}_N - \mathbf{AA}_N \mathbf{B}_{N-1}) \quad (41)$$

where  $N = i$  for row calculations and, by inference,  $N = j$  for column calculations. In addition,  $\mathbf{AA}_N$  is set equal to zero for the first node of a row or column, and  $\mathbf{CC}_N$  is set equal to zero for the last node of the row or column.

Sets of equations of the form given by equations 40 and 41 for the nodes along a row or column are what Peaceman and Rachford term the G and B arrays. The use of the G and B arrays greatly reduces both the needed computer core storage and execution time.

Examination of equations 18, 25, 32, and 39 reveals the general form for *row* calculations as

$$h_{i,j} = G_i - B_i h_{i+1,j} \quad (42)$$

Similarly, the general form of the head equations for *column* calculations is

$$h_{i,j} = G_j - B_j h_{i,j+1} \quad (43)$$

In summary, the process of calculating heads along columns or rows in the digital model includes first computing the values of G and B (equations 40 and 41) for the nodes of a column or row in order of increasing  $j$  or  $i$ , respectively. In the course of accomplishing this, the head at the last node of the column or row is found. Equations 42 and 43 are then used to solve for all other heads in the column or row in order of decreasing  $j$  or  $i$ , respectively. After completing the calculation of heads in an individual column or row, the computer proceeds to the next column or row until all have been processed satisfactorily.

## Part 2. Basic Aquifer Simulation Program

The basic aquifer simulation program listing given in figure 5 was coded in FORTRAN IV to solve the sets of column and row node equations 11 and 12, respectively. The program listing of figure 5 is a modification of that given by Prickett and Lonquist (1968).

Figure 6 illustrates the parameters included in the basic program. Briefly, the basic aquifer simulation program is intended for use when analyzing cause and effect relationships involving drawdowns or heads in a nonsteady-state, nonhomogeneous and/or homogeneous, isotropic aquifer system under nonleaky artesian conditions. Under these conditions it is possible to include special boundaries such as irregular barrier or recharge boundaries (constant head) and constant withdrawal or recharge rates.

The program listing of figure 5 is written in such a way that it will operate with any consistent set of units. However, the comment cards preceding the actual program list the variables in the gallon-day-foot system of units. This system of units is made consistent (exterior to the program) by working with a so-called storage factor defined as

$$SF_{1,i,j} = 7.48S\Delta x\Delta y \quad (44)$$

where

$SF_{1,i,j}$  = storage factor for node located at model coordinates  $i,j$ , in gallons per foot (gal/ft)

$S$  = the aquifer storage coefficient for artesian conditions, a fraction

7.48 = number of gallons in a cubic foot of water, in gallons per cubic foot (gal/ft<sup>3</sup>)

$y, x$  = finite difference grid intervals, in feet

The vector volume of the storage factor is the same as was defined in figure 3. The storage factor of equation 44 is substituted for the terms  $Sx^2$  in equations 11 and 12, which transforms these equations so that all combinations of terms have consistent units of gallons per day (gpd).

### Job Setup

As the first step in the computer job setup, the aquifer system properties are discretized by superimposing a square-mesh finite difference grid over maps of the aquifer properties as illustrated in figure 7. The total dimensions of the grid are defined by NC, the number of columns of the model, and by NR, the number of rows in the model. Next, the parameter card and default value card are prepared according to the formats illustrated in figure 8a and b; terms on these cards are defined in figure 5 (for ERROR, see page 14). The parameter and default value cards provide data for simulating a NC by NR aquifer system having homogeneous-isotropic properties with identical initial heads and net withdrawal rates at all nodes.

Each vector volume of the discrete portions of the aquifer

associated with a numbered node of the grid is outlined (see examples in figure 7) and assigned an average value of transmissivity, storage factor, initial head, and net withdrawal rate. A node card deck is then prepared according to the node card format illustrated in figure 8c. The node deck contains one card for each node that has any aquifer system properties differing from those defined on the default value card. If a node card is included, all values must be punched on it even if some of the values are equal to the default values. The boundary of the aquifer is approximated in a stepwise fashion as shown in figure 7, and zero transmissivities are assigned to nodes outside that boundary. (Do not assign zero storage factors outside of the aquifer boundary since this will cause a divide check error in the program.)

The basic aquifer simulation program deck, parameter card, default value card, and node deck are assembled in the order illustrated in figure 9. Appropriate computer installation job control cards are included, and the program is ready to run. The computer output will be in the form of printed numerical values of heads for all nodes at the end of every time increment according to FORMAT statements 310 and 330 of the basic program shown in figure 5.

### Program Operational Sequences

The step-by-step operation of the basic aquifer simulation program is explained in the following discussion according to the listing of figure 5 and the flow chart given in figure 10.

The basic program has three main sections. First, statements with ID numbers 0034 through 0071 (see figure 5) prepare the computer for the problem and handle the data input. The second section includes the actual simulation, ID numbers 0073 through 0178. The last section, with ID statements 0180 through 0187, controls printing of the results.

#### Computer Preparation and Data Input

In the first main section the dimension statement (ID 0034-0036) reserves core storage sufficient for models that contain not more than 50 columns and 50 rows. The first subscript limits the number of columns and the second limits the number of rows. These limits need not be the same and can be changed according to available computer storage or individual problems. The G and B arrays are used in both the column and row calculations and must be dimensioned to the maximum column and row limits. The array  $DL_{i,j}$ , to be explained in the second main section, is dimensioned to match the limits on the number of columns and rows.

The IBM 360 system subroutine ERRSET (ID number 0040) is called to avoid termination of the program because of

	ID		ID
	NUMBER		NUMBER
C	ILLINOIS STATE WATER SURVEY	C	REFINE ESTIMATES OF HEADS BY IADI METHOD
C	BASIC AQUIFER SIMULATION PROGRAM	C	0094
C		C	0095
C		C	0096
C		80	0097
C	DEFINITION OF VARIABLES	C	0098
C		C	0099
C	HO(I,J)-----HEADS AT START OF TIME	C	0100
C	INCREMENT (I,J)	C	0101
C	H(I,J)-----HEADS AT END OF TIME	C	0102
C	INCREMENT (FT)	C	0103
C	SF1(I,J)-----STORAGE FACTOR FOR	C	0104
C	ARTESIAN CONDITIONS	C	0105
C	(GAL/FT)	C	0106
C	Q(I,J)-----CONSTANT WITHDRAWAL	C	0107
C	RATES (GPD)	C	0108
C	T(I,J,1)-----AQUIFER TRANSMISSIVITY	C	0109
C	BETWEEN I,J AND I,J+1	C	0110
C	(GAL/DAY/FT)	C	0111
C	T(I,J,2)-----AQUIFER TRANSMISSIVITY	C	0112
C	BETWEEN I,J AND I+1,J	C	0113
C	(GAL/DAY/FT)	90	0114
C	AA,BB,CC,DD-----COEFFICIENTS IN WATER	C	0115
C	BALANCE EQUATIONS	100	0116
C	NR-----NO. OF ROWS IN MODEL	110	0117
C	NC-----NO. OF COLUMNS IN MODEL	C	0118
C	NSTEPS-----NO. OF TIME INCREMENTS	120	0119
C	DELTA-----TIME INCREMENTS (DAYS)	130	0120
C	HH,SI,QQ,TT-----DEFAULT VALUES	C	0121
C	I-----MODEL COLUMN NUMBER	140	0122
C	J-----MODEL ROW NUMBER	150	0123
C		160	0124
C		170	0125
C		C	0126
C		C	0127
C		C	0128
C		C	0129
C		C	0130
C		C	0131
C		C	0132
C		C	0133
C		180	0134
C		C	0135
C		C	0136
C		C	0137
C		C	0138
C		190	0139
C		C	0140
C		C	0141
C		C	0142
C		C	0143
C		C	0144
C		C	0145
C		C	0146
C		C	0147
C		C	0148
C		C	0149
C		C	0150
C		200	0151
C		C	0152
C		C	0153
C		210	0154
C		C	0155
C		220	0156
C		C	0157
C		230	0158
C		240	0159
C		250	0160
C		260	0161
C		C	0162
C		270	0163
C		C	0164
C		280	0165
C		C	0166
C		C	0167
C		C	0168
C		290	0169
C		C	0170
C		C	0171
C		C	0172
C		C	0173
C		C	0174
C		C	0175
C		C	0176
C		300	0177
C		C	0178
C		C	0179
C		C	0180
C		C	0181
C		310	0182
C		C	0183
C		C	0184
C		C	0185
60		320	0186
70		330	0187
C		C	0188
C		C	0189
C		C	0190

Figure 5. Basic aquifer simulation program listing

possible underflows. (The subroutine ERRSET may be called by a different name on other computers.) Drawdowns, on

occasion, are extremely small at great distances from centers of pumping. If drawdowns are small enough to cause an underflow, the subroutine ERRSET automatically sets these small numbers equal to zero and allows the processing to continue.

The input-output storage unit numbers are defined only once in the program, in statements of ID 0044-0046. This allows for their use in all READ and WRITE statements of the program.

Statements of ID 0048-0071 handle the data input to the computer. The storage arrays are filled with the default values which create a NR by NC aquifer model of uniform properties, each node having identical initial heads and net withdrawal rates. This feature avoids the inconvenience of individually specifying node-related properties when they are the same for large portions of the model. The second READ statement (ID 0068) offers the opportunity to replace any of the default values on a node-by-node basis.

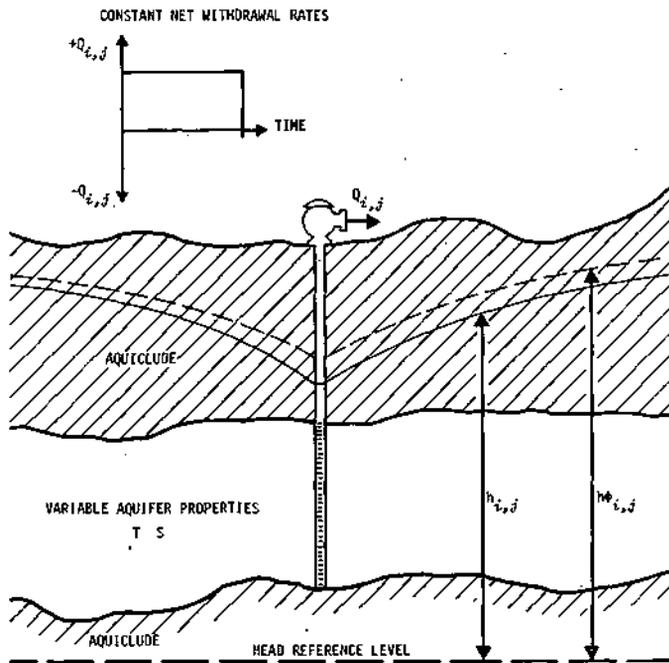


Figure 6. Generalized aquifer cross section for basic aquifer simulation program parameters

### Simulation

The statement ID 0075 sets TIME equal to zero at the start of the simulation, and ID 0077 updates TIME as the simulation progresses. TIME is used only for labeling the printed output and is not used in the simulation that follows. ID 0076

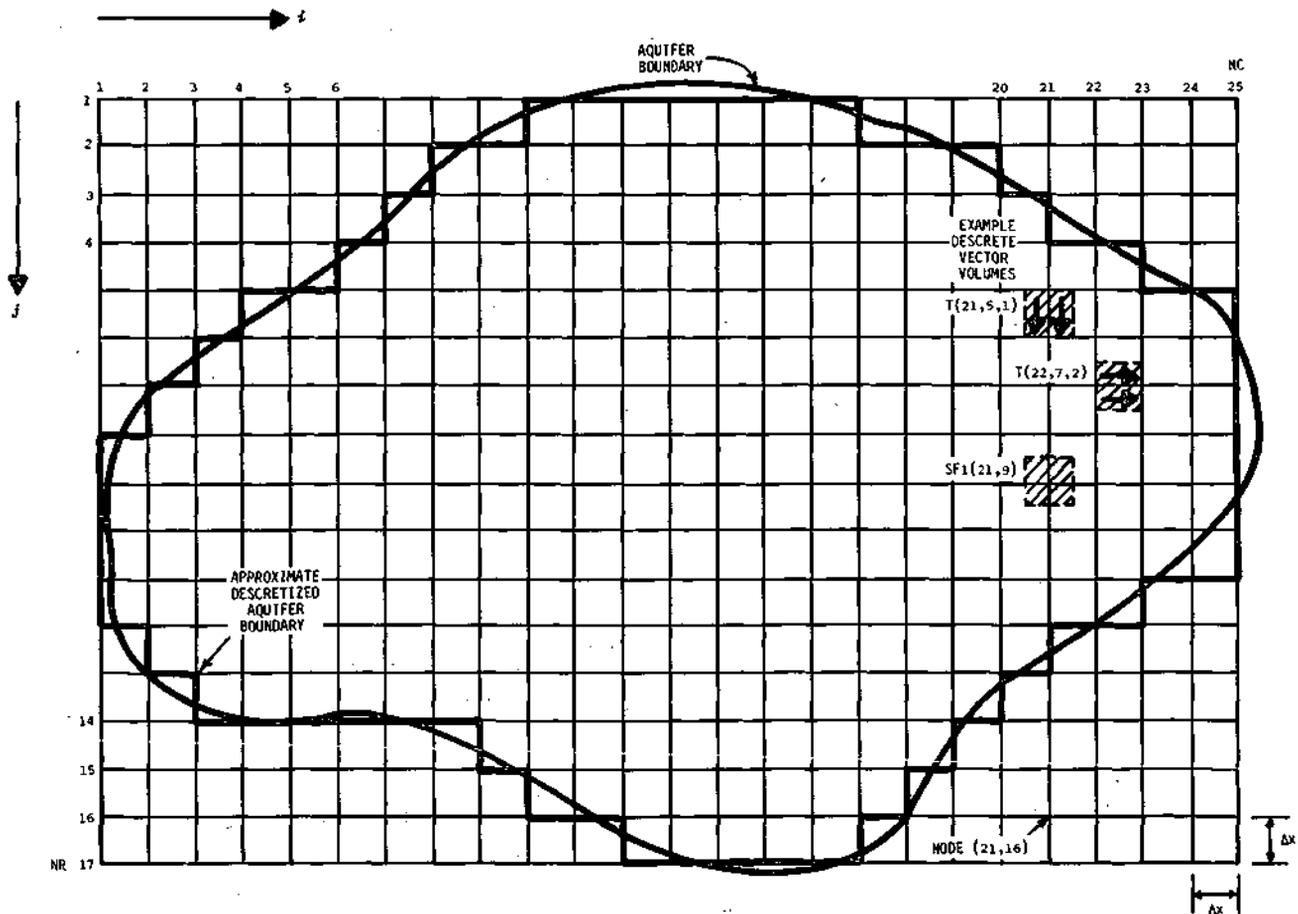


Figure 7. Plan view of finite difference grid over map of aquifer system

RIGHT JUSTIFY ALL DATA WITHIN THEIR FIELDS

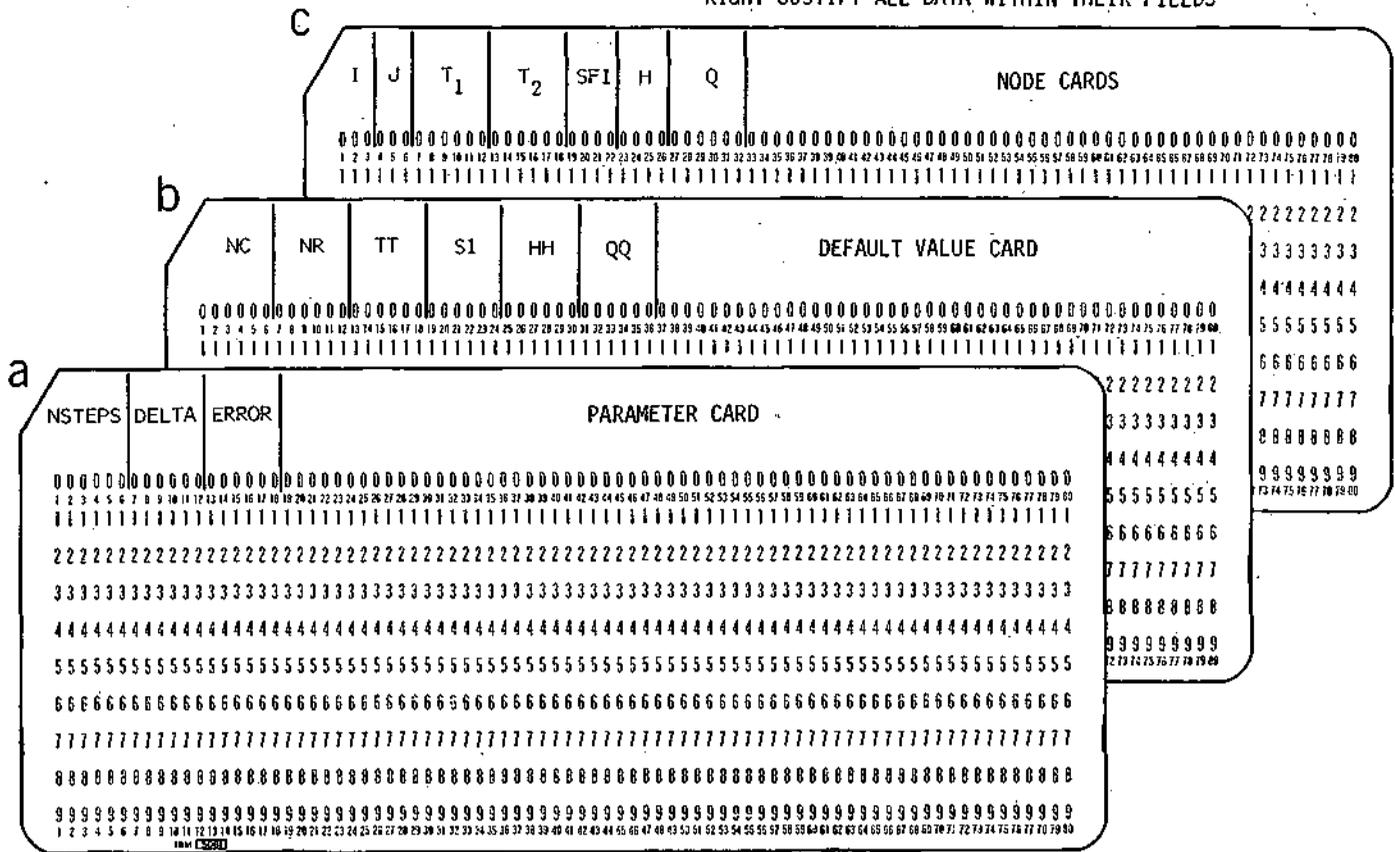


Figure 8. Input data formats for basic aquifer simulation program

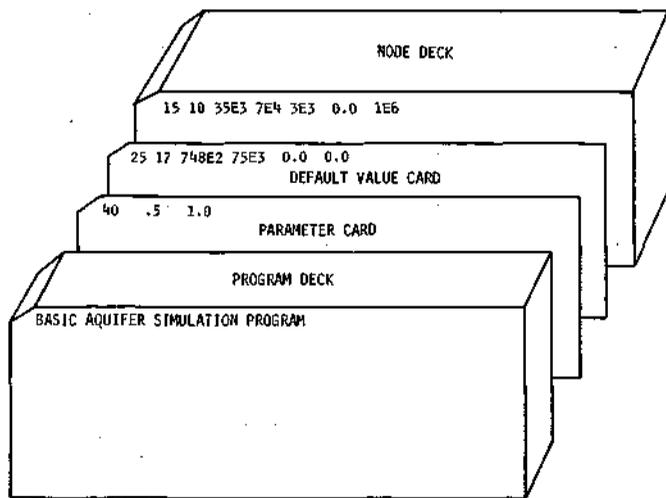


Figure 9. Typical job setup for basic aquifer simulation program

is a DO loop that causes all of the remaining instructions in the program to be repeated for each time step. The remaining portion of the simulation is divided into three subsections.

**Preliminary Head Predictor.** The first subsection is titled PREDICT HEADS FOR NEXT TIME INCREMENT and includes ID numbers 0082-0092. In essence, this subsection provides the computer with the capability of predicting the model response in the future on the basis of the response of the model

during the previous time steps. The inclusion of this predictor section significantly increases the convergence rate of the solution. Figure 11 illustrates the basic components of the predictor. The assumption is made that the ratio of present to past differences in head,  $F$ , will equal the ratio of the future to present differences in head. In equation form, the prediction relationship is

$$F = D/DL_{i,j} = \text{predicted future difference}/D \quad (45)$$

where the difference terms  $D$  and  $DL_{i,j}$  are defined as illustrated in figure 11.

When  $F$  is undefined because  $DL_{i,j}$  is zero, as in the first two time steps,  $F$  is set equal to 1.0. The additional IF statements (ID 0087-0090) insure that  $DL_{i,j}$  is non-zero (as in the second time step) to prevent a zero divide error and to restrict  $F$  to a reasonable size. The predictor, as defined by equation 45, is made fully operational for the third and all subsequent time steps.

The statement ID 0085 places the heads  $h_{i,j}$  calculated from the last time increment in  $h\phi_{i,j}$ , in preparation for the next time increment. ID 0096 and 0098 are iteration counter statements.

**Column Calculations.** The second subsection of the simulation is set up to solve the column equations of the form given as equation 11.

Statements of ID 0102-0105 govern the order in which the column equations are processed. During odd-numbered itera-

tions the columns are processed in order of increasing column number; during even-numbered iterations the columns are processed in order of decreasing column numbers. This type of processing is much more efficient than continually processing columns in the same order.

Statements of ID 0107-0127 form the G and B arrays one column at a time. The terms AA, BB, CC, and DD of the program are the same as those defined in equations 11a through 11d. After the calculations of the G and B arrays, the heads of the column are calculated in order of the row numbers. The absolute value of any change made in head

is added to the term E. The term E maintains a summation of all such head-change values. After all columns have been processed in the above manner, the program proceeds to calculating heads at the nodes by rows.

*Row Calculations.* The row calculations form the third subsection of the simulation which includes ID numbers 0141-0178.

The procedures for calculating heads along rows are the same as those for the columns, except the subscripts are changed to fit the row equations 12a through 12d. The G and B arrays for the rows are calculated, the heads are then calculated, and the absolute value of any changes in head that have occurred since the column calculations is further summed in E.

At the end of the row calculations a comparison is made between E and ERROR (see ID 0178). If E is greater than ERROR (a chosen error tolerance read from the parameter card), the program assumes that the solution has not converged and returns control to ID 0097 to start another iteration. When the error tolerance is met, the solution results for that time step are ready for printing.

*Printout of Results*

The third main section of the basic aquifer simulation program handles printing of the heads for each time step (see ID numbers 0180-0187). All heads are printed for the model after every time step. Between the WRITE statements of the PRINT RESULTS section is the instruction DELTA = DELTA \* 1.2, which indicates that the time increment DELTA is made to increase in size as the simulation progresses. The reasons for this, to be explained later, are related to increased accuracy during the early part of the simulation and increased efficiency as time progresses.

After printing the heads at the end of the time increment, control is transferred to ID number 0076 to begin the computations for the next time increment.

**Illustrative Examples**

Several comparisons can be made between known theoretical solutions and computer-simulated solutions to gain an understanding of how the basic program operates. These comparisons will serve as examples of the use of the basic aquifer simulation program and will show its sensitivity to parameter changes. The first comparisons to be illustrated were all accomplished with uniform grid spacing, homogeneous-isotropic aquifer system properties, initial heads set at the zero reference level, and equal time increments. Equal time increments were simulated by removing the instruction DELTA = DELTA \* 1.2 in ID 0184 in the listing of figure 5. Other comparisons will be made to show the use of various boundary conditions, nonuniform time increments, and nonuniform grid spacings.

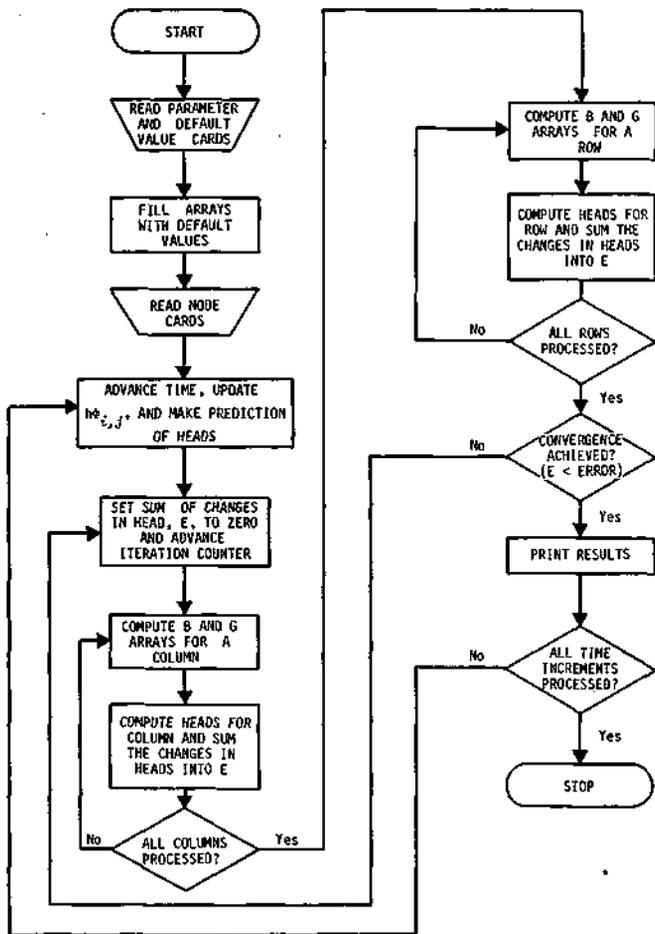


Figure 10. Flow chart for basic aquifer simulation program

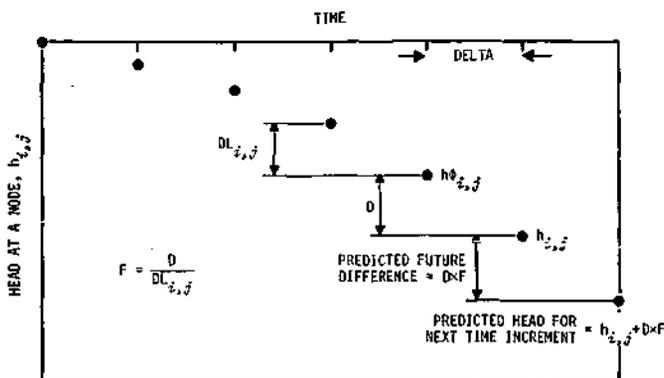


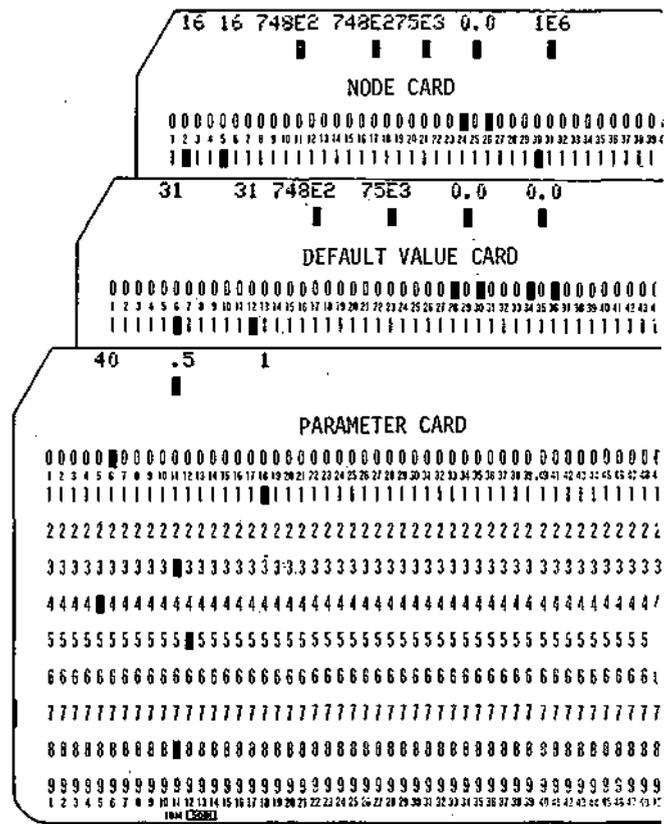
Figure 11. Definition of predictor difference terms

*Infinite Aquifer Conditions*

Theis (1935), who introduced the nonequilibrium formula for nonleaky artesian conditions, provides the theoretical solution comparison for the first computer example. Theis' formula was partly based on the assumptions that the aquifer is infinite in areal extent, is of the same thickness throughout, and is confined by impermeable beds (aquicludes) through which no leakage can take place.

An effectively infinite aquifer was computer-simulated by making the number of model rows and columns sufficiently large that the heads at points of interest were not affected by the presence of the boundary edges of the model.

The input to the computer consisted of the data on the parameter and default value cards and on one node card (which represents the addition of a well pumping at a constant rate located in the center of the model), as illustrated in figure 12. The computer output was in the form of negative numbers representing drawdowns, or heads, below the assigned zero head reference level.



(See figure 8 for card formats)

**AQUIFER VALUES**

$T = 74,800$  gpd/ft  
 $S = 1.003 \times 10^{-2}$   
 $SF_1 = 75,000$  gal/ft  
 $HH = 0.0$  ft  
 $QQ = 0.0$  gpd  
 Grid interval = 1000 ft  
 $NR = 31$   
 $NC = 31$   
 $DELTA = 0.5$  days  
 $NSTEPS = 40$   
 $Q(16,16) = 10^6$  gpd

Figure 12. Sample computer input data for Theis conditions

Uniform Time Increments, DELTA. A time-drawdown graph in figure 13 illustrates the sensitivity of the computer-simulated drawdowns for Theis conditions as a result of varying the size of the discrete time increment DELTA. Figure 14 shows a similar comparison but at a point more distant from the pumped well. It should be noted that the computer solutions required about six time steps before the drawdowns fell on the theoretical curves, irrespective of the DELTA value. However, as the time increment (DELTA) was reduced, the computer solution merged with the theoretical curve at earlier times, and the magnitude of the difference between theoretical and computer solutions became smaller. Thus the accuracy becomes better by decreasing DELTA.

Also, it should be noted that the computer-simulated cone of depression is slightly distorted during the early portions of pumping. In the area near the pumped well the drawdowns are less than theory, and in areas distant from the pumped well the drawdowns are in excess of theory. This phenomenon is mainly due to the effects of discretizing time. Errors due to discretizing time become critical when water levels fluctuate rapidly as when a pump is first turned on. Several methods are available for reducing errors of this

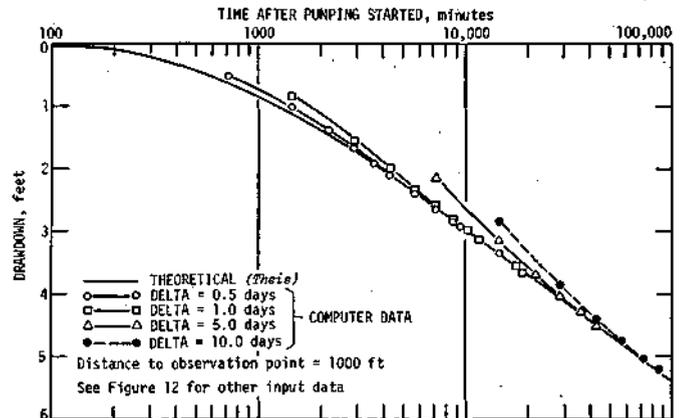


Figure 13. Comparison of theoretical and digital computer solutions near a pumped well with DELTA as a variable

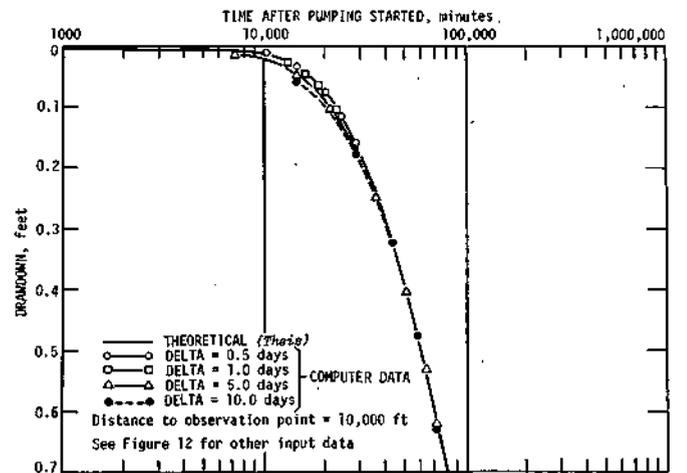


Figure 14. Comparison of theoretical and digital computer solutions distant from a pumped well with DELTA as a variable

type; however, decreasing the size of the time increment DELTA is the presently preferred method.

A distance-drawdown graph for the computer input data given in figure 12 is shown in figure 15. A comparison between theory and computed drawdowns indicates excellent agreement after 6 days of pumping.

A general rule for choosing an initial DELTA value, based on the comparisons given above, is to decide at what minimum TIME drawdowns or heads of interest occur and then precede this time by at least six time increments. For instance, if it is desired to have drawdowns or heads for periods of time in excess of 6 years, then an initial DELTA of about 1 year should be chosen.

On a practical basis, there is a lower limit to the choice of an initial DELTA. Discretizing the aquifer storage properties into vector volumes and lumping those properties at the finite difference grid nodes is only an approximation of the actual continuously distributed system. Choosing an initial DELTA so small that the cone of depression does not go significantly beyond the pumping node of the model would be inconsistent with the assumptions made in finite difference approximations.

*Convergence Test, ERROR.* All computer programs for aquifer evaluations should include an internal check on the errors that are inherently present in solving finite difference equations. Many types of error checks are possible, depending on the problem under study, and these could fall into four main categories.

The first type might be to keep a running check on the water balance of the system as the program operates and to insure that balance is maintained. A second type is to set an upper limit for the maximum change in water level between iterations at any node. A third type of check might entail substituting the drawdowns or heads back into the finite difference equations and evaluating the error that may exist. The fourth type of error check, which is the type included in the basic aquifer simulation program, takes into account the total system by controlling the sum

of the changes in heads during iterations over the entire model. This type works well with aquifer problems concerning regional analyses. In any particular situation, it is possible that more than one of the above error checks might be included.

The error check in the basic aquifer simulation program assures that computed heads or drawdowns have converged to acceptable answers within a specified tolerance. The specified tolerance is predetermined and entered on the parameter card as the term ERROR. The term ERROR, in units of feet, represents the maximum allowable sum of the absolute values of the changes in head for all node points of the model during an iteration. If the heads or drawdowns have not changed more than the error tolerance during an iteration, then the solution has converged to acceptable accuracy.

Figure 16 shows how drawdowns vary with changes in the ERROR term. As the ERROR term is decreased, the drawdowns come closer to the theoretical curve. However, there is a point of diminishing returns wherein further reductions in ERROR will not yield significant improvements in the accuracy of the computed drawdowns. For instance, figure 16 shows great improvement between computer drawdowns when the ERROR term is reduced from 100 to 50, and continued similar improvement on a further reduction in the ERROR term to 5. However, very little difference is indicated by the change between 5 and 0.5.

The ERROR term should be set at the highest possible value without degrading the desired accuracy of the solution because large numbers of iterations may be needed to provide highly accurate results (a point covered later in this report). This, in turn, greatly increases the computer time needed for the desired solution.

Usually, a particular program is 'tested' by making a few computer runs with differing values of ERROR, and then an appropriate ERROR term is chosen at the point where reduction in that term does not significantly change the solution. The question immediately arises as to where to start in choosing a first value of ERROR. A 'rule of thumb' for

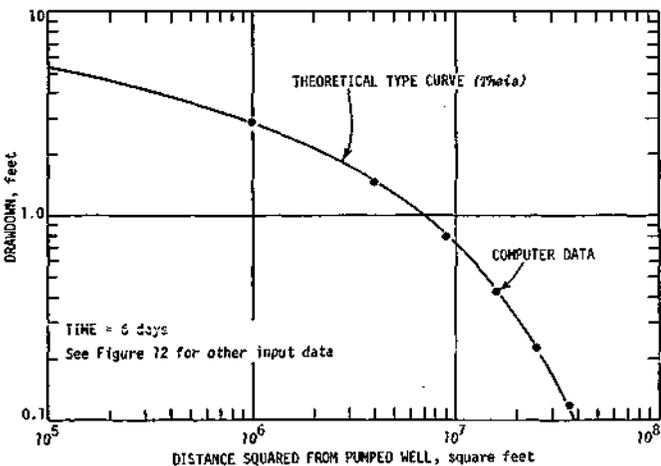


Figure 15. Distance-drawdown comparison of theoretical and digital computer solutions

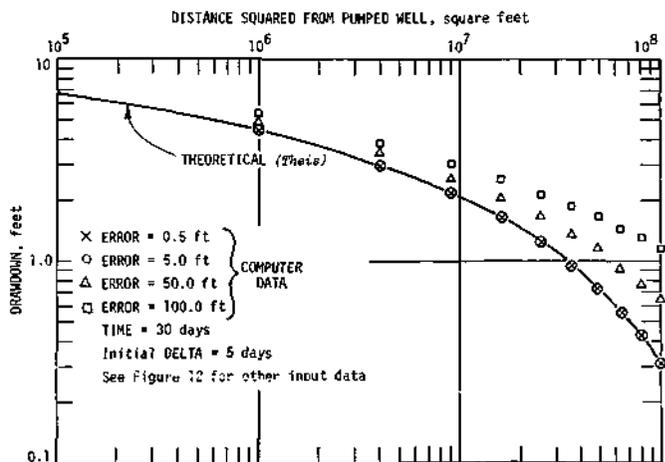


Figure 16. Distance-drawdown comparison of theoretical and digital computer solutions with ERROR as a variable

choosing the initial value of ERROR, which has been found useful for both uniform and nonuniform time increments, is given in the following empirical equation

$$\text{ERROR} = Q \times \text{DELTA}/10 \times \text{SF1}$$

where

- Q = total net withdrawal rate of model, in gpd
- DELTA = initial time increment, in days
- SF1 = average storage factor of model, in gal/ft

Applying the above equation to the problem illustrated in figure 16 yields an initial ERROR value of about 6.7 feet. That this would be a reasonable starting value is shown by the accuracy of the solution in figure 16 with ERROR set at 5.0 feet. It should be pointed out again that the above equation is a rule-of-thumb first approximation only, and the final choice of ERROR should be based on the response of the computer program as it applies to the particular problem under study.

*Program Iterations, ITER.* AS mentioned earlier, highly accurate answers require more computer work which may be in the form of an increase in the number of program iterations. Figure 17 shows the number of iterations required of the basic aquifer simulation program to meet the standards set by the ERROR term. As the ERROR term decreases, the number of iterations increases. However, as TIME increases, the number of iterations per time increment will eventually decrease regardless of the value of ERROR because the head predictor increases in its effectiveness.

As an example, for the curve with ERROR of 5.0 feet in figure 17, the first time increment required 5 iterations to converge to the desired solution. Then the number of iterations, per time increment, reduced to 3 at 10 days, reached a peak of 5 again at 20 days, and finally declined to 1 at a

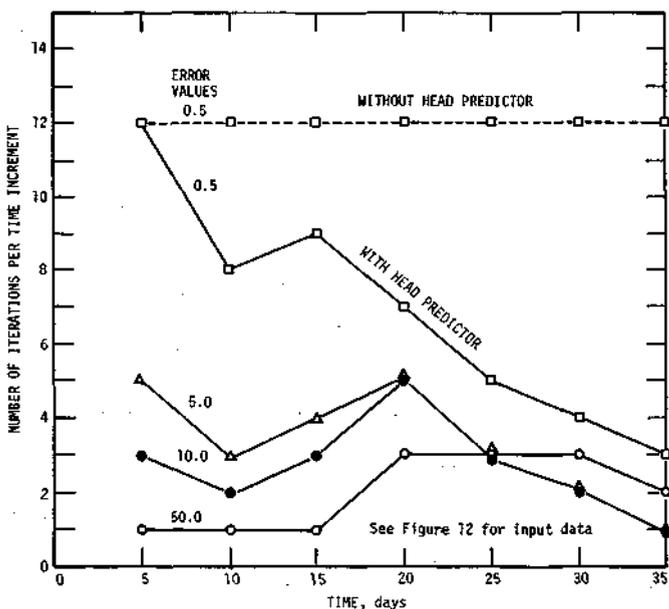


Figure 17. Number of program iterations versus time for various values of ERROR

TIME of 35 days. The peak noted at 20 days is the result of starting the full operation of the head predictor.

The worth of the predictor section in reducing the number of program iterations can be noted in figure 17 by comparing the curves with the ERROR value of 0.5 feet. The number of program iterations without the head predictor remains constant in time but, with the head predictor, reduces rapidly as time progresses.

*Nonuniform Time Increments, DELTA.* Although small time increments are needed for accuracy when water levels are fluctuating rapidly (as when a well first starts pumping), the small time increment becomes less and less important as time goes on because water level fluctuations slow in their rate of decline. Thus, as rapid fluctuations in water levels dissipate, it is desirable to attempt to use larger time increments because the total number of time steps (NSTEPS) and program iterations (ITER) can then be reduced, for a more efficient program. The most desirable scheme becomes one that uses small time increments for greater accuracy during times of rapidly fluctuating water levels, and uses progressively larger increments for efficiency (without loss of accuracy) when water levels move more slowly.

In most program listings of this report, rapidly fluctuating water levels are caused only by turning pumps on or off and by making changes in recharge rates at the start of a time step. These changes in pumpage and recharge rates will be made at specified times. Therefore rapidly changing water levels, of the type discussed above, occur at known times. Under these conditions, the desired accuracy and efficiency can be included in the basic aquifer simulation program by the instruction DELTA = DELTA\* 1.2, ID 0184 in the listing of figure 5. An initial small DELTA is chosen for the first time increment. Thereafter, DELTA is increased between time steps in a geometric progression by a multiplying factor of 1.2. The progression of TIME in the computer simulation is as follows.

ISTEP	TIME
1	DELTA
2	DELTA + DELTA × 1.2
3	DELTA + DELTA × 1.2 + (DELTA × 1.2) × 1.2
·	·
·	·
ISTEP	DELTA [1 + 1.2 <sup>1</sup> + ... + 1.2 <sup>(ISTEP-1)</sup> ]
·	·
·	·
NSTEPS	DELTA [1 + 1.2 <sup>1</sup> + 1.2 <sup>2</sup> + ... + 1.2 <sup>(NSTEPS-1)</sup> ]

where

ISTEP = time increment number

NSTEPS = total number of time increments in simulation

DELTA = initial time increment

An equation can be written giving TIME as a function of any ISTEP as

$$\text{TIME} = \text{DELTA} \times \quad (46)$$

where

$$\delta = 1 + 1.2^1 + 1.2^2 + \dots + 1.2^{(ISTEP-1)}$$

Values of the function versus ISTEP are given in table 1 and can be used to calculate TIME for any given initial DELTA.

A similar approach of using nonuniform time increments was given by Pinder and Bredehoeft (1968) and produced excellent results. However, it should be mentioned that the factor 1.2 is problem-dependent and may require adjustment to fit individual aquifer conditions.

Figure 18 shows a time-drawdown graph for the nonuniform time increment scheme. The number of time increments out to about 21 days (~ 30,000 minutes) was 9. This can be compared with the 21 uniform time incre-

Table 1. Values of the Function versus ISTEP

ISTEP	$\delta$
1	1.00
2	2.20
3	3.64
4	5.37
5	7.44
6	9.93
7	12.92
8	16.50
9	20.80
10	25.96
11	32.15
12	39.58
13	48.50
14	59.20
15	72.03
16	87.44
17	105.93
18	128.12
19	154.74
20	186.69
21	225.02
22	271.03
23	326.23
24	392.48
25	471.98
26	567.37
27	681.84
28	819.21
29	984.05
30	1181.87
31	1419.24
32	1704.08
33	2045.90
34	2456.08
35	2948.29
36	3538.95
37	4247.74
38	5098.28
39	6118.93
40	7343.71
41	8813.45
42	10577.14
43	12693.57
44	15233.28
45	18280.93
46	21938.11
47	26326.73
48	31593.07
49	37912.68
50	45496.21

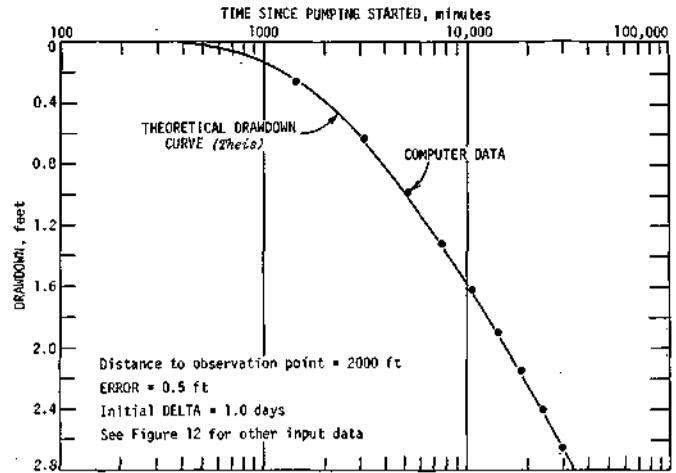


Figure 18. Comparison of theoretical and digital computer solutions using nonuniform time increments

ments of 1.0 day each needed to provide 21-day drawdowns.

Although the number of time steps has been reduced by using the nonuniform time increments, the number of program iterations must also reflect the desired efficiency as TIME progresses. Figure 19 illustrates how the program iterations vary with TIME as a function of using either the uniform or nonuniform time increments. Again, the worth of the predictor section is shown by the reduction in iterations from 23 to 5 at a TIME of about 59 days. With the predictor, the difference in efficiency between the uniform and nonuniform time increment schemes is less obvious. However, a graph was prepared (figure 20) showing the number of days simulated per iteration as TIME progresses. The data points of figure 20 have been computed from data taken

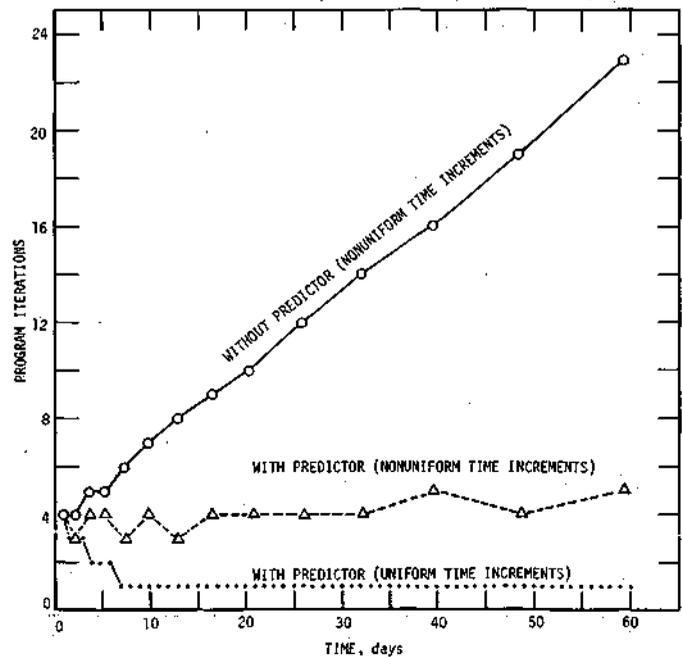


Figure 19. Program iterations for uniform versus nonuniform time increments

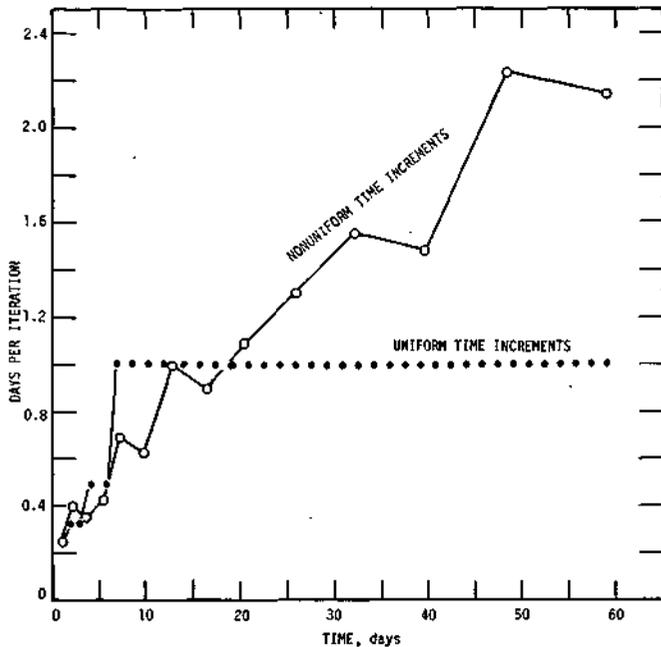


Figure 20. Number of days simulated per iteration versus time

from figure 19. As shown in figure 20, the nonuniform time increment scheme becomes more efficient as TIME progresses. For example, after about 20 days of pumping, the nonuniform time increment scheme shows marked increases in days simulated per iteration over the number for the uniform time increment scheme.

One undesirable feature of the nonuniform time increment scheme is that computed drawdowns or heads do not correspond to even days, but are printed at fractions of days. If DELTA is given an initial value of 1 day, the values of table 1 are representative of TIME values, in days, for which drawdowns or heads are computed. Except for the initial time step, none of the values are in even days.

The following procedure can be used to obtain drawdowns or heads at a specified TIME other than those listed in table 1. From an alternate form of equation 46, an initial DELTA can be computed such that the drawdowns or heads at a specified TIME will appear in the series of program printouts. The alternate form of equation 46 is

$$\text{DELTA} = \text{TIME}/\delta \quad (47)$$

If, for example, drawdowns or heads are needed at the end of 30 days of pumping, then TIME equals 30 in equation 47. Next, the value (as a function of ISTEP) which is just greater than the desired TIME is obtained from table 1. The values of TIME and  $\delta$  are then substituted into equation 47 resulting in

$$\text{DELTA} = 30/32.15 = 0.933 \text{ days}$$

The initial DELTA of 0.933 days is entered on the parameter card, and the drawdowns or heads at a TIME of 30 days will be printed after the 11th time step. The program then continues to operate until the total number of time increments (NSTEPS) is completed.

### Barrier Boundary Conditions

A barrier boundary is defined as a boundary across which there is no flow of water. This type of boundary can be formed in the program either by assigning zero transmissivities outside the boundary of interest or by assigning modified transmissivities and storage factors along the boundary according to the vector volume concept. However, storage factors should not be set equal to zero outside the boundary of interest because this would cause a zero divide check error, and the computer would terminate processing the program.

It should be realized that the edges of the grid, defined by NC and NR, also represent barrier boundaries located (by the vector volume concept) one half of a grid interval beyond the end node points.

Figure 21 illustrates a theoretical time-drawdown graph comparison with the digitally simulated barrier boundary. This simulation was made with homogeneous aquifer properties under artesian conditions, with nonuniform time increments. The theoretical barrier boundary type curve was obtained from a report by Stallman (1963).

### Recharge Boundary Conditions

A recharge boundary is defined as a boundary along which there is no drawdown or change in water level. This boundary is further assumed to fully penetrate the aquifer. This type of boundary is most easily handled in the computer simulation by setting the storage factors of the nodes along the position of the recharge boundary to extremely large values. A typical storage factor to produce a recharge boundary as described above is  $10^{21}$  gal/ft.

Figure 22 illustrates a comparison between a theoretical time-drawdown graph and one obtained by the digital computer for the case of a single recharge boundary. The theoretical recharge boundary type curve was obtained from Stallman (1963).

### Models with Variable Grid Spacings

Most often the computer simulations are accomplished with models of uniform grid spacing as illustrated in figure 7, but such spacings are not always necessary nor desirable. For example, uniform grid spacings may waste computer core storage and consume excessive amounts of computer execution time when dealing with aquifers covering large areas. For this type of problem it may be possible to design a variable grid spacing model that allows a great reduction in both the needed core storage and execution time.

Variable grid spacings have been successfully used for several years in electric analog model designs, and numerous examples of these can be found in the literature (Liebman, 1954; MacNeal, 1953; Blair, 1966; and Karplus, 1958). Bittinger et al. (1967) suggested a method for nonuniform

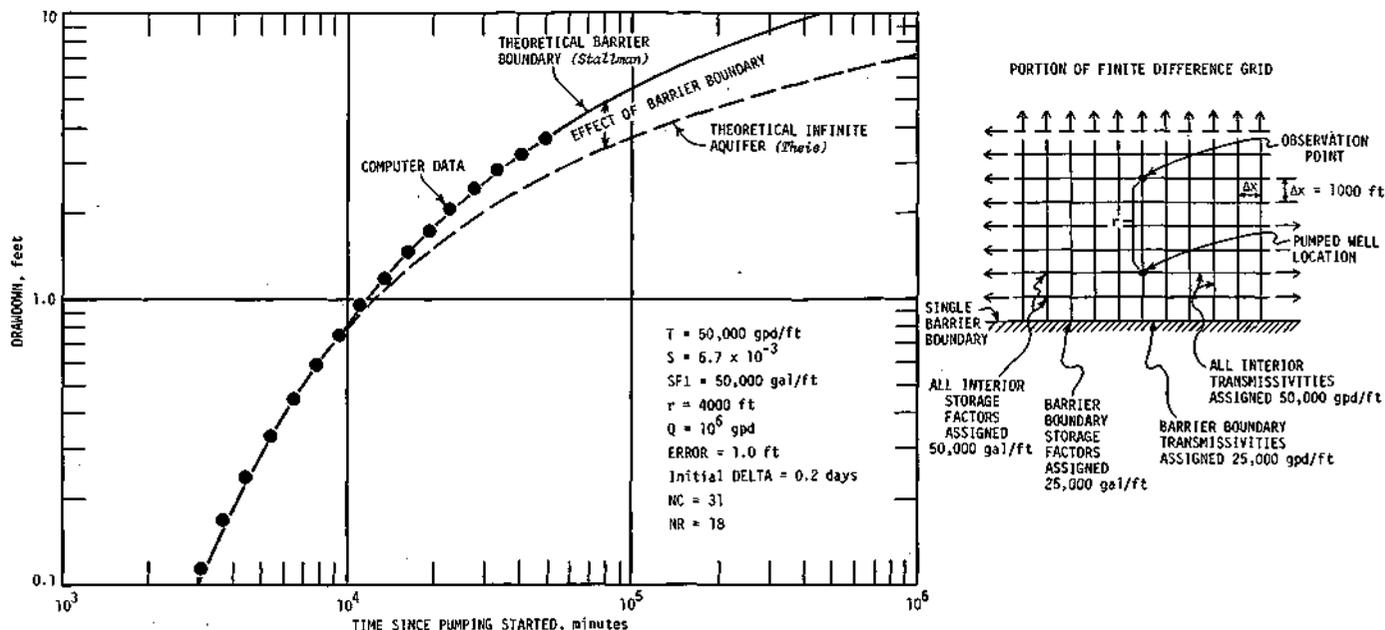


Figure 21. Comparison of theoretical and digital computer solutions for barrier boundary conditions

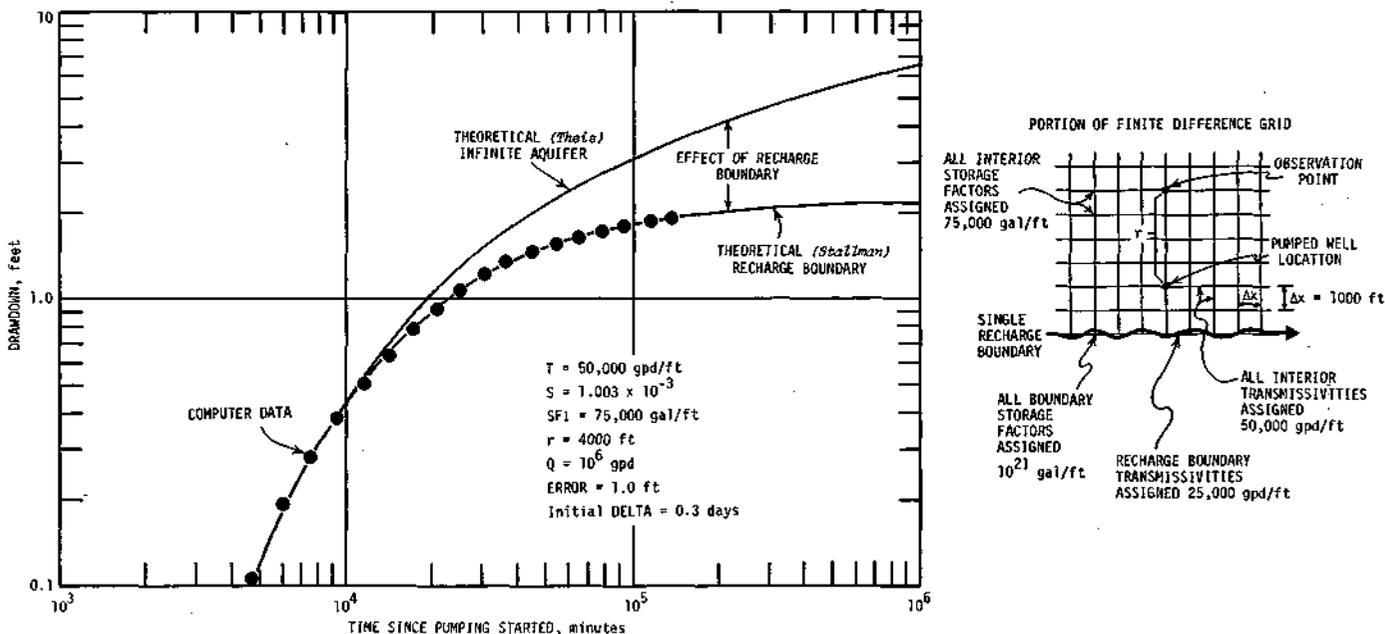


Figure 22. Comparison of theoretical and digital computer solutions for recharge boundary conditions

grid spacing design as shown in figure 23. Here, fine grid spacings were used in areas of concentrated pumpage where steep hydraulic gradients exist, and progressively coarser grid spacings were used in areas distant from pumpage where gradients are more uniform.

Actually, using variable grid spacings is somewhat like using variable time increments. When water levels are changing rapidly, small time increments are necessary for an accurate solution, but when water levels move slowly, larger time increments are possible with little loss in model accuracy. Similarly, when the hydraulic gradients, or the aquifer properties, or both, are changing rapidly in space, small grid spacings are necessary to give an accurate solution. On

the other hand, when gradients or aquifer properties change more smoothly in space, larger grids are possible without any great loss in the accuracy of the solution.

An example variable grid spacing model shown in figure 24 illustrates how a 61 by 61 uniform grid spacing model can be replaced with a 17 by 17 variable grid spacing model. The uniform grid model would represent 3721 node points, whereas the variable grid model has only 289 nodes or a saving of 3432 nodes. Figure 24 also illustrates how the vector volume concept is applied to the variable grid spacing model.

The variable grid spacing model of figure 24 was set up for Theis conditions with a single well pumping at a con-

stant rate at node coordinates  $i = 9$  and  $j = 9$ . Node cards were prepared for all nodes of the model. Figure 25 illustrates a distance-drawdown comparison of computer and theoretical solutions for the model shown in figure 24. The comparison indicates good agreement; however, the computer data falls slightly below the theoretical curve. Nonetheless, the accuracy of variable grid spacing model solution

is encouraging, and under certain conditions its use can be highly beneficial.

Some amount of experience is necessary in defining the magnitude of the variable sized grids, and this experience can be gained only by making a few computer runs with different grid configurations and observing the effects.

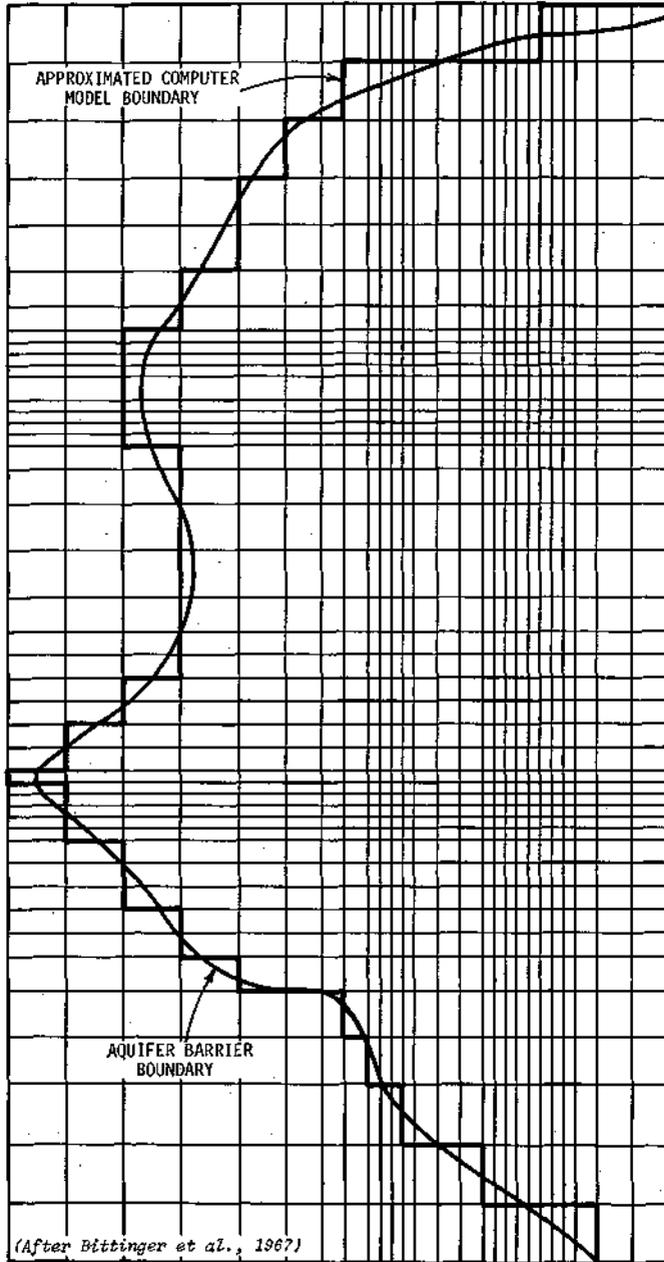


Figure 23. Finite difference grid using variable spacing

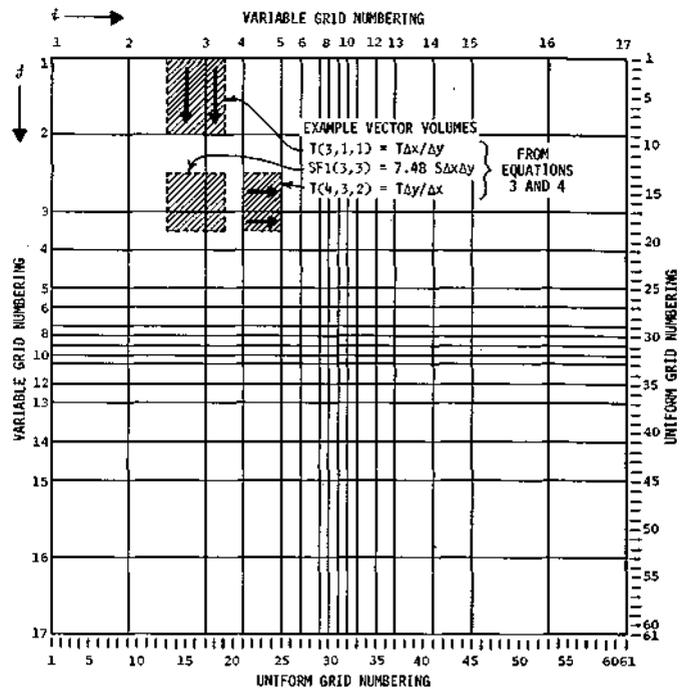


Figure 24. Example variable grid spacing model

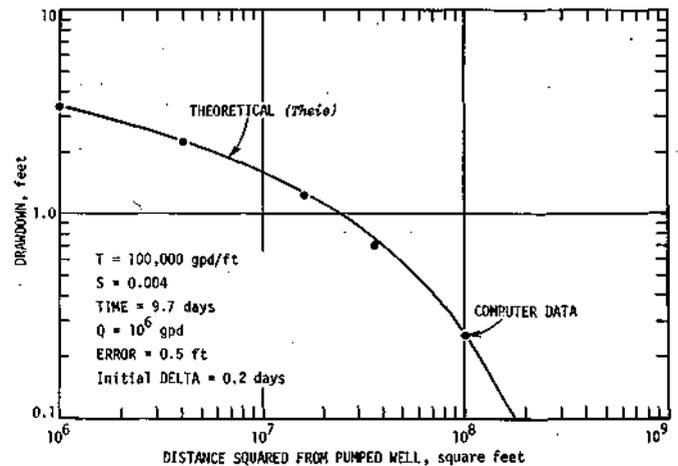


Figure 25. Distance-drawdown comparison of theoretical and computer solutions with data from variable grid spacing model

## Part 3. Modifications of Basic Aquifer Simulation Program

The basic aquifer simulation program of figure 5 forms the nucleus of all the program listings that follow. Various modifications can be made to the basic program that allow incorporating additional features such as specialized types of printouts and differing boundary conditions. The printout options are presented first, followed by separate program presentations for seven modifications and a combination of these. Examples of special problems are then given.

### VARIOUS PRINTOUT OPTIONS

#### Numerical Printout

Most program listings of this report, including the basic aquifer simulation program, incorporate WRITE statements necessary for printing heads or drawdowns in numerical form. The numerical output is used for most analyses. Figure 26a illustrates a typical printout of numerical drawdowns as a result of pumping a single well in the center of a test model containing 7 columns and 15 rows. The computer data cards for this problem illustrated in figure 26b are used as input to the basic aquifer simulation program.

The WRITE statement 320 and the FORMAT statement 330 of the basic program of figure 5 are programmed so that the drawdowns for models containing more than 10 columns are printed on indented lines.

#### Category Printout

Although numerical data are important in some analyses, other types of printout are occasionally desirable. Figure 27a shows a printout where the drawdowns have been put into categories representing differing ranges of heads or drawdowns. The category printout is programmed as illustrated in the listing of figure 28. The program of figure 28 is the same as the basic aquifer simulation program except that the printout programming is modified as indicated by the additional boxed-in instructions. The program modifications required adding two small arrays to the DIMENSION statement, including another READ statement for reading in the various categories, modifying the PRINT RESULTS section, and adding a category card to the input data.

The category card contains the information necessary to define the ranges of heads or drawdowns represented by the numbered printout. As illustrated on the category card in figure 27b, 11 categories ( $NT = 11$ ) are defined. The first category specifies that if the aquifer is missing (no heads computed) then the printout remains blank. The second category, represented on the printout as 0, indicates that the head at the node is between zero and  $-1$  foot. The third category, represented as a printed 1, indicates that the head at that node is between  $-1$  and  $-5$  feet. Finally,

Each presentation includes a description of the modification, job setup instructions, data sets for a sample problem with comparisons of computer and theoretical results, and the program listing. Changes in the listing have been marked, and duplications of statement numbers from one listing to another have been avoided so that each modification can be considered as a non-interfering separate module to be added to the basic program as needed.

the last category defined is a 9 which, if printed, indicates that the head at the node is between  $-150$  and  $-200$  feet. Water level heads are punched on the category card in order of highest to lowest. These categories can be changed from one problem to the next to fit the particular situation. The example category printout of figure 27a can be compared with the numerical printout of figure 26a for the identical problem.

It should be mentioned that the listing of figure 28 is also set up so that letters and symbols, as well as numbers, can be printed as the category representation.

#### Time-Water Level Graphs

Figure 29 is a typical graph showing the variation of water levels with time, at particular points in a aquifer. Figure 29a illustrates declining water levels versus ISTEP at a head scale interval (AL) of 5 feet per line. The plotted points, shown as asterisks, represent water levels within the range of the scaled ordinate. For instance, the plotted asterisks for ISTEP 25 in figure 29a indicates a head of  $-90.000$  feet; however, this should be interpreted as meaning the water level is at least  $-90.000$  but not greater than  $-95.000$  feet below the reference level. A table is also printed (figure 29b) to serve as a reference to the progress of TIME as a function of ISTEP.

Time-water level graphs can be obtained with the modified basic aquifer simulation program listing given in figure 30. In this case, the basic program was modified by adding four small arrays in the DIMENSION statement, by including a READ statement for reading in the necessary time-water level information, and by replacing the entire PRINT RESULTS section by the statements needed to form the time table and water level graphs.

Example input data cards for the time-water level curve printout are illustrated, in the proper order, in figure 31. A time-water level card is prepared in addition to the usual parameter, default value, and node cards. The number of nodes (NN) for which time-water level curves are desired, the scale factor (AL), and the pairs of  $i,j$  coordinates of the





```

C      ILLINOIS STATE WATER SURVEY
C      BASIC AQUIFER SIMULATION PROGRAM
C      WITH CATEGORY PRINTOUT OPTION
C
C      DEFINITION OF VARIABLES
C      HO(I,J)-----HEADS AT START OF TIME
C                   INCREMENT (I,J)
C      H(I,J)-----HEADS AT END OF TIME
C                   INCREMENT (FT)
C      SF1(I,J)----STORAGE FACTOR FOR
C                   ARTESIAN CONDITIONS
C                   (GAL/FT)
C      Q(I,J)-----CONSTANT WITHDRAWAL
C                   RATES (GPD)
C      T(I,J,1)----AQUIFER TRANSMISSIVITY
C                   BETWEEN I,J AND I,J+1
C                   (GAL/DAY/FT)
C      T(I,J,2)----AQUIFER TRANSMISSIVITY
C                   BETWEEN I,J AND I+1,J
C                   (GAL/DAY/FT)
C      AA,BB,CC,DD--COEFFICIENTS IN WATER
C                   BALANCE EQUATIONS
C      NR-----NO. OF ROWS IN MODEL
C      NC-----NO. OF COLUMNS IN MODEL
C      NSTEPS-----NO. OF TIME INCREMENTS
C      DELTA-----TIME INCREMENTS (DAYS)
C      HH,S1,QQ,TT--DEFAULT VALUES
C      I-----MODEL COLUMN NUMBER
C      J-----MODEL ROW NUMBER
C      NT-----NO. OF HEAD CATEGORIES
C      TAB-----HEAD VALUE OF CATEGORY
C
C
C      DIMENSION H(50,50),HO(50,50),
C      1SF1(50,50),Q(50,50),T(50,50,2),
C      2B(50),G(50),DL(50,50)
C      3,ROW(50),TAB(2,20)
C
C      TURN OFF UNDERFLOW TRAP
C
C      CALL ERRSET(208,256,-1,1)
C
C      DEFINE INPUT AND OUTPUT DEVICE NUMBERS
C
C      INTEGER OUT
C      IN=5
C      OUT=6
C
C      READ PARAMETER CARD AND
C      DEFAULT VALUE CARD
C
C      READ(IN,10)NSTEPS,DELTA,ERROR,
C      INC,NR,TT,S1,HH,QQ
C      FORMAT(I6,2F6.0,2I6,4F6.0)
C
C      READ CATEGORY PRINTOUT DATA
C
C      READ(IN,11)NT,((TAB(I,J),I=1,2),J=1,NT)
C      FORMAT(I2,13(A1,F5.0))
C
C      FILL ARRAYS WITH DEFAULT VALUES
C
C      DO 20 I=1,NC
C      DO 20 J=1,NR
C      T(I,J,1)=TT
C      T(I,J,2)=TT
C      SF1(I,J)=S1
C      H(I,J)=HH
C      HO(I,J)=HH
C      Q(I,J)=QQ
C
C      READ NODE CARDS
C
C      READ(IN,40)END=50,I1,J,T(I,J,1),
C      I2,I,J,2,SF1(I,J),H(I,J),Q(I,J)
C      FORMAT(I2I3,2F6.0,2F4.0,1F6.0)
C      GO TO 30
C
C      START OF SIMULATION
C
C      TIME=0
C      DO 320 ISTEP=1,NSTEPS
C      TIME=TIME+DELTA
C
C      PREDICT HEADS FOR NEXT
C      TIME INCREMENT
C
C      DO 70 I=1,NC
C      DO 70 J=1,NR
C      D=H(I,J)-HO(I,J)
C      HO(I,J)=H(I,J)
C      F=1.0
C      IF(DL(I,J).EQ.0.01)GO TO 60
C      IF(ISTEP.GT.2)F=D/DL(I,J)
C      IF(F.GT.5)F=5.0
C      IF(F.LT.0.01)F=0.0
C      DL(I,J)=D
C      H(I,J)=H(I,J)+D*F
C
C      REFINES ESTIMATES OF HEADS BY IADI METHOD
C
C      ITER=0
C      E=0.0
C      ITER=ITER+1
C
C      COLUMN CALCULATIONS
C
C      DO 190 I=1,NC
C      I=11
C      IF(MOD(ISTEP+ITER,2).EQ.1) I=NC-1+1
C      DO 170 J=1,NR
C
C      CALCULATE B AND G ARRAYS
C
C      DD=HO(I,J)*SF1(I,J)/DELTA-Q(I,J)
C      BB=SF1(I,J)/DELTA
C      AA=0.0
C      CC=0.0
C      IF(J-1)90,100,90
C      AA=-T(I,J-1,1)
C      BB=BB+T(I,J-1,1)
C      IF(J-NR)110,120,110
C      CC=-T(I,J,1)
C      BB=BB+T(I,J,1)
C      IF(I-1)130,140,130
C      BB=BB+T(I-1,J,2)
C      DD=DD+H(I-1,J)*T(I-1,J,2)
C      IF(I-NC)150,160,150
C      BB=BB+T(I,J,2)
C      DD=DD+H(I+1,J)*T(I,J,2)
C      W=BB-AA*B(I-1)
C      B(I)=CC/W
C      G(I)=(DD-AA*G(I-1))/W
C
C      RE-ESTIMATE HEADS
C
C      E=E+ABS(H(I,NR)-G(INR))
C      H(I,NR)=G(INR)
C      N=NR-1
C      HA=G(N)-B(N)*H(I,N+1)
C      E=E+ABS(HA-H(I,N))
C      H(I,N)=HA
C      N=N-1
C      IF(N)190,190,180
C      CONTINUE
C
C      ROW CALCULATIONS
C
C      DO 300 JJ=1,NR
C      J=JJ
C      IF(MOD(ISTEP+ITER,2).EQ.1) J=NR-J+1
C      DO 280 I=1,NC
C      BB=SF1(I,J)/DELTA
C      DD=HO(I,J)*SF1(I,J)/DELTA-Q(I,J)
C      AA=0.0
C      CC=0.0
C      IF(J-1)200,210,200
C      BB=BB+T(I,J-1,1)
C      DD=DD+H(I,J-1)*T(I,J-1,1)
C      IF(J-NR)220,230,220
C      DD=DD+H(I,J+1)*T(I,J,1)
C      BB=BB+T(I,J,1)
C      IF(I-1)240,250,240
C      BB=BB+T(I-1,J,2)
C      AA=-T(I-1,J,2)
C      IF(I-NC)260,270,260
C      BB=BB+T(I,J,2)
C      CC=-T(I,J,2)
C      W=BB-AA*B(I-1)
C      B(I)=CC/W
C      G(I)=(DD-AA*G(I-1))/W
C
C      RE-ESTIMATE HEADS
C
C      E=E+ABS(H(INC,J)-G(INC))
C      H(INC,J)=G(INC)
C      N=NC-1
C      HA=G(N)-B(N)*H(N+1,J)
C      E=E+ABS(HA-H(N,J))
C      H(N,J)=HA
C      N=N-1
C      IF(N)300,300,290
C      CONTINUE
C      IF(E.GT.ERROR) GO TO 80
C
C      PRINT RESULTS
C
C      WRITE(OUT,310)TIME,E,ITER
C      FORMAT(16M2TIME=F6.2///,E20.7,15)
C      DELTA=DELTA*1.2
C      DO 320 J=1,NR
C
C      DO 312 I=1,NC
C      DO 311 K=1,NT
C      IF(-H(I,J)+TAB(K))312,312,311
C      CONTINUE
C      ROW(I)=TAB(I,K)
C      WRITE(OUT,330)(ROW(K),K=1,NC)
C      FORMAT(2X,50A2)
C
C      STOP
C      END

```

Figure 28. Basic aquifer simulation program with category printout option

I, J COORDINATES  
OF NODE  
NODE( 4, 6)

a

b

HEAD, IN FEET	ISTEP					
	5	10	15	20	25	30
0.0	*****	1	1	1	1	1
-5.000	1	****	1	1	1	1
-10.000	1	1	***1	1	1	1
-15.000	1	1	**	1	1	1
-20.000	1	1	1 *	1	1	1
-25.000	1	1	1 *	1	1	1
-30.000	1	1	1 *1	1	1	1
-35.000	1	1	1 *	1	1	1
-40.000	1	1	1	1	1	1
-45.000	1	1	1	1*	1	1
-50.000	1	1	1	1 *	1	1
-55.000	1	1	1	1	1	1
-60.000	1	1	1	1 *	1	1
-65.000	1	1	1	1	1	1
-70.000	1	1	1	1	1	1
-75.000	1	1	1	1	*1	1
-80.000	1	1	1	1	1	1
-85.000	1	1	1	1	1	1
-90.000	1	1	1	1	*	1
-95.000	1	1	1	1	1	1
-100.000	1	1	1	1	1	1
-105.000	1	1	1	1	1	1
-110.000	1	1	1	1	1*	1
-115.000	1	1	1	1	1	1
-120.000	1	1	1	1	1	1
-125.000	1	1	1	1	1	1
-130.000	1	1	1	1	1 *	1
-135.000	1	1	1	1	1	1
-140.000	1	1	1	1	1	1
-145.000	1	1	1	1	1	1
-150.000	1	1	1	1	1	1
-155.000	1	1	1	1	1 *	1
-160.000	1	1	1	1	1	1
-165.000	1	1	1	1	1	1
-170.000	1	1	1	1	1	1
-175.000	1	1	1	1	1	1
-180.000	1	1	1	1	1	1
-185.000	1	1	1	1	1	1
-190.000	1	1	1	1	1	*1
-195.000	1	1	1	1	1	1
-200.000	1	1	1	1	1	1
-205.000	1	1	1	1	1	1
-210.000	1	1	1	1	1	1
-215.000	1	1	1	1	1	1
-220.000	1	1	1	1	1	1
-225.000	1	1	1	1	1	*

ISTEP	TIME(DAYS)
1	0.50
2	1.10
3	1.82
4	2.68
5	3.72
6	4.96
7	6.46
8	8.25
9	10.40
10	12.98
11	16.08
12	19.79
13	24.25
14	29.60
15	36.02
16	43.72
17	52.97
18	64.06
19	77.37
20	93.34
21	112.51
22	135.51
23	163.12
24	196.24
25	235.99
26	283.69
27	340.92
28	409.61
29	492.03
30	590.94

Figure 29. Time-water level graph for sample problem (a) and a table of ISTEP versus TIME (b)

```

C ILLINOIS STATE WATER SURVEY
C BASIC AQUIFER SIMULATION PROGRAM
C WITH TIME-WATER LEVEL PRINTOUT OPTION
C
C DEFINITION OF VARIABLES
C
C HD(I,J)-----HEADS AT START OF TIME
C INCREMENT (I,J)
C HI(I,J)-----HEADS AT END OF TIME
C INCREMENT (I,J)
C SF(I,J)-----STORAGE FACTOR FOR
C ARTESIAN CONDITIONS
C (GAL/FT)
C Q(I,J)-----CONSTANT WITHDRAWAL
C RATES (GPD)
C T(I,J,1)-----AQUIFER TRANSMISSIVITY
C BETWEEN I,J AND I,J+1
C (GAL/DAY/FT)
C T(I,J,2)-----AQUIFER TRANSMISSIVITY
C BETWEEN I,J AND I+1,J
C (GAL/DAY/FT)
C AA,BB,CC,DD-----COEFFICIENTS IN WATER
C BALANCE EQUATIONS
C NR-----NO. OF ROWS IN MODEL
C NC-----NO. OF COLUMNS IN MODEL
C NSTEPS-----NO. OF TIME INCREMENTS
C DELTA-----TIME INCREMENTS (DAYS)
C HH,SP,QQ,TT-----DEFAULT VALUES
C I-----MODEL COLUMN NUMBER
C J-----MODEL ROW NUMBER
C
C NN-----NO. OF NODES FOR WHICH
C TIME-WATER LEVEL CURVES
C ARE DESIRED
C NODE(I,J)-----I,J COORDINATES OF
C NODE WHERE T-WL CURVE
C IS DESIRED
C AL-----SCALE FACTOR FOR T-WL
C CURVES
C
C DIMENSION H(50,50),HD(50,50),
C SF(1:50,50),Q(1:50,50),T(1:50,50,2),
C ZB(50),G(50),OL(1:50,50)
C 4,NODE(10,2),HEAD(10,100),LINE(100),ALINE(100)
C
C TURN OFF UNDERFLOW TRAP
C
C CALL ERRSET(208,256,-1,1)
C
C DEFINE INPUT AND OUTPUT DEVICE NUMBERS
C
C INTEGER OUT
C IN=5
C OUT=6
C
C READ PARAMETER CARD AND
C DEFAULT VALUE CARD
C
C READ(IN,10)NSTEPS,DELTA,ERROR,
C INC,NR,TT,SI,HH,QQ
C FORMAT(16,2F6.0,2I6.0,4F6.0)
C
C READ NODE DATA FOR TIME-
C WATER LEVEL CURVES
C
C READ(IN,14)NN,AL,(NODE(I,J),J=1,2,I=1,NN)
C FORMAT(15,F5.0,10I4,13)
C
C FILL ARRAYS WITH DEFAULT VALUES
C
C DO 20 I=1,NC
C DO 20 J=1,NR
C T(I,J,1)=TT
C T(I,J,2)=TT
C SF(I,J)=SI
C HI(I,J)=HH
C HD(I,J)=HD
C Q(I,J)=QQ
C
C 20
C
C READ NODE CARDS
C
C 30 READ(IN,40,END=50)T,J,T(I,J,1),
C T(I,J,2),SF(1:J),H(1:J),Q(1:J)
C FORMAT(2I3,2F6.0,2F4.0,1F6.0)
C GO TO 30
C
C START OF SIMULATION
C
C 50 TIME=0
C DO 320 ISTEP=1,NSTEPS
C TIME=TIME+DELTA
C
C PREDICT HEADS FOR NEXT
C TIME INCREMENT
C
C DO 70 I=1,NC
C DO 70 J=1,NR
C H=HI(I,J)-HO(I,J)
C HO(I,J)=HI(I,J)
C F=1.0
C IF(OL(I,J).EQ.0.0)GO TO 60
C IF(ISTEP.GT.2)F=0/DL(I,J)
C IF(F.GT.5)F=5.0
C IF(F.LT.0.0)F=0.0
C DL(I,J)=0
C H(I,J)=H(I,J)+F*H
C
C 70
C
C REFINE ESTIMATES OF HEADS BY IADI METHOD
C
C ITER=0
C E=0.0
C ITER=ITER+1

```

```

C COLUMN CALCULATIONS
C
C DO 190 I=1,NC
C I=I
C IF(MOD(I)STEP+ITER,2).EQ.1)I=NC+1+I
C DO 170 J=1,NR
C
C CALCULATE B AND G ARRAYS
C
C BB=SF(I,J)/DELTA
C DD=HO(I,J)*SF(I,J)/DELTA-Q(I,J)
C AA=0.0
C CC=0.0
C IF(J-1)90,100,90
C AA=-T(I,J-1,1)
C BB=BB+T(I,J-1,1)
C 100 IF(J-NR)110,120,110
C CC=-T(I,J,1)
C BB=BB+T(I,J,1)
C 120 IF(I-1)130,140,130
C BB=BB+T(I-1,J,2)
C DD=DD+H(I-1,J)*T(I-1,J,2)
C 140 IF(I-NC)150,160,150
C BB=BB+T(I,J,2)
C DD=DD+H(I-1,J)*T(I,J,2)
C 160 W=BB-AA*B(J-1)
C B(I)=CC/W
C 170 G(I)=DD-AA*G(I-1)/W
C
C RE-ESTIMATE HEADS
C
C E=E+ABS(H(I,NR)-G(NR))
C H(I,NR)=G(NR)
C NN=NR-1
C HA=G(NR)-B(NR)*H(I,N+1)
C E=E+ABS(HA-H(I,N))
C H(I,N)=HA
C NN=N-1
C IF(N)190,190,180
C
C 190 CONTINUE
C
C ROW CALCULATIONS
C
C DO 300 JJ=1,NR
C JJ=JJ
C IF(MOD(ISTEP+ITER,2).EQ.1)J=NR+1+J
C DO 280 I=1,NC
C BB=SF(I,J)/DELTA
C DD=HO(I,J)*SF(I,J)/DELTA-Q(I,J)
C AA=0.0
C CC=0.0
C IF(J-1)200,210,200
C BB=BB+T(I,J-1,1)
C DD=DD+H(I,J-1)*T(I,J-1,1)
C 210 IF(J-NR)220,230,220
C DD=DD+H(I,J-1)*T(I,J,1)
C BB=BB+T(I,J,1)
C 230 IF(I-1)240,250,240
C BB=BB+T(I-1,J,2)
C AA=-T(I-1,J,2)
C 250 IF(I-NC)260,270,260
C BB=BB+T(I,J,2)
C CC=-T(I,J,2)
C 270 W=BB-AA*B(I-1)
C B(I)=CC/W
C 280 G(I)=(DD-AA*G(I-1))/W
C
C RE-ESTIMATE HEADS
C
C E=E+ABS(H(INC,J)-G(INC))
C H(INC,J)=G(INC)
C NN=NC-1
C HA=G(INC)-B(INC)*H(N+1,J)
C E=E+ABS(HA-H(INC,J))
C H(N,J)=HA
C NN=N-1
C IF(N)300,300,290
C
C 300 CONTINUE
C IF(E.GT.ERROR) GO TO 80
C
C PRINT TIME TABLE
C
C IF(ISTEP.EQ.1)WRITE(OUT,301)
C 301 FORMAT('1STEP TIME(DAYS)')
C DELTA=DELTA*1.2
C WRITE(OUT,302)ISTEP,TIME
C 302 FORMAT(16,6X,F8.2)
C
C STORE RESULTS
C
C DO 320 K=1,NN
C J=NODE(K,1)
C J=NODE(K,2)
C 320 HEAD(K,ISTEP)=H(I,J)
C
C PRINT RESULTS
C
C DO 345 K=1,NN
C WRITE(OUT,340)(NODE(K,J),J=1,2),(I,I=5,NSTEPS,5)
C 340 FORMAT('1NODE(',I3,',',I3,')////12X20I5)')
C 1BOT=1000000
C 1TOP=1000000
C DO 341 I=1,NSTEPS
C LINE(I)=HEAD(K,I)/AL
C IF(LINE(I).GT.1TOP)1TOP=LINE(I)
C 341 IF(LINE(I).LT.1BOT)1BOT=LINE(I)
C DO 343 I=1,NSTEPS
C DATA BLANK,VERT,PLQT/' ','1','*'/
C ALINE(I)=BLANK
C IF(I/5.EQ.I/5.0)ALINE(I)=VERT
C 343 IF(LINE(I).EQ.1TOP)ALINE(I)=PLQT
C X=AL*1TOP
C WRITE(OUT,344)X,(ALINE(I),I=1,NSTEPS)
C 344 FORMAT(F10.3,2X,100A1)
C ITOP=1TOP-1
C IF(1TOP.GE.1BOT)GO TO 342
C
C 345 CONTINUE
C
C STOP
C END

```

Figure 30. Basic aquifer simulation program with time-water level printout option



## VARIABLE PUMPING RATES

Pumping rates or net withdrawal rates can be programmed to change in a stepwise fashion as a function of time. An example problem and the job setup are illustrated in figures 32-35. The computer listing for the variable pumping rates is given in figure 36 and is a modification of the basic aquifer simulation program.

A stepwise pumping schedule, as exemplified in figure 32, is set up for each well of the model. The computer program can manipulate positive (pumping), negative (recharge), or zero withdrawal rates for a total number of wells (NP). Between changes in pumping rate, the computer program operates with nonuniform time steps within each period of pumping. However, the initial DELTA is reset at each change in pumping rate for the reasons outlined in the nonuniform time increment section of this report.

As an example, figure 32 shows four pumping rate changes (NRT = 4). Each pumping rate is in effect for 11 time increments (NSP = 11), and the total number of time increments is 44 (NSTEPS = 44). The length of time that each pumping rate is effective is the same for all wells.

The length of time of pumping for each pumping rate is known and usually corresponds to field inventory periods. Equation 47 is used, along with table 1, to calculate an initial DELTA and number of ISTEPS to fit the inventory period.

Once the pumping schedules have been set up, the rates and well location coordinates are entered on pumping schedule cards according to FORMAT statement 13 of the listing of figure 36. A pump parameter card is included to define NP, NSP, and NRT according to FORMAT statement 11. Figure 33 shows the pumping schedules for five wells in an aquifer model with 9 columns and 31 rows. The pumping rates were plotted on an inventory period of 30 days. Equation 47 and table 1 were used to compute an initial DELTA of 0.933 days so that TIME equals 30 days after 11 time in-

crements are processed. After 11 time increments, the pumping rates are changed, DELTA is reset to its initial value, and another group of 11 time increments is processed to TIME equals 60 days. This continues until all pumping changes have been processed.

Figure 34 illustrates a time-drawdown plot of water levels, as a result of the pumping scheme of figure 33, at an observation point with coordinates  $i = 4$  and  $j = 15$ .

Figure 35 shows the input data cards and their order for the variable pumping schedules set up in figure 33. Any node cards that might be included are put immediately following the pumping schedule cards, although there were no node cards for this particular problem.

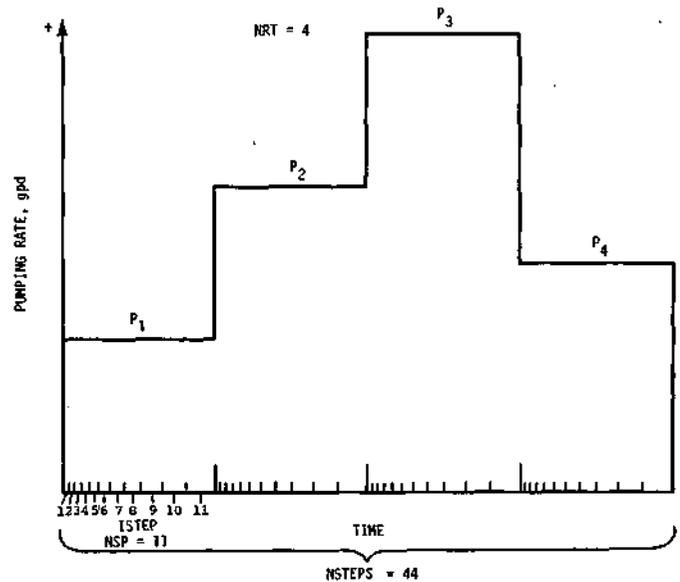


Figure 32. Example variable pumping rate schedule

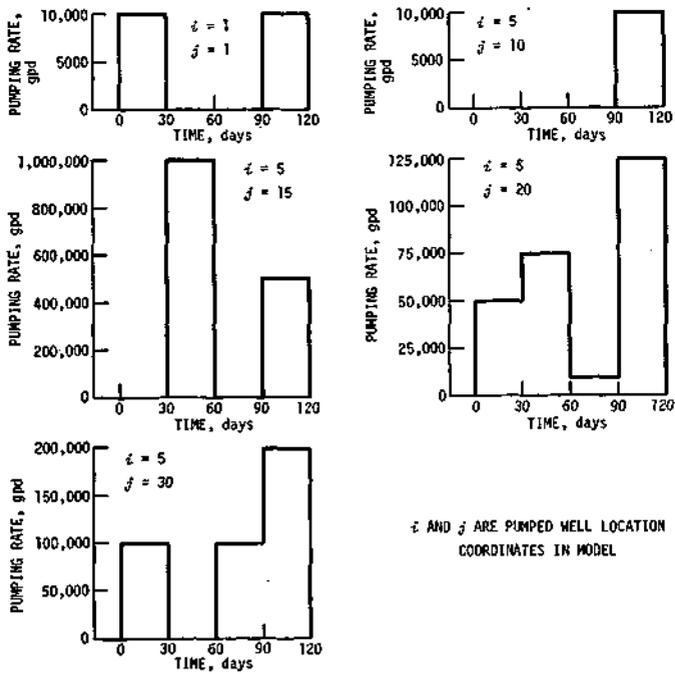


Figure 33. Example pumping schedules for five wells located in a model of 9 columns by 31 rows

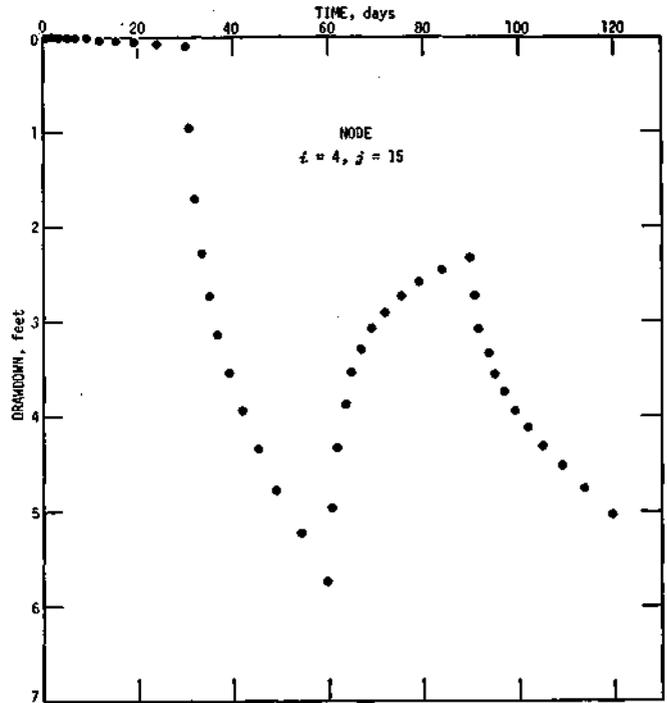


Figure 34. Example time-drawdown graph for variable pumping scheme

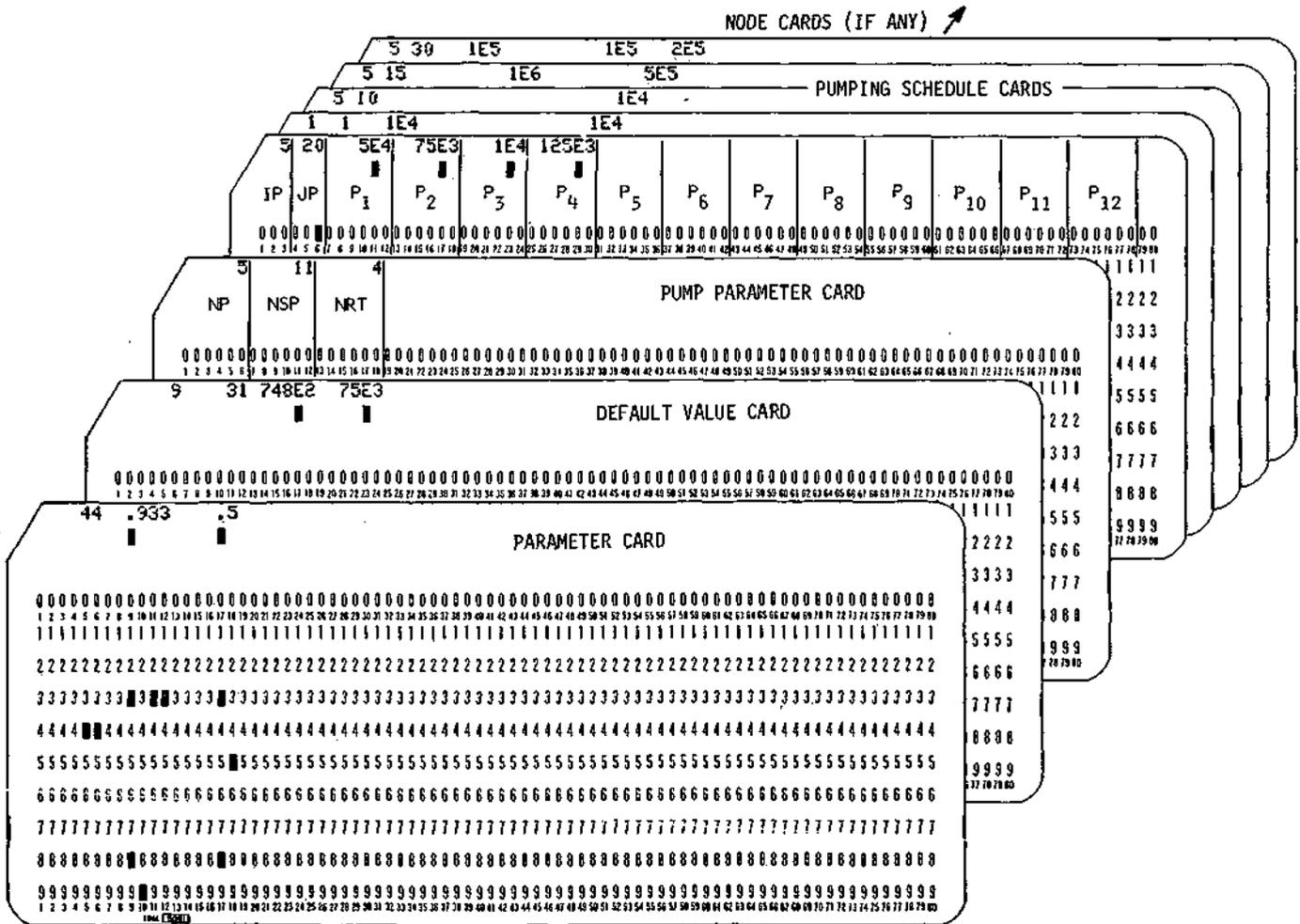


Figure 35. Data deck setup for variable pumping scheme



## LEAKY ARTESIAN CONDITIONS

The basic aquifer simulation program can be modified to include leaky artesian aquifer conditions as defined below. The modifications and an example problem are illustrated in figures 37-40, and the modified computer program listing is given in figure 41.

Leaky artesian conditions exist (Hantush and Jacob, 1955) where aquifers are overlain by confining beds (aquitards) which impede the vertical flow of groundwater as illustrated in figure 37. It is assumed that the leakage through the confining bed into the aquifer is vertical and proportional to the difference in head between the aquifer and the source bed above the confining layer. It is further assumed that the hydraulic head in the source bed supplying leakage remains constant, that the storage in the confining bed is neglected, and that the head in the aquifer does not fall below the bottom of the confining layer.

Walton (1960) gives an equation (a modified form of Darcy's law) that describes the rate of leakage through such confining beds. The equation in the notation of this report is

$$Q_n = (P'/m') \Delta h A_c \quad (48)$$

where

$Q_n$  = leakage rate through the confining bed, in gpd

$P'$  = vertical hydraulic conductivity of confining bed, in gallons per day per square foot (gpd/ft<sup>2</sup>)

$m'$  = thickness of confining bed in feet

$h$  = difference between the head in the aquifer and in the source bed overlying the confining layer, in feet

$A_c$  = area of confining bed through which leakage takes place, in square feet

In terms of the head notation of figure 37

$$\Delta h = RH_{i,j} - h_{i,j} \quad (49)$$

The vector volume of the portion of the confining bed represented at each node of the digital model is illustrated in figure 3. This vector volume extends the full depth of the confining layer  $m'$  and has horizontal dimensions of  $x$   $y$ , the volume being centered around the node point  $i,j$ . Therefore the area term of equation 48 may be expressed as

$$A_c = \Delta x \Delta y \quad (50)$$

Substitution of equations 49 and 50 into 48 yields

$$Q_n = (P'/m') \Delta x \Delta y (RH_{i,j} - h_{i,j}) \quad (51)$$

A recharge factor, in gallons per day per foot (gpd/ft), is defined as

$$R_{i,j} = (P'/m') \Delta x \Delta y \quad (52)$$

Substituting equation 52 into 51 and carrying out the multiplication result in

$$Q_n = R_{i,j} RH_{i,j} - R_{i,j} h_{i,j} \quad (53)$$

To include leaky artesian conditions, as defined above, in the basic aquifer simulation program requires substitution of equation 53 into the column and row equations 11 and 12, respectively. Inspection of equations 11, 12, and 53 shows that the component  $R_{i,j}RH_{i,j}$  remains as a positive constant term in DD. Since the head at the node  $h_{i,j}$  is involved in the component  $R_{i,j}h_{i,j}$ , the term  $R_{i,j}$  is included in BB, the sign of which will be positive because of its shift to the left hand side of equations 11 and 12.

The computer program listing for the above case of leaky artesian conditions is given in figure 41; modifications made to the basic aquifer simulation program are boxed. As indicated in figure 41, two additional arrays are dimensioned to accommodate the source bed heads  $RH_{i,j}$  and the recharge factors  $R_{i,j}$ . Also the source bed heads and the recharge factors are added to the default value and node cards and read into the computer by the changes in the READ statements. Finally, the addition of the recharge factors and source bed heads are taken into account by the additional terms in the BB and DD coefficients of the ROW and COLUMN calculation sections of the program.

The procedures leading to the job setup are the same as those described for the basic aquifer simulation program; however, the confining bed properties and source bed heads are also discretized with a finite difference grid that is congruent with the aquifer property grid. Each vector volume of the discrete portions of the confining layer, associated with a numbered node of the grid, is then outlined and assigned an average recharge factor (see equation 52) and source bed water level elevation. An appropriate recharge factor is entered on the default value card and differing values on the node cards.

Figure 38 illustrates a theoretical versus computer-simulated time-drawdown curve for leaky artesian conditions with homogeneous aquifer and confining bed properties. The theoretical curve of figure 38 is from data published by Hantush (1956) as applied by Walton (1960). The computer input data cards related to figure 38 are given in figure 40. The good agreement between theory and computer data should be noted. A distance-drawdown comparison is also given in the graph of figure 39 for steady-state conditions, and this again indicates excellent agreement between theory and computer-simulated data. The theoretical curve of figure 39 was obtained from Jacob (1946).



```

C      ILLINOIS STATE WATER SURVEY
C      BASIC AQUIFER SIMULATION PROGRAM
C      WITH LEAKY ARTESIAN CONDITIONS
C
C      DEFINITION OF VARIABLES
C
C      HO(I,J)-----HEADS AT START OF TIME
C                   INCREMENT (I,J)
C      HI(I,J)-----HEADS AT END OF TIME
C                   INCREMENT (PT)
C      SF1(I,J)-----STORAGE FACTOR FOR
C                   ARTESIAN CONDITIONS
C                   (GAL/FT)
C      Q(I,J)-----CONSTANT WITHDRAWAL
C                   RATES (GPD)
C      T(I,J,1)-----AQUIFER TRANSMISSIVITY
C                   BETWEEN I,J AND I,J+1
C                   (GAL/DAY/FT)
C      T(I,J,2)-----AQUIFER TRANSMISSIVITY
C                   BETWEEN I,J AND I+1,J
C                   (GAL/DAY/FT)
C      R(I,J)-----RECHARGE FACTOR
C                   (GAL/DAY/FT)
C      RH(I,J)-----ELEVATION OF WATER
C                   LEVEL IN SOURCE
C                   BED (FT)
C
C      AA,BB,CC,DD--COEFFICIENTS IN WATER
C                   BALANCE EQUATIONS
C      NR-----NO. OF ROWS IN MODEL
C      NC-----NO. OF COLUMNS IN MODEL
C      NSTEPS-----NO. OF TIME INCREMENTS
C      DELTA-----TIME INCREMENTS (DAYS)
C      HH,S1,QQ,TT--DEFAULT VALUES
C      I-----MODEL COLUMN NUMBER
C      J-----MODEL ROW NUMBER
C
C      DIMENSION H(50,50),HO(50,50),
C      SF1(50,50),Q(50,50),T(50,50,2),
C      ZB(50),G(50),DL(50,50),R(50,50),
C      BRH(50,50)
C
C      TURN OFF UNDERFLOW TRAP
C
C      CALL ERRSET(208,256,-1,1)
C
C      DEFINE INPUT AND OUTPUT DEVICE NUMBERS
C
C      INTEGER OUT
C      IN=5
C      OUT=6
C
C      READ PARAMETER CARD AND
C      DEFAULT VALUE CARD
C
C      READ(IN,10)NSTEPS,DELTA,ERROR,
C      INC,RR,TT,S1,HH,QQ,RR,RRH
C      10  FORMAT(16,2F6.0,2I6,6F6.0)
C
C      FILL ARRAYS WITH DEFAULT VALUES
C
C      DO 20 I=1,NC
C      DO 20 J=1,NR
C      T(I,J,1)=TT
C      T(I,J,2)=TT
C      SF1(I,J)=S1
C      H(I,J)=HH
C      HO(I,J)=HH
C      R(I,J)=RR
C      RH(I,J)=RRH
C      20  Q(I,J)=QQ
C
C      READ NODE CARDS
C
C      30  READ(IN,40,END=50)I,J,TII,J,1,
C      1T(I,J,2),SF1(I,J),H(I,J),Q(I,J),
C      2R(I,J),RH(I,J)
C      40  FORMAT(2I3,2F6.0,2F4.0,3F6.0)
C      GO TO 30
C
C      START OF SIMULATION
C
C      50  TIME=0
C      DO 320 ISTEP=1,NSTEPS
C      TIME=TIME+DELTA
C
C      PREDICT HEADS FOR NEXT
C      TIME INCREMENT
C
C      DO 70 I=1,NC
C      DO 70 J=1,NR
C      D=H(I,J)-HO(I,J)
C      HO(I,J)=H(I,J)
C      F=1.0
C      IF(DL(I,J).EQ.0.0)GO TO 60
C      IF(ISTEP.GT.2)F=D/DL(I,J)
C      IF(F.GT.5)F=5.0
C      IF(F.LT.0.0)F=0.0
C      60  DL(I,J)=D
C      70  H(I,J)=H(I,J)+D*F
C
C      REFINE ESTIMATES OF HEADS BY IADI METHOD
C
C      ITER=0
C      80  E=0.0
C      ITER=ITER+1
C
C      COLUMN CALCULATIONS
C
C      DO 190 I=1,NC
C      I=1
C      IF(MOD(ISTEP+ITER,2).EQ.1) I=NC-I+1
C      DO 170 J=1,NR
C
C      CALCULATE B AND G ARRAYS
C
C      BB=SF1(I,J)/DELTA+R(I,J)
C      DD=HO(I,J)*SF1(I,J)/DELTA-Q(I,J)+
C      1RH(I,J)*R(I,J)
C      AA=0.0
C      CC=0.0
C      IF(J-1)90,100,90
C      AA=-T(I,J-1,1)
C      BB=BB+T(I,J-1,1)
C      100  IF(J-NR)110,120,110
C      110  CC=-T(I,J,1)
C      BB=BB+T(I,J,1)
C      120  IF(I-1)130,140,130
C      130  DD=DD+H(I-1,J)*T(I-1,J,2)
C      140  IF(I-NC)150,160,150
C      150  BB=BB+T(I,J,2)
C      DD=DD+H(I+1,J)*T(I,J,2)
C      160  W=BB-AA*B(J-1)
C      B(I)=CC/W
C      170  G(I)=(DD-AA*G(I-1))/W
C
C      RE-ESTIMATE HEADS
C
C      E=E+ABS(H(I,NR)-G(INR))
C      H(I,NR)=G(INR)
C      N=NR-1
C      180  HA=G(N)-B(N)*H(I,N+1)
C      E=E+ABS(HA-H(I,N))
C      H(I,N)=HA
C      N=N-1
C      190  IF(N)190,190,180
C      CONTINUE
C
C      ROW CALCULATIONS
C
C      DO 300 JJ=1,NR
C      J=JJ
C      IF(MOD(ISTEP+ITER,2).EQ.1) J=NR-J+1
C      DO 280 I=1,NC
C      BB=SF1(I,J)/DELTA+R(I,J)
C      DD=HO(I,J)*SF1(I,J)/DELTA-Q(I,J)+
C      1RH(I,J)*R(I,J)
C      AA=0.0
C      CC=0.0
C      IF(J-1)200,210,200
C      BB=BB+T(I,J-1,1)
C      DD=DD+H(I,J-1)*T(I,J-1,1)
C      210  IF(J-NR)220,230,220
C      220  DD=DD+H(I,J+1)*T(I,J,1)
C      BB=BB+T(I,J,1)
C      230  IF(I-1)240,250,240
C      240  BB=BB+T(I-1,J,2)
C      AA=-T(I-1,J,2)
C      250  IF(I-NC)260,270,260
C      260  BB=BB+T(I,J,2)
C      CC=-T(I,J,2)
C      270  W=BB-AA*B(I-1)
C      B(I)=CC/W
C      280  G(I)=(DD-AA*G(I-1))/W
C
C      RE-ESTIMATE HEADS
C
C      E=E+ABS(H(INC,J)-G(INC))
C      H(INC,J)=G(INC)
C      N=NC-1
C      290  HA=G(N)-B(N)*H(N+1,J)
C      E=E+ABS(H(N,J)-HA)
C      H(N,J)=HA
C      N=N-1
C      300  IF(N)300,300,290
C      CONTINUE
C      IF(E.GT.ERROR) GO TO 80
C
C      PRINT RESULTS
C
C      WRITE(OUT,310)TIME,E,ITER
C      310  FORMAT(6H2TIME=,F6.2///,E20.7,15)
C      DELTA=DELTA*1.2
C      DO 320 J=1,NR
C      320  WRITE(OUT,330)J,(H(I,J),I=1,NC)
C      330  FORMAT(15,5X,10F10.4/(12X10F10.4))
C
C      STOP
C      END

```

Figure 41. Basic aquifer simulation program with leaky artesian conditions

## INDUCED INFILTRATION

Modifications for induced infiltration conditions can be included in the basic aquifer simulation program. The modifications and an example problem are illustrated in figures 42-47, and the modified computer program listing is given in figure 48.

Induced infiltration can take place when wells are pumped in aquifers which are near and in hydraulic connection with streams, rivers, or other surface water bodies. In the course of pumping, water levels in an aquifer may be lowered below surface water levels and the aquifer is recharged by influent seepage from the surface water body. Several publications are available (for example, Boulton, 1942; Rorabaugh, 1951; Hantush, 1965; and Walton and Ackroyd, 1966) which describe the flow interconnection phenomenon between surface water bodies and aquifers.

The conditions for which the computer program for induced infiltration is set up are shown in figure 42. It is assumed that wells fully penetrate the aquifer, that the drawdown in the flow field is small compared with the saturated thickness of the aquifer, and that the well is far enough from the stream that the effects of partial penetration of the river will be negligible. Further, it is assumed that the head in the surface water body remains constant.

As shown in figure 42a, as the head in the aquifer  $h_{i,j}$  declines below the head in the stream  $RH_{i,j}$ , water is induced to flow from the stream  $Q_n$ , across the streambed, and into the aquifer. According to Walton and Ackroyd (1966) the rate of flow is directly proportional to the streambed area, the permeability of the streambed, and the head difference between that of the aquifer and the stream; it is inversely proportional to the streambed thickness. In equation form the rate of flow through the streambed can be expressed by the following modified form of Darcy's law.

$$Q_n = (P'/m') \Delta h A_s \quad (54)$$

where

- $Q_n$  = infiltration rate through the streambed, in gpd
- $P'$  = hydraulic conductivity of the streambed, in gpd/ft<sup>2</sup>
- $m'$  = thickness of the streambed, in feet
- $A_s$  = area of the streambed assigned to node, in square feet
- $h$  = head difference between the level of water in the stream and that in the aquifer beneath the stream, in feet

However, the rate of flow through the streambed reaches a limit (figure 42b) as maximum hydraulic gradients are created when the head in the aquifer,  $h_{i,j}$ , directly beneath the stream falls below the bottom of the streambed,  $RD_{i,j}$  (Walton et al., 1967). In terms of the head notation of figure 42

$$\Delta h = RH_{i,j} - h_{i,j} \quad (55)$$

Maximum hydraulic gradients occur when  $h_{i,j} = RD_{i,j}$ . Thus, the maximum head loss is given by

$$\Delta h_{\max} = RH_{i,j} - RD_{i,j}$$

The vector volumes of the individual streambed portions represented at each node of the digital model are illustrated in figure 43. These vector volumes extend the full depth of the streambed layer,  $m'$ , and have horizontal areas denoted by  $A_s$ .

Substitution of equation 55 into equation 54 yields

$$Q_n = (P'/m') A_s (RH_{i,j} - h_{i,j}) \quad (56)$$

A recharge factor, in gallons per day per foot, is defined at all stream nodes as

$$R_{i,j} = (P'/m') A_s \quad (57)$$

in which average values of streambed hydraulic conductivities and thicknesses, and the areas within the vector volumes illustrated in figure 43 are used. Substituting equation 57 into 56 and carrying out the multiplication result in

$$Q_n = R_{i,j} RH_{i,j} - R_{i,j} h_{i,j} \quad (58)$$

Equation 58 is identical to equation 53 (see leaky artesian conditions) except the terms now pertain to streambed properties and surface water heads instead of confining bed and source bed heads as in the leaky artesian case.

To include the effects of induced infiltration, as described above, in the basic aquifer simulation program requires two main modifications. First, just as with leaky artesian conditions, the components  $R_{i,j}RH_{i,j}$  and  $R_{i,j}$  are entered into the finite difference equations (see equations 11 and 12) in DD and BB respectively. Secondly, STREAMBED INFILTRATION CONTROL sections are programmed to check whether or not aquifer water levels have fallen below the bottom of the streambed and to control the infiltration rate according to the nonlinear graph shown in figure 42b.

The computer program for induced infiltration conditions is given in figure 48; modifications to the basic aquifer simulation program are boxed. As indicated in figure 48 three additional arrays are dimensioned to accommodate the stream water level elevations  $RH_{i,j}$ , the streambed recharge factors  $R_{i,j}$ , and the elevations of the streambed bottoms  $RD_{i,j}$ . Also, the stream water level elevations, recharge factors, and streambed bottom elevations are added to the default value and node cards as shown in figure 47, and are read into the computer by the changes in the READ statements. Finally, the STREAMBED INFILTRATION CONTROL sections are added to the ROW and COLUMN calculations.

The computer job setup is illustrated by the following example problem which considers an infinite aquifer of uniform properties with a straight line river crossing it. This river is assumed to have a semipervious streambed with a thickness of 6 inches, a vertical hydraulic conductivity of 0.5 gpd/ft<sup>2</sup>, and a stream width of 75 feet; it is discretized in 1000-foot length intervals. The initial static water level everywhere in the aquifer is at an elevation of 700 feet. The water level in the stream is also everywhere at an elevation

of 700 feet. The elevations of the bottom of the streambed are everywhere equal to 699 feet. Recharge factors for all nodes along the stream are identical and are calculated from equation 57 as

$$R_{i,j} = (P'/m')A_s$$

$$R_{i,j} = (5 \times 10^{-1} / 5 \times 10^{-1}) 75 \times 1000$$

$$R_{i,j} = 75,000 \text{ gpd/ft}$$

It is further assumed that a fully penetrating well will be located a distance of 2000 feet from the river and pumped at a constant rate of 2 million gallons per day (mgd). The aquifer is homogeneous and isotropic and has a transmissivity of 74,800 gpd/ft and a storage coefficient of  $1.003 \times 10^{-2}$ .

The data cards for this problem are as shown in figure 47. The formats of the data cards are the same as those of the basic aquifer simulation program except for the addition of the three fields defining R values and RH and RD elevations.

An example time-drawdown curve is shown in figure 44. The computer data were obtained with the data deck parameters outlined in figure 47. Since there is no closed form solution known to the authors for the water level distribution that includes the nonlinear effects of induced infiltration described above, the time-water level data are presented in comparison with the response of an infinite aquifer with no recharge and also with a fully penetrating recharge boundary. As shown in figure 44, the water level declines in the observation well are coincident with both theoretical curves during the early portion of pumping, then for a time follow the fully penetrating recharge boundary curve reflecting the effects of recharge from the river. The computer data then deviate from the recharge boundary curve as maximum head losses across the streambed begin to occur.

Two cross-sectional views of water levels in the above stream-aquifer system are shown in figures 45 and 46. The cross section shown in figure 45 is illustrated along a line perpendicular to the stream and passing through the pumped well location. The curves for the response of an infinite aquifer and a fully penetrating recharge boundary are also shown for comparison.

Figure 46 shows the cross section of water levels in the

aquifer directly beneath and along the line of the stream. Maximum head losses across the streambed have occurred at seven node positions beneath the stream.

It should be pointed out that the program of figure 48 can also be used for limiting the amount of recharge to an aquifer under leaky artesian conditions when aquifer water levels fall below the bottom of a confining layer. Under these conditions, the recharge factors are defined from confining layer properties instead of streambed properties. The RH array then defines the heads in the source bed overlying the confining layer, and the RD array defines the elevations of the bottom of the confining layer. Both leaky artesian conditions and induced infiltration conditions can be handled simultaneously in the same model since the two conditions never overlap.

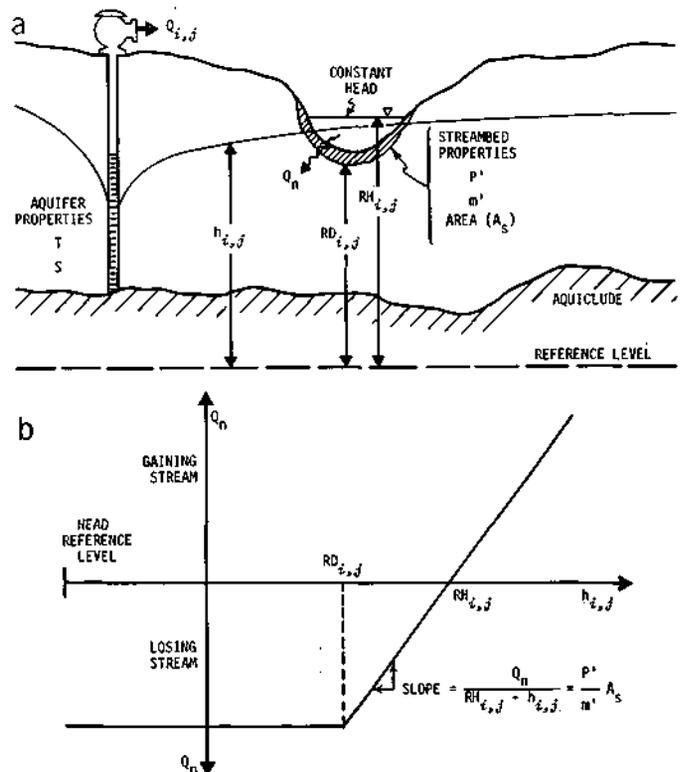


Figure 42. Basic aquifer simulation program parameters with induced infiltration (a) and head-infiltration rate curve (b)

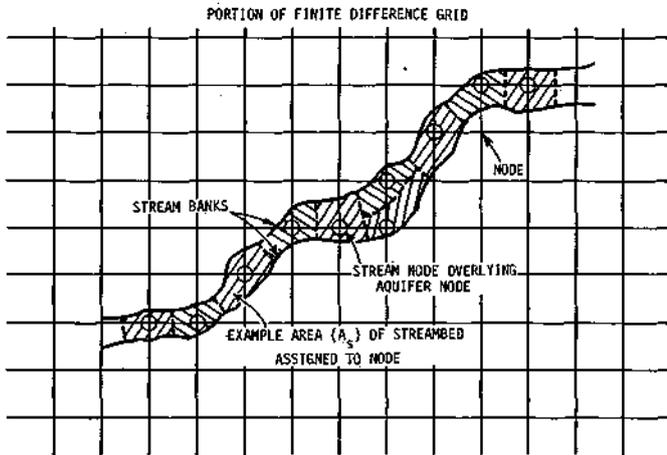


Figure 43. Example streambed vector volumes

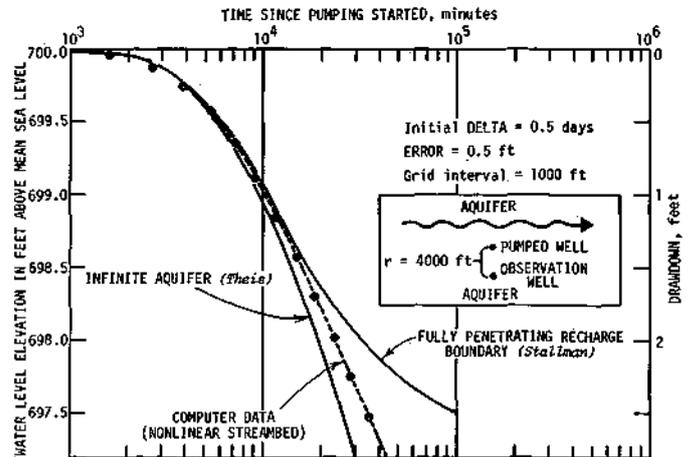


Figure 44. Time-drawdown curves for induced infiltration problem

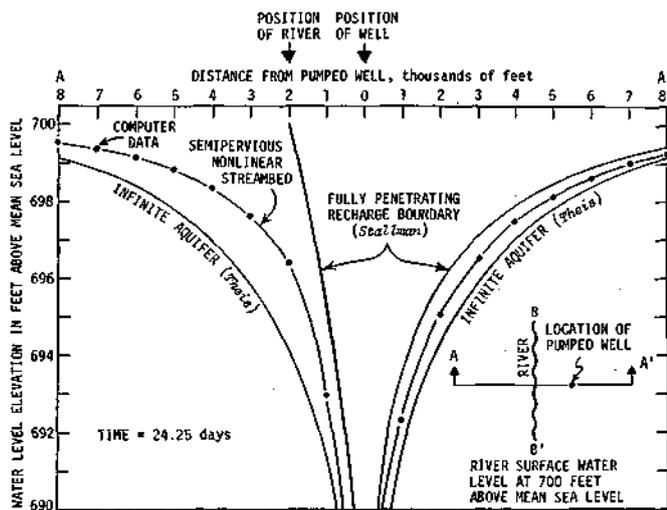


Figure 45. Cross section distance-drawdown graphs for induced infiltration problem

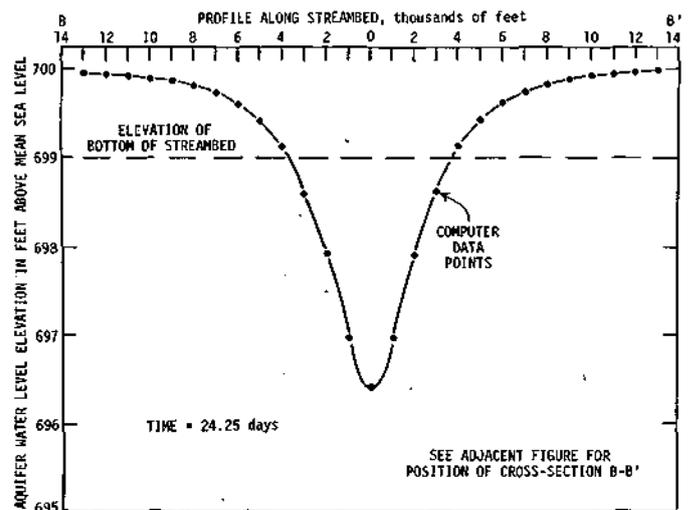
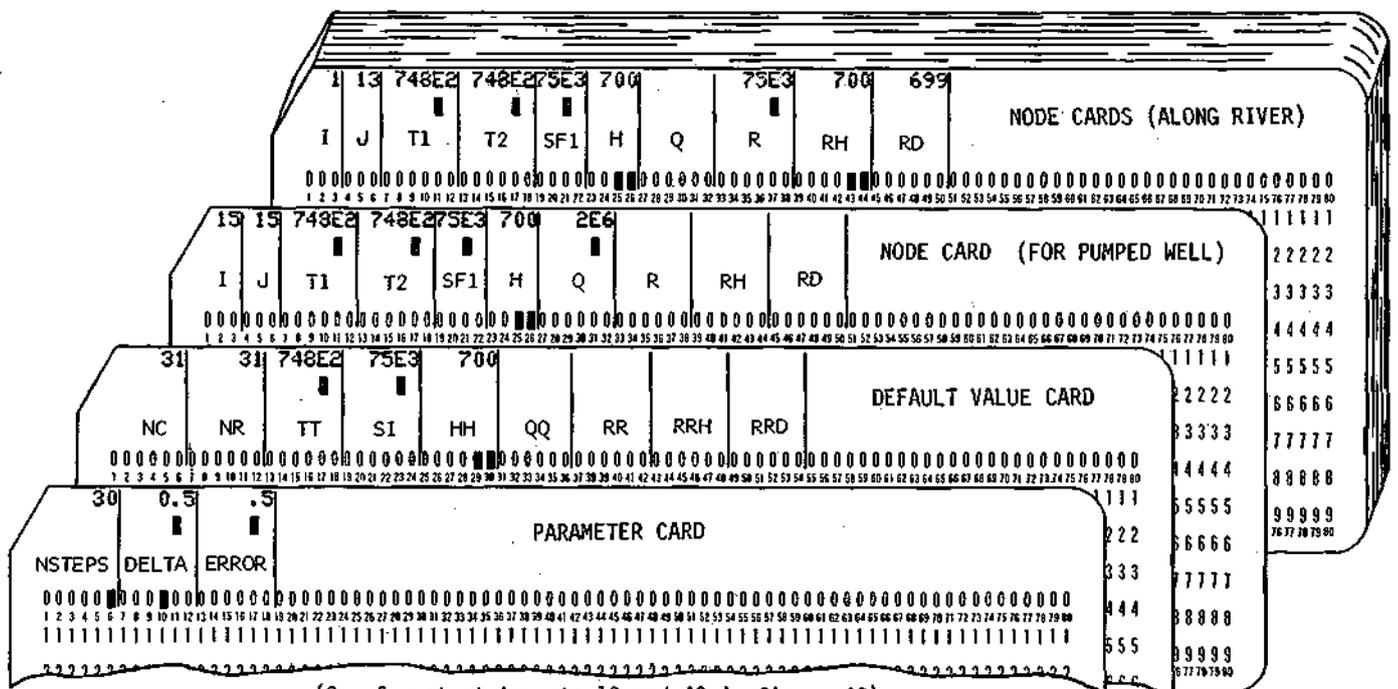


Figure 46. Head distribution in aquifer beneath streambed for induced infiltration problem



(See format statements 10 and 40 in figure 48)

Figure 47. Data deck setup for induced infiltration problem



## GROUNDWATER EVAPOTRANSPIRATION

This section describes the effects of evapotranspiration and the modifications needed for their inclusion in the basic aquifer simulation program. These problems and the program listing are illustrated in figures 49-53.

In areas where the water table is near the land surface, water can be discharged from the groundwater reservoir to the atmosphere by the process of evapotranspiration, a term combining evaporation from the capillary fringe above the water table and transpiration from plants whose roots penetrate to the water table or the capillary fringe (Meinzer, 1923). In some cases the water escaping to the atmosphere through the process of evapotranspiration may be stopped by lowering groundwater levels to such a depth that the process ceases (Robinson, 1964; Skibitzke, 1963).

The conditions and assumptions upon which the computer program is written are illustrated in figure 49. In this aquifer cross section the water levels,  $h_{i,j}$ , are near the land surface. The land surface elevations are specified by  $RH_{i,j}$ , and the elevations of the water table below which evapotranspiration ceases are defined by  $RD_{i,j}$ . The rate of evapotranspiration  $Q_{et}$  for each node of the digital model is made a linear function of the difference between the elevation of the land surface and the elevation of the water table (see figure 51c) until the critical depth  $RD_{i,j}$  is reached and  $Q_{et}$  ceases. A more complex relationship can be used if desired. The vector volume of each node is defined in figure 3.

It is further assumed that wells fully penetrate the aquifer and that the drawdown in the flow field is small compared with the saturated thickness of the aquifer.

The computer program listing for the evapotranspiration process described above is given in figure 53 in which the needed modifications to the basic aquifer simulation program are boxed. The modifications include expanding the dimension statement to reserve core storage for the R, RH, and RD arrays. The format statements for the default value and node cards are modified to add individual node values of R, RH, and RD. The EVAPOTRANSPIRATION CONTROL sections of figure 53 regulate the rate of evapotranspiration leaving the groundwater reservoir according to the relationships outlined in figure 51c.

The evapotranspiration control sections are identical to the induced infiltration control sections of figure 48. The evapotranspiration rates are effectively controlled by applying the principle of superposition between constant net withdrawal rates and the effects of induced infiltration, as illustrated in figure 51. Figure 51a shows that the maximum rate at which evapotranspiration could take place for the areas represented by each node of the model is entered as a constant net withdrawal rate in the  $Q_{i,j}$  array. If any constant pumpage or recharge (negative pumpage) takes place at the same node where evapotranspiration occurs, that additional pumpage or recharge is summed vectorially with the evapotranspiration rate and entered as a total in  $Q_{i,j}$ . In-

dividual evapotranspiration factors, representing the slope of the straight lines in figure 51b and c, are specified for each node and entered in the  $R_{i,j}$  array according to

$$R_{i,j} = Q_{et(max)} / (RH_{i,j} - RD_{i,j}) \quad (59)$$

The data deck setup procedures can be explained with an example problem pertaining to the geohydrologic setting in West Pakistan described by Greenman et al. (1967).

Water level fluctuations in the Punjab region of West Pakistan can be generally illustrated as shown in figure 50. Water levels began to rise about 1902 in response to the introduction of a widespread irrigation system. Leakage from the irrigation distribution system was recharging the underlying aquifer at a fairly uniform rate until about 1920. Speaking in general terms regarding several hydrographs, Greenman states: "Following this period of linear rise there is an interval of progressive flattening of the hydrographs, indicating an increasing evapotranspiration loss as the water table approached land surface." Eventually, the hydrographs stabilized when the effects of evapotranspiration balanced the recharge rate.

Simulation of this phenomenon requires, beyond the usual computer input data, values for  $RH_{i,j}$ ,  $RD_{i,j}$ ,  $R_{i,j}$ , and  $Q_{i,j}$ . Figure 50 shows that the land surface elevation  $RH_{i,j}$  is about 643 feet. Evapotranspiration effects begin when water levels are above 30 feet below the land surface or at an elevation  $RD_{i,j}$  of 613 feet.

Measurements taken from the hydrograph of figure 50 indicate that the rate of water level rise during the linear portion averaged about 2.3 feet per year. This rise, in terms of the daily recharge rate from irrigation leakage, amounts to  $9.45 \times 10^{-3}$  gpd/ft<sup>2</sup> while assuming a storage coefficient of 0.20. With 1000-foot grid intervals, the recharge rate is  $9.45 \times 10^3$  gpd for each node.

The slope of the line  $R_{i,j}$  of figure 51b or c can now be computed since the evapotranspiration rate equaled the recharge rate when water levels were at an elevation of 640 feet and the evapotranspiration rate was zero when water levels were at an elevation of 613. The slope of that line is computed as the difference in evapotranspiration rates divided by the difference in water table elevations; therefore  $9.45 \times 10^3$  divided by (640 - 613) equals 350 gpd/ft.

From equation 59,  $Q_{et(max)}$  is computed to be  $1.05 \times 10^4$  gpd. The only remaining data input needed is a value for  $Q_{i,i}$ , which is calculated as  $Q_{et(max)}$  minus the recharge rate (negative pumpage). Thus  $Q_{i,j}$  is computed as  $1.05 \times 10^4 - 9.45 \times 10^3$  which equals  $1.05 \times 10^3$  gpd.

The data deck setup for the West Pakistan evapotranspiration problem is shown in figure 52. Initial heads were set equal to a water table elevation of 572 when the irrigation system was constructed about 1902, and a uniform time step DELTA of 365 days was used in the simulation. The computer output and well hydrograph are compared in figure 50.



```

C      ILLINOIS STATE WATER SURVEY
C      BASIC AQUIFER SIMULATION PROGRAM
C      WITH EVAPOTRANSPIRATION CONDITIONS
C
C      DEFINITION OF VARIABLES
C
C      HQ(I,J)-----HEADS AT START OF TIME
C                   INCREMENT (I,J)
C      H(I,J)-----HEADS AT END OF TIME
C                   INCREMENT (FT)
C      SF(I,J)-----STORAGE FACTOR FOR
C                   ARTESIAN CONDITIONS
C                   (GAL/FT)
C      Q(I,J)-----CONSTANT WITHDRAWAL
C                   RATES (GPD)
C      T(I,J,1)-----AQUIFER TRANSMISSIVITY
C                   BETWEEN I,J AND I,J+1
C                   (GAL/DAY/FT)
C      T(I,J,2)-----AQUIFER TRANSMISSIVITY
C                   BETWEEN I,J AND I+1,J
C                   (GAL/DAY/FT)
C
C      R(I,J)-----EVAPOTRANSPIRATION
C                   FACTOR (GAL/DAY/FT)
C
C      AA,BB,CC,DD--COEFFICIENTS IN WATER
C                   BALANCE EQUATIONS
C      NR-----NO. OF ROWS IN MODEL
C      NC-----NO. OF COLUMNS IN MODEL
C      NSTEPS-----NO. OF TIME INCREMENTS
C      DELTA-----TIME INCREMENTS (DAYS)
C      HH,S1,QQ,TT--DEFAULT VALUES
C      I-----MODEL COLUMN NUMBER
C      J-----MODEL ROW NUMBER
C
C      RH(I,J)-----ELEVATION OF LAND (FT)
C      RD(I,J)-----ELEVATION BELOW WHICH
C                   EVAPOTRANSPIRATION
C                   LOSSES CEASE (FT)
C
C
C      DIMENSION H(50,50),HQ(50,50),
C      SF(1,50,50),Q(50,50),T(50,50,2),
C      ZB(50),G(50),R(50,50),DL(50,50)
C      S,RH(50,50),RD(50,50)
C
C      TURN OFF UNDERFLOW TRAP
C
C      CALL ERRSET(208,256,-1,1)
C
C      DEFINE INPUT AND OUTPUT DEVICE NUMBERS
C
C      INTEGER OUT
C      IN=5
C      OUT=6
C
C      READ PARAMETER CARD AND
C      DEFAULT VALUE CARD
C
C      READ(IN,10)NSTEPS,DELTA,ERROR,
C      INC,NR,TT,S1,HH,QQ,RR,RRH,RRD
C      FORMAT(16,2F6.0/216,7F6.0)
C
C      FILL ARRAYS WITH DEFAULT VALUES
C
C      DO 20 I=1,NC
C      DO 20 J=1,NR
C      T(I,J,1)=TT
C      T(I,J,2)=TT
C      SF(I,J)=S1
C      H(I,J)=HH
C      HQ(I,J)=HH
C      R(I,J)=RR
C      RH(I,J)=RRH
C      RD(I,J)=RRD
C
C      Q(I,J)=QQ
C
C      READ NODE CARDS
C
C      30 READ(IN,40,END=50)I,J,T(I,J,1),
C      T(I,J,2),SF(I,J),H(I,J),Q(I,J),
C      ZB(I,J),RH(I,J),RD(I,J)
C      FORMAT(2I3,2F6.0,2F4.0,4F6.0)
C      GO TO 30
C
C      START OF SIMULATION
C
C      50 TIME=0
C      DO 320 ISTEP=1,NSTEPS
C      TIME=TIME+DELTA
C
C      PREDICT HEADS FOR NEXT
C      TIME INCREMENT
C
C      DO 70 I=1,NC
C      DO 70 J=1,NR
C      D=H(I,J)-HQ(I,J)
C      HQ(I,J)=H(I,J)
C      F=1.0
C      IF(DL(I,J).EQ.0.0)GO TO 60
C      IF(ISTEP.GT.2)F=D/DL(I,J)
C      IF(F.GT.5)F=5.0
C      IF(F.LT.0.0)F=0.0
C      DL(I,J)=D
C      70 H(I,J)=H(I,J)+D*F
C
C      REFINE ESTIMATES OF HEADS BY JADI METHOD
C
C      ITER=0
C      E=0.0
C      80 ITER=ITER+1
C
C      COLUMN CALCULATIONS
C
C      DO 190 I=1,NC
C      I=I
C      IF(MOD(ISTEP+ITER,2).EQ.1) I=NC-I+1
C      DO 170 J=1,NR
C
C      CALCULATE B AND G ARRAYS
C
C      EVAPOTRANSPIRATION CONTROL
C
C      IF(H(I,J).LT.RD(I,J))GO TO 81
C      RE=RH(I,J)*R(I,J)
C      RB=1.0
C      GO TO 82
C      81 RE=(RH(I,J)-RD(I,J))*R(I,J)
C      RB=0.0
C      82 DD=HQ(I,J)*SF(I,J)/DELTA-Q(I,J)+RE
C      BB=SF(I,J)/DELTA+R(I,J)*RB
C
C      AA=0.0
C      CC=0.0
C      IF(J-1)90,100,90
C      90 AA=T(I,J-1,1)
C      BB=BB+T(I,J-1,1)
C      100 IF(J-NR)110,120,110
C      110 CC=T(I,J,1)
C      BB=BB+T(I,J,1)
C      120 IF(I-1)130,140,130
C      130 DD=DD+H(I-1,J,2)
C      DD=DD+H(I-1,J)*T(I-1,J,2)
C      140 IF(I-NC)150,160,150
C      150 BB=BB+T(I,J,2)
C      DD=DD+H(I,J)*T(I,J,2)
C      160 W=BB-AA*B(I-1)
C      B(I)=CC/W
C      170 G(I)=(DD-AA*G(I-1))/W
C
C      RE-ESTIMATE HEADS
C
C      E=E+ABS(H(I,NR)-G(NR))
C      H(I,NR)=G(NR)
C      N=NR-1
C      180 HA=G(N)-B(N)*H(I,N+1)
C      E=E+ABS(HA-H(I,N))
C      H(I,N)=HA
C      N=N-1
C      IF(N)190,190,180
C      190 CONTINUE
C
C      ROW CALCULATIONS
C
C      DO 300 JJ=1,NR
C      J=JJ
C      IF(MOD(ISTEP+ITER,2).EQ.1) J=NR-J+1
C      DO 280 I=1,NC
C
C      EVAPOTRANSPIRATION CONTROL
C
C      IF(H(I,J).LT.RD(I,J))GO TO 191
C      RE=RH(I,J)*R(I,J)
C      RB=1.0
C      GO TO 192
C      191 RE=(RH(I,J)-RD(I,J))*R(I,J)
C      RB=0.0
C      192 DD=HQ(I,J)*SF(I,J)/DELTA-Q(I,J)+RE
C      BB=SF(I,J)/DELTA+R(I,J)*RB
C
C      AA=0.0
C      CC=0.0
C      IF(J-1)200,210,200
C      200 BB=BB+T(I,J-1,1)
C      DD=DD+H(I,J-1)*T(I,J-1,1)
C      210 IF(J-NR)220,230,220
C      220 DD=DD+H(I,J+1)*T(I,J,2)
C      BB=BB+T(I,J,1)
C      230 IF(I-1)240,250,240
C      240 BB=BB+T(I-1,J,2)
C      AA=T(I-1,J,2)
C      250 IF(I-NC)260,270,260
C      260 BB=BB+T(I,J,2)
C      CC=T(I,J,2)
C      270 W=BB-AA*B(I-1)
C      B(I)=CC/W
C      280 G(I)=(DD-AA*G(I-1))/W
C
C      RE-ESTIMATE HEADS
C
C      E=E+ABS(H(INC,J)-G(INC))
C      H(INC,J)=G(INC)
C      N=NC-1
C      290 HA=G(N)-B(N)*H(N+1,J)
C      E=E+ABS(H(N,J)-HA)
C      H(N,J)=HA
C      N=N-1
C      IF(N)300,300,290
C      300 CONTINUE
C      IF(E.GT.ERROR) GO TO 80
C
C      PRINT RESULTS
C
C      WRITE(OUT,310)TIME,E,ITER
C      FORMAT(6H2TIME=F6.2///,E20.7,15)
C      DO 320 J=1,NR
C      310 WRITE(OUT,330)J,H(I,J),I=1,NC)
C      320 FORMAT(15,5X,10F10.4/(12X)0F10.4)
C      330
C
C      STOP
C      END

```

Figure 53. Basic aquifer simulation program with evapotranspiration conditions

## STORAGE COEFFICIENT CONVERSION

The conversion of the storage coefficient from artesian to water table conditions can be included in the basic aquifer simulation program by the modifications described in this section. Figures 54-57 illustrate these procedures, and figure 58 gives the modified computer program listing.

The basic aquifer simulation program operates when the groundwater system is under strictly artesian conditions in which water level heads remain everywhere above the top of the aquiclude overlying the aquifer. However, because of possible overpumping, excessive interference between wells and well fields, or prolonged periods of drought, water level heads may fall below the top of the aquiclude in parts of the aquifer and, in these areas, the storage coefficient converts from artesian to water table.

Figure 54 illustrates a typical cross section where a portion of an aquifer has converted to water table conditions in the vicinity of a pumped well. Artesian conditions still exist beyond the boundary of the aquifer portion being dewatered. As time progresses, the cone of depression expands, the portion of the aquifer being dewatered becomes larger, and more of the aquifer converts from artesian to water table conditions.

If it is assumed that the aquifer thickness dewatered is small compared with the total thickness of the aquifer then the modified basic aquifer simulation program listing of figure 58 will simulate the storage coefficient conversion process described above.

The first modification made to the basic program is to define additional storage factors according to the following formula

$$SF2_{i,j} = 7.48S_{wt}\Delta x\Delta y \quad (60)$$

where

$SF2_{i,j}$  = storage factor for node located at model coordinates  $i,j$ , in gal/ft

$S_{wt}$  = aquifer storage coefficient for water table conditions for node located at model coordinates  $i,j$ , a fraction

Next an array,  $CH_{i,j}$ , is added to define the elevations of the top of the aquifer as illustrated in figure 54. The DIMENSION statement, the READ statements, and the default value section are expanded to include the additional  $SF2_{i,j}$  and  $CH_{i,j}$  arrays. Finally the STORAGE COEFFICIENT CONTROL sections are added to the COLUMN and ROW calculation sections of the program.

The STORAGE COEFFICIENT CONTROL section (figure 58) operates as follows. The storage factor is first set equal to that for the artesian condition. The first IF statement checks to determine whether the water level at the node is below the top of the aquifer and, if it is, replaces the artesian

storage factor with the water table storage factor. The DD and BB terms are then calculated as usual with the proper storage factor. The second IF statement checks to determine whether the storage coefficient conversion has taken place during that particular time increment. If conversion has taken place, then the storage factors are proportioned as illustrated in figure 56. Since the water table storage factor is already in DD, by virtue of the first IF statement, then that portion given by  $h\phi_{i,j} - CH_{i,j}$  must be subtracted out and replaced by the artesian storage factor.

The data deck setup for an example problem is illustrated in figure 57. A single well is being pumped in the center of an aquifer which has an artesian head 2 feet above the top of the aquifer. The head reference level is assumed to be at zero, the initial heads are set at zero, and the elevation of the top of the aquifer is set at  $-2$  feet. Two values of storage factors are defined, one for the artesian case and the other for the water table case.

The computer results are shown in figure 55. Since there is no closed form solution known to the authors for the drawdown effects for the storage coefficient conversion situation, the computed data are shown in comparison with two theoretical Theis curves, one for strictly artesian conditions and the other for strictly water table conditions. As indicated in figure 55, the computer data follow the artesian curve during the early portion of pumping, then begin to deviate toward the water table curve as dewatering effects begin. As time progresses, more and more of the aquifer converts to water table conditions, and the computed drawdowns fall closer to the water table theoretical curve.

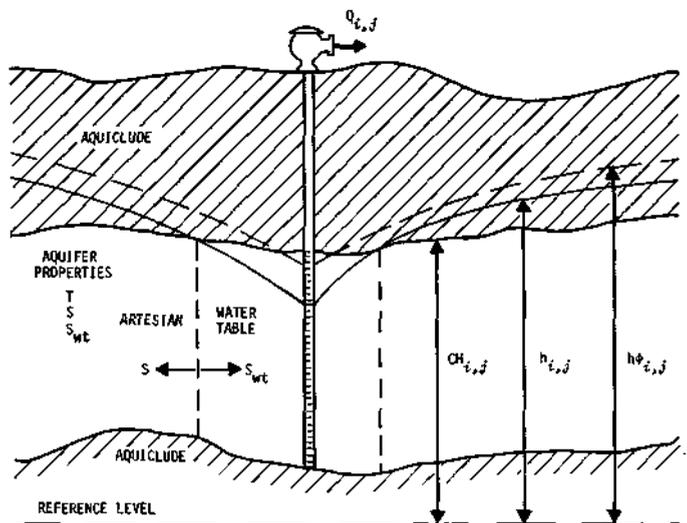


Figure 54. Basic aquifer simulation program parameters with storage coefficient conversion conditions





## WATER TABLE CONDITIONS

Modifications of the basic aquifer simulation program to include water table conditions are described below and are illustrated in figures 59-62. The modified computer program listing is given in figure 63.

Water table conditions are illustrated in figure 59 which shows a well pumping from an aquifer that is unconfined on the top with the water being released from storage by gravity drainage of the interstices in the portion of the aquifer being dewatered. Gravity drainage of the interstices decreases the saturated thickness of the aquifer and therefore the aquifer transmissivity.

In designing the computer model the aquifer is discretized in the usual manner, and values of hydraulic conductivity  $PERM_{i,j}$  are assigned to the individual vector volumes representing the different portions of the aquifer. In addition, values of aquifer bottom elevations  $BOT_{i,j}$  are assigned to the individual node points of the model.

Figure 61 shows a typical aquifer vector volume of the model in which the flow of water is passing through a vector volume that is wedge shaped. The equivalent aquifer transmissivity of the wedge shaped vector volume, between the node points  $i,j$  and  $i + 1,j$ , can be approximated by the following formula (Butler, 1957)

$$T_{i,j,2} = PERM_{i,j,2} \sqrt{(h_{i,j} - BOT_{i,j})(h_{i+1,j} - BOT_{i+1,j})} \quad (61)$$

where

$T_{i,j,2}$  = aquifer transmissivity of the vector volume between  $i,j$  and  $i + 1,j$

$PERM_{i,j,2}$  = hydraulic conductivity of aquifer within the vector volume between  $i,j$  and  $i + 1,j$

Similarly, the equivalent aquifer transmissivity of the vector volume between node points  $i,j$  and  $i,j + 1$  is given by

$$T_{i,j,1} = PERM_{i,j,1} \sqrt{(h_{i,j} - BOT_{i,j})(h_{i,j+1} - BOT_{i,j+1})} \quad (62)$$

Transmissivities calculated with equations 61 and 62 represent geometric means, which are more accurate than values computed as an average between nodes, especially when dealing with steep gradients near pumping centers.

The computer program listing for water table conditions is given in figure 63. As shown by the boxed modifications, two additional arrays are dimensioned, and the READ statements and default value section are modified to accommodate values of hydraulic conductivities and aquifer bottom elevations. Two TRANSMISSIVITY GONTROL sections are included to compute values of transmissivity from equations 61 and 62, after any changes in head have occurred.

The remaining modifications show what to do when the aquifer dries up. Some reasons for an aquifer to dry up would be a well being overpumped, water levels dropping below a rise in the aquifer bottom, or water levels dropping along the edges of an aquifer. The computer program checks to see if heads have fallen below the bottom of the aquifer. If so, those heads are set equal to the bottom elevation

plus 0.01 foot. Thus the aquifer transmissivity always has some positive value, and this allows refilling of the aquifer if the opportunity ever occurs.

The listing of figure 63 includes modifications to handle drying up or filling the aquifer at any node of the computer model, except those nodes where pumpage is taking place. The pumping nodes are excepted, because of the many alternatives available if a well runs dry (e.g., turn the pump off, reduce the pumpage to match the aquifer capabilities, shift pumpage to other wells, or use some combination of these). Additional programming to deal with the pumped well node conditions should be added to the listing of figure 63 by the reader for his particular situation. If the job is set up properly, the program will sense when a pumping well runs dry and will cease to print out results; that is, the computed heads will not converge. (It may be possible to save execution time by putting a limit on the program iterations, say 20, to terminate processing.)

The job setup for water table conditions is similar to that for the basic aquifer simulation program. The hydraulic conductivities, aquifer bottom elevations, water table storage factors (see equation 60), heads, and net withdrawal rates of the aquifer are discretized and assigned to the individual vector volumes of the computer model. Appropriate aquifer properties are entered on the default value card and any differing values on the node cards.

Figure 60 illustrates a theoretical versus computer-simulated distance-drawdown graph for a single well pumping in a homogeneous isotropic aquifer under water table conditions. The input data are given in figure 62. Two sets of data are shown in figure 60. The first was taken directly from the computer output and represents the drawdown curves for water table conditions. The second set was derived by adjusting the computer data for the effects of aquifer thinning according to Jacob's (1944) correction factor  $s^2/2m$  in which  $s$  is the observed drawdown and  $m$  is the initial aquifer thickness. The corrected drawdowns can then be compared with the theoretical Theis distance-drawdown curve.

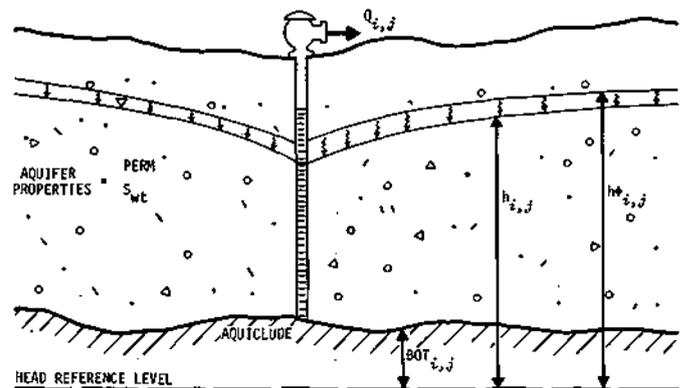


Figure 59. Basic aquifer simulation program parameters with water table conditions

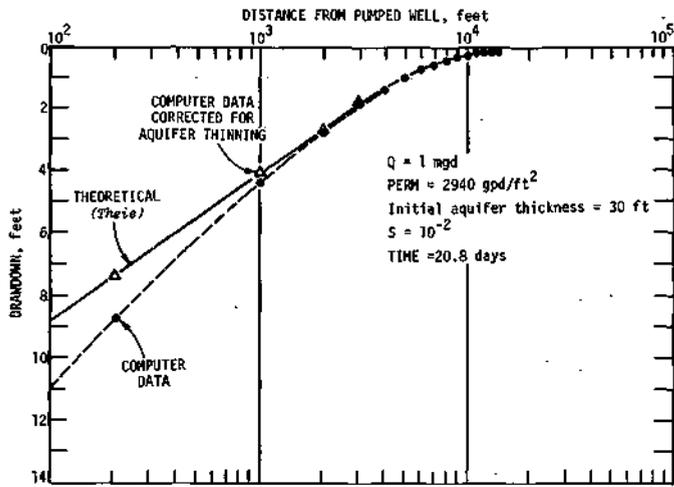


Figure 60. Comparison of computed and theoretical distance-drawdown curves for water table conditions

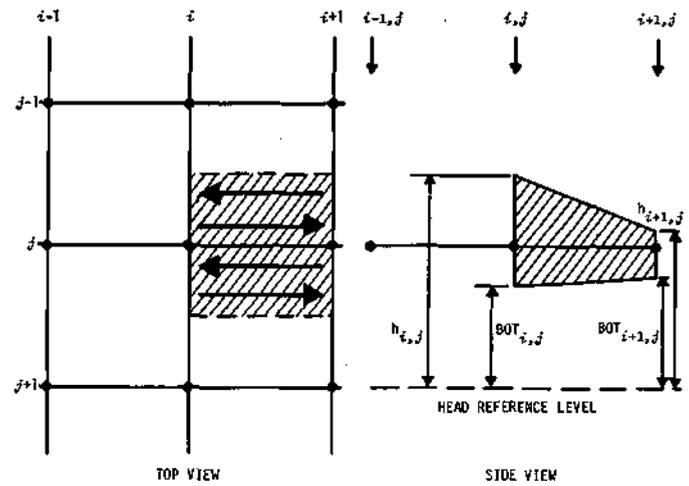


Figure 61. Typical vector volume of aquifer model for water table conditions

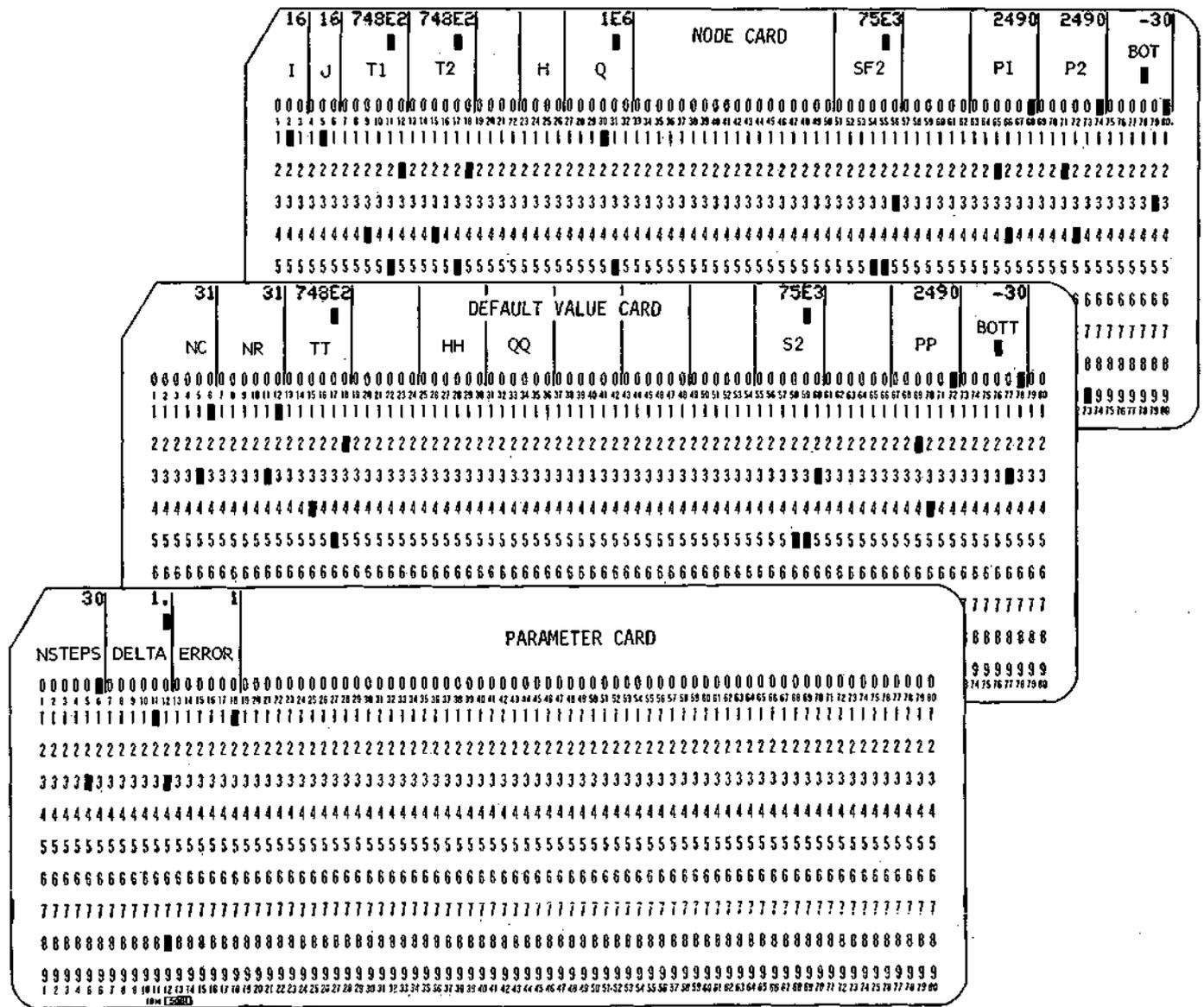


Figure 62. Sample input data for water table conditions

```

C ILLINOIS STATE WATER SURVEY
C BASIC AQUIFER SIMULATION PROGRAM
C WITH WATER-TABLE CONDITIONS
C
C DEFINITION OF VARIABLES
C HD(I,J)-----HEADS AT START OF TIME
C INCREMENT (I,J)
C HI(I,J)-----HEADS AT END OF TIME
C INCREMENT (FT)
C SF2(I,J)-----STORAGE FACTOR FOR WATER-
C TABLE CONDITIONS (GAL/FT)
C Q(I,J)-----CONSTANT WITHDRAWAL
C RATES (GPD)
C T(I,J,1)-----AQUIFER TRANSMISSIVITY
C BETWEEN I,J AND I+1,J
C (GAL/DAY/FT)
C T(I,J,2)-----AQUIFER TRANSMISSIVITY
C BETWEEN I,J AND I+1,J
C (GAL/DAY/FT)
C AA,BB,CC,DD-COEFFICIENTS IN WATER
C BALANCE EQUATIONS
C NR-----NO. OF ROWS IN MODEL
C NC-----NO. OF COLUMNS IN MODEL
C NSTEPS-----NO. OF TIME INCREMENTS
C DELTA-----TIME INCREMENTS (DAYS)
C HH,52,QQ,TT-DEFAULT VALUES
C I-----MODEL COLUMN NUMBER
C J-----MODEL ROW NUMBER
C PERM(I,J,1)-AQUIFER PERMEABILITY
C BETWEEN I, J AND I+1, J+1
C (GAL/DAY/SQ FT)
C PERM(I,J,2)-AQUIFER PERMEABILITY
C BETWEEN I, J AND I+1, J
C (GAL/DAY/SQ FT)
C BOT(I,J)-----ELEVATION OF BOTTOM OF
C AQUIFER (FT)
C
C DIMENSION H(50,50),HD(50,50),
C SF2(50,50),Q(50,50),T(50,50,2),
C B(150),G(150),DL(50,50)
C 3,PERM(50,50,2),BOT(50,50)
C
C TURN OFF UNDERFLOW TRAP
C CALL ERRSET(208,256,-1,1)
C
C DEFINE INPUT AND OUTPUT DEVICE NUMBERS
C
C INTEGER OUT
C IN=5
C OUT=6
C
C READ PARAMETER CARD AND
C DEFAULT VALUE CARD
C
C READ(IN,10,NSTEPS,DELTA,ERROR,
C INC,NR,TT,HH,QQ,52,PP,BOTT
C 10 FORMAT(16,2F6.0,2I6,0,2F6.0,6X,2F6.0
C 1,18X,1F6.0,6X,2F6.0)
C
C FILL ARRAYS WITH DEFAULT VALUES
C
C DO 20 I=1,NC
C DO 20 J=1,NR
C SF2(I,J)=52
C PERM(I,J,1)=PP
C PERM(I,J,2)=PP
C BOT(I,J)=BOTT
C T(I,J,1)=TT
C T(I,J,2)=TT
C H(I,J)=HH
C HD(I,J)=HH
C Q(I,J)=QQ
C
C 20
C
C READ NODE CARDS
C
C 30 READ(5,40,END=50)I,J,T(I,J,1),
C T(I,J,2),H(I,J),Q(I,J),
C 40 FORMAT(2I3,2F6.0,6X,1F4.0,1F6.0,18X,1F6.0,6X,3F6.0)
C GO TO 30
C
C START OF SIMULATION.
C
C 50 TIME=0
C DO 320 ISTEP=1,NSTEPS
C TIME=TIME+DELTA
C
C PREDICT HEADS FOR NEXT
C TIME INCREMENT
C
C DO 70 J=1,NC
C DO 70 I=1,NR
C B=H(I,J)-HD(I,J)
C HD(I,J)=H(I,J)
C F=1.0
C IF(DL(I,J).EQ.0.0)GO TO 60
C IF(ISTEP,GT.2)F=D/DL(I,J)
C IF(F,GT.5)F=5.0
C IF(F,LT.0.0)F=0.0
C 60 DL(I,J)=D
C H(I,J)=H(I,J)+F
C 70 IF(H(I,J).LE.BOT(I,J))H(I,J)=BOT(I,J)+0.01
C
C REFINE ESTIMATES OF HEADS BY JACOBI METHOD
C
C ITER=0
C E=0.0
C 80 ITER=ITER+1
C
C TRANSMISSIVITY CONTROL
C
C DO 83 J=1,NC
C DO 83 I=1,NR
C 1F(I,LT,NC)T(I,J,2)=PERM(I,J,2)*SQRT((H(I,J)-
C 83 1BOT(I,J))*H(I+1,J)-BOT(I+1,J)))
C 1F(J,LT,NR)T(I,J,1)=PERM(I,J,1)*SQRT((H(I,J)-
C 1BOT(I,J))*H(I,J+1)-BOT(I,J+1)))
C
C COLUMN CALCULATIONS
C
C DO 190 I=1,NC
C I=I
C 1F(MOD(ISTEP+ITER,2).EQ.1) I=NC-I+1
C DO 170 J=1,NR
C
C CALCULATE B AND G ARRAYS
C
C BB=SF2(I,J)/DELTA
C DD=HD(I,J)*SF2(I,J)/DELTA-Q(I,J)
C AA=0.0
C CC=0.0
C IF(J-1)90,100,90
C 90 AA=TT(I,J-1,1)
C BB=BB+T(I,J-1,1)
C 100 IF(J-NR)110,120,110
C 110 CC=TT(I,J,1)
C BB=BB+T(I,J,1)
C 120 IF(I-1)130,140,130
C BB=BB+T(I-1,J,2)
C DD=DD+H(I-1,J)*T(I-1,J,2)
C 140 IF(I-NC)150,160,150
C BB=BB+T(I,J,2)
C DD=DD+H(I+1,J)*T(I,J,2)
C 160 N=BB-AA*B(I-1)
C B(I)=CC/W
C 170 G(I)=(DD-AA*G(I-1))/W
C
C RE-ESTIMATE HEADS
C
C E=E+ABS(H(I,NR))-G(NR))
C H(I,NR)=G(NR)
C NR=NR-1
C 180 HA=G(NI)-B(NI)*H(NI,N+1)
C E=E+ABS(HA-H(NI,N))
C H(I,N)=HA
C N=N-1
C IF(N,GT.0)GO TO 180
C DO 190 N=1,NR
C 1F(H(N,1).GT.BOT(N,1))GO TO 190
C E=E+BOT(N,1)+0.01-H(N,1)
C H(I,N)=BOT(N,1)+0.01
C 190 CONTINUE
C
C TRANSMISSIVITY CONTROL
C
C DO 193 J=1,NR
C DO 193 I=1,NC
C 1F(I,LT,NC)T(I,J,2)=PERM(I,J,2)*SQRT((H(I,J)-
C 1BOT(I,J))*H(I+1,J)-BOT(I+1,J)))
C 193 1F(J,LT,NR)T(I,J,1)=PERM(I,J,1)*SQRT((H(I,J)-
C 1BOT(I,J))*H(I,J+1)-BOT(I,J+1)))
C
C ROW CALCULATIONS
C
C DO 300 JJ=1,NR
C J=JJ
C 1F(MOD(ISTEP+ITER,2).EQ.1) J=NR-J+1
C DO 280 I=1,NC
C BB=SF2(I,J)/DELTA
C DD=HD(I,J)*SF2(I,J)/DELTA-Q(I,J)
C AA=0.0
C CC=0.0
C IF(J-1)200,210,200
C 200 BB=BB+T(I,J-1,1)
C DD=DD+H(I,J-1)*T(I,J-1,1)
C 210 IF(J-NR)220,230,220
C DD=DD+H(I,J+1)*T(I,J,1)
C BB=BB+T(I,J,1)
C 230 IF(I-1)240,250,240
C BB=BB+T(I-1,J,2)
C 240 AA=TT(I-1,J,2)
C 250 IF(I-NC)260,270,260
C BB=BB+T(I,J,2)
C CC=TT(I,J,2)
C 260 N=BB-AA*B(I-1)
C B(I)=CC/W
C 270 G(I)=(DD-AA*G(I-1))/W
C 280
C RE-ESTIMATE HEADS
C
C E=E+ABS(H(NC,J)-G(NC))
C H(NC,J)=G(NC)
C NC=NC-1
C 290 HA=G(NI)-B(NI)*H(NI,J)
C E=E+ABS(H(NI,J)-HA)
C H(N,J)=HA
C N=N-1
C IF(N,GT.0)GO TO 290
C DO 300 N=1,NC
C 1F(H(N,1).GT.BOT(N,1))GO TO 300
C E=E+BOT(N,1)+0.01-H(N,1)
C H(N,J)=BOT(N,1)+0.01
C 300 CONTINUE
C
C PRINT RESULTS
C 1F(E,GT.ERROR) GO TO 80
C
C WRITE(OUT,330)TIME,E,ITER
C 310 FORMAT(6H2TIME=,F6.2///,E20.7,15)
C DELTA=DELTA*.2
C DO 320 J=1,NR
C 320 WRITE(OUT,330)J,(H(I,J),I=1,NC)
C 330 FORMAT(15,5X,10F10.4/(12X10F10.4))
C
C STOP
C END

```

Figure 63. Basic aquifer simulation program listing with water table conditions

### THREE-DIMENSIONAL PROBLEM

The type of three-dimensional problem for which a computer program is given is illustrated in figures 64-66. The modified computer program listing is given in figure 67.

Figure 64 shows an aquifer system made up of four 'stacks,' each composed of an aquifer and an associated overlying confining bed. The stacks are numbered in order from top to bottom. Above the uppermost stack is a source bed which is assumed to have a constant head. It is also assumed that the leakage through a confining bed is vertical and proportional to the difference between heads in the adjacent aquifers, and that storage in the confining beds is negligible.

The program listing of figure 67 will simulate the effects of pumping from the multiple aquifer system described above up to a maximum of four stacks. The program listing of figure 67 is a modification of the basic aquifer simulation program with leaky artesian conditions as was given in figure 41. The main modifications include increasing the size of the arrays by one more dimension to include the additional vertical flow components. Further, the method of allocating leakage rates to an aquifer is expanded to include simultaneous leakage from above and below. Thus computing the G and B arrays involves including an additional BB and DD statement (boxed in figure 67) to take into account leakage from below. (Leakage from above was already included in the program for ordinary leaky artesian conditions.)

The job setup for the three-dimensional problem can be illustrated by an example. Hantush (1967) made a theoretical study of the effects of pumping from one aquifer of a multiple layered system composed of two infinite homogeneous and isotropic aquifers separated by a single leaky

confining bed. The two aquifers are otherwise assumed to be bounded by aquicludes above the uppermost aquifer and below the bottom aquifer.

For this example it is assumed that both aquifers have identical transmissivities of 10,000 gpd/ft and storage coefficients of 0.000401.

The separating confining bed has a vertical hydraulic conductivity of 0.04 gpd/ft<sup>2</sup> and a thickness of 10 feet. It is also assumed that pumpage will take place at a constant rate of 100,000 gpd from the upper aquifer (stack 1) only.

If a square grid interval of 500 feet is assumed, the storage factors for both aquifers can be computed from equation 44 as

$$\begin{aligned} SF1_{i,j} &= 7.48 S \Delta x \Delta y \\ SF1_{i,j} &= 7.48 (4.01 \times 10^{-4})(5 \times 10^2)(5 \times 10^2) \\ SF1_{i,j} &= 750 \text{ gal/ft} \end{aligned}$$

Since the upper aquifer (stack 1) has an aquiclude instead of a leaky confining bed above it, the recharge factors for the upper stack are set equal to zero. The recharge factors for the separating confining bed above the bottom aquifer (stack 2) is computed from equation 52 as

$$\begin{aligned} R_{i,j} &= (P'/m') \Delta x \Delta y \\ R_{i,j} &= (4 \times 10^{-2}/10)(5 \times 10^2)(5 \times 10^2) \\ R_{i,j} &= 1000 \text{ gpd/ft} \end{aligned}$$

The computer input data deck for the above problem is shown in figure 66. There is one default value card for each stack. Selected computer output data are shown in figure 65 in comparison with the theoretical curves derived by Hantush (1967). The agreement between the computer simulation and that from theory is good.

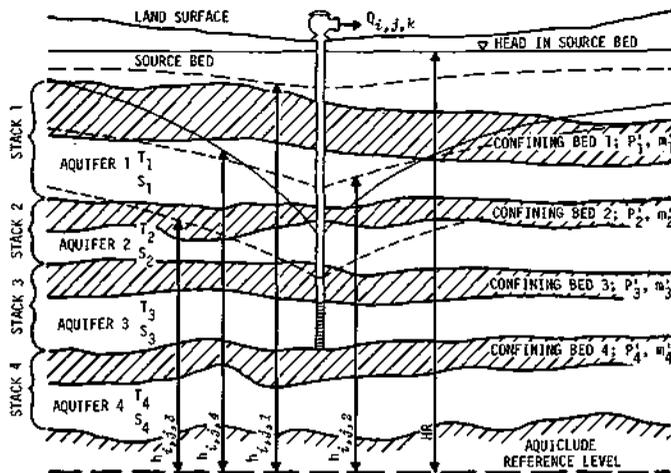


Figure 64. Aquifer simulation program parameters in three dimensions

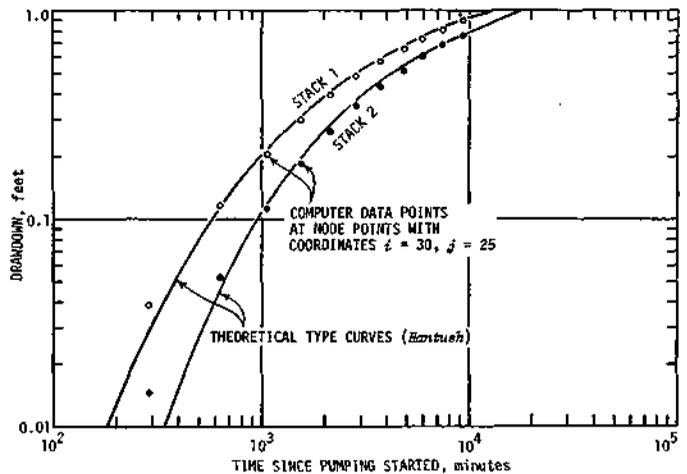


Figure 65. Comparisons of theoretical and computer simulated curves for three-dimensional problem

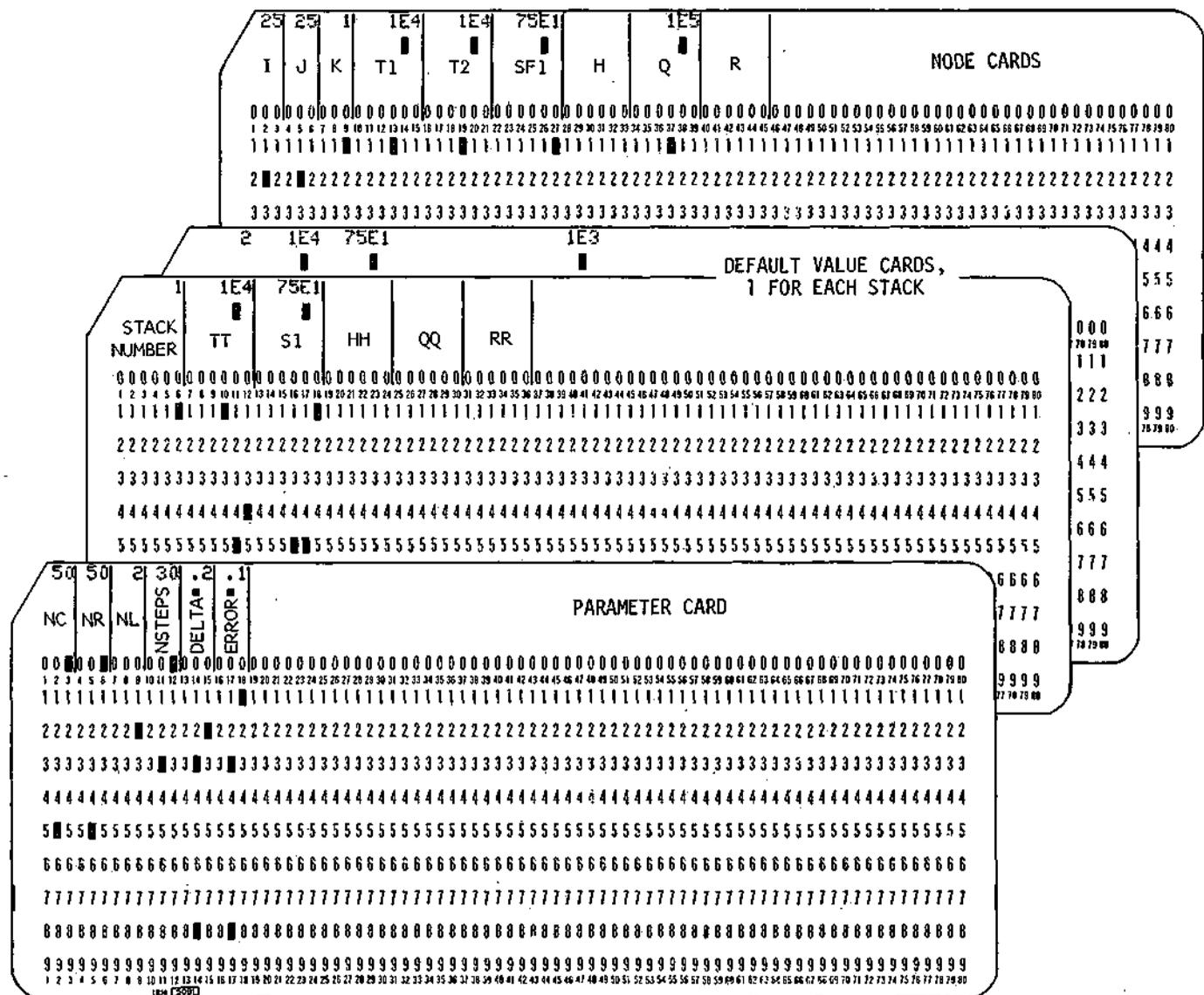


Figure 66. Input data deck for three-dimensional problem

```

C ILLINOIS STATE WATER SURVEY
C AQUIFER SIMULATION PROGRAM FOR
C THREE DIMENSIONAL PROBLEM
C
C DEFINITION OF VARIABLES
C
C HD(I,J,K)-----HEADS AT START OF TIME
C INCREMENT
C
C HI(I,J,K)-----HEADS AT END OF TIME
C INCREMENT (FT)
C
C SF1(I,J,K)-----STORAGE FACTORS FOR
C ARTESIAN CONDITIONS (GAL)
C
C Q(I,J,K)-----CONSTANT WITHDRAWAL
C RATES (GPD)
C
C T(I,J,1,K)-----AQUIFER TRANSMISSIVITY
C BETWEEN I,J AND I,J+1
C (GAL/DAY/FT)
C
C T(I,J,2,K)-----AQUIFER TRANSMISSIVITY
C BETWEEN I,J AND I+1,J
C (GAL/DAY/FT)
C
C R(I,J,K)-----RECHARGE FACTOR
C FOR CONFINING BED ABOVE
C LAYER K (GPD/FT)
C
C AA,BB,CC,DD-----COEFFICIENTS IN WATER
C BALANCE EQUATIONS
C
C NC-----NO. OF COLUMNS IN MODEL
C
C NR-----NO. OF ROWS IN MODEL
C
C NL-----NO. OF STACKS IN MODEL
C
C NSTEPS-----NO. OF TIME INCREMENTS
C
C DELTA-----TIME INCREMENTS (DAYS)
C
C HH,S1,QQ,TT-----DEFAULT VALUES
C
C I-----MODEL COLUMN NUMBER
C
C J-----MODEL ROW NUMBER
C
C K-----MODEL STACK NUMBER
C
C
C DIMENSION H(50,50,4),HD(50,50,4),
C ISF1(50,50,4),Q(50,50,4),T(50,50,2,4),
C 2B(50,4),G(50,4),R(50,50,4),DL(50,50,4),
C 3TT(4),S1(4),HH(4),QQ(4),RR(4)
C
C TURN OFF UNDERFLOW TRAP
C
C CALL ERRSET(208,256,-1,1)
C
C DEFINE INPUT AND OUTPUT DEVICE NUMBERS
C
C INTEGER OUT
C IN=5
C OUT=6
C
C READ PARAMETER AND DEFAULT
C VALUE CARDS
C
C READ(IN,10)NC,NR,NL,NSTEPS,DELTA,
C IERROR,(K,TT(K),S1(K),HH(K),QQ(K),RR(K),KK=1,NL)
C FORMAT(4I3,2F3.0/,1I6,5F6.0)
C
C 10 FILL ARRAYS WITH DEFAULT VALUES
C
C DO 20 K=1,NL
C DO 20 I=1,NC
C DO 20 J=1,NR
C T(I,J,1,K)=TT(K)
C T(I,J,2,K)=TT(K)
C SF1(I,J,K)=S1(K)
C HI(I,J,K)=HH(K)
C HD(I,J,K)=HD(K)
C R(I,J,K)=RR(K)
C Q(I,J,K)=QQ(K)
C
C 20 READ NODE CARDS
C
C 30 READ(IN,40,END=50)I,J,K,T(I,J,1,K),
C T(I,J,2,K),SF1(I,J,K),H(I,J,K),
C 2Q(I,J,K),R(I,J,K)
C 40 FORMAT(3I3,6F6.0)
C HD(I,J,K)=HI(I,J,K)
C GO TO 30
C
C START OF SIMULATION
C
C 50 TIME=0.0
C DO 280 ISTEP=1,NSTEPS
C TIME=TIME+DELTA
C
C PREDICT HEADS FOR NEXT
C TIME INCREMENT
C
C DO 51 K=1,NL
C DO 51 I=1,NC
C DO 51 J=1,NR
C D=HI(I,J,K)-HD(I,J,K)
C HD(I,J,K)=H(I,J,K)
C F=1.0
C IF(DL(I,J,K).EQ.0.0) GO TO 500
C IF(ISTEP.GT.2)F=D/DL(I,J,K)
C IF(F.GT.5)F=5.0
C IF(F.LT.0.0)F=0.0
C DL(I,J,K)=D
C 51 H(I,J,K)=H(I,J,K)+D*F
C
C REFINE ESTIMATES OF HEADS
C BY IADI METHOD
C
C ITER=0
C 52 E=0.0
C ITER=ITER+1

```

```

C STACK AND COLUMN CALCULATIONS
C
C DO 160 K=1,NL
C DO 160 I=1,NC
C DO 140 J=1,NR
C
C CALCULATE B AND G ARRAYS
C
C HR=H(I,J,K-1)
C IF(K.EQ.1)HR=0.0
C 59 DD=HD(I,J,K)+SF1(I,J,K)/DELTA-Q(I,J,K)+
C IR(I,J,K)*HR
C BB=SF1(I,J,K)/DELTA+R(I,J,K)
C AA=0.0
C CC=0.0
C IF(K-NL.EQ.0.0)GO TO 55
C BB=BB+R(I,J,K+1)
C DD=DD+R(I,J,K+1)*H(I,J,K+1)
C
C 55 IF(J-1)60,70,80
C 60 AA=TT(I,J-1,K)
C BB=BB+T(I,J-1,K)
C 70 IF(J-NR)80,90,80
C 80 CC=TT(I,J,K)
C BB=BB+T(I,J,K)
C 90 IF(I-1)100,110,100
C 100 BB=BB+T(I-1,J,2,K)
C DD=DD+H(I-1,J,K)*T(I-1,J,2,K)
C 110 IF(I-NC)120,130,120
C 120 BB=BB+T(I,J,2,K)
C DD=DD+H(I,J,K)*T(I,J,2,K)
C 130 W=BB-AA*B(J-1,K)
C B(I,K)=CC/W
C 140 G(I,K)=(DD-AA*G(J-1,K))/W
C
C RE-ESTIMATE HEADS
C
C E=E+ABS(H(I,NR,K)-G(INR,K))
C H(I,NR,K)=G(INR,K)
C N=NR-1
C 150 HA=G(N,K)-B(N,K)*H(I,N+1,K)
C E=E+ABS(HA-H(I,N,K))
C H(I,N,K)=HA
C N=N-1
C IF(N)160,160,150
C CONTINUE
C
C STACK AND ROW CALCULATIONS
C
C DO 270 K=1,NL
C DO 270 J=1,NR
C DO 250 I=1,NC
C HR=H(I,J,K-1)
C 169 DD=HD(I,J,K)+SF1(I,J,K)/DELTA-Q(I,J,K)+
C IR(I,J,K)*HR
C AA=0.0
C CC=0.0
C BB=SF1(I,J,K)/DELTA+R(I,J,K)
C IF(K-NL.EQ.0.0)GO TO 165
C BB=BB+R(I,J,K+1)
C DD=DD+R(I,J,K+1)*H(I,J,K+1)
C
C 165 IF(J-1)170,180,170
C 170 BB=BB+T(I,J-1,K)
C DD=DD+H(I,J-1,K)*T(I,J-1,K)
C 180 IF(J-NR)190,200,190
C 190 DD=DD+H(I,J,K)*T(I,J,K)
C BB=BB+T(I,J,K)
C 200 IF(I-1)210,220,210
C 210 BB=BB+T(I-1,J,2,K)
C AA=TT(I-1,J,2,K)
C 220 IF(I-NC)230,240,230
C 230 BB=BB+T(I,J,2,K)
C CC=TT(I,J,2,K)
C 240 W=BB-AA*B(I-1,K)
C B(I,K)=CC/W
C 250 G(I,K)=(DD-AA*G(I-1,K))/W
C
C RE-ESTIMATE HEADS
C
C E=E+ABS(H(INC,J,K)-G(INC,K))
C H(INC,J,K)=G(INC,K)
C N=NC-1
C 260 HA=G(N,K)-B(N,K)*H(N+1,J,K)
C E=E+ABS(HA-H(N,J,K))
C H(N,J,K)=HA
C N=N-1
C IF(N)270,270,260
C 270 CONTINUE
C IF(E.GT.ERROR)GO TO 52
C
C PRINT RESULTS
C
C WRITE(OUT,300)TIME,E,ITER
C 300 FORMAT(6H1TIME=,F6.2//,E20.7,15)
C DELTA=DELTA*1.2
C DO 310 K=1,NL
C WRITE(OUT,305)K
C 305 FORMAT('OSTACK',12,/)
C DO 310 J=1,NR
C 310 WRITE(OUT,320)J,(H(I,J,K),I=1,NC)
C 320 FORMAT(15,5X,10F10.4/(12X,10F10.4))
C 280 CONTINUE
C STOP
C END

```

Figure 67. Aquifer simulation program listing for three-dimensional problem

## COMPOSITE AQUIFER SIMULATION PROGRAM

Most often an analysis of cause and effect relationships in a groundwater reservoir involves simulating several aquifer conditions simultaneously. Such an aquifer situation is depicted in figure 68 and an example problem is illustrated in figures 69-71. The modified basic aquifer simulation program listing is given in figure 72.

The program listing of figure 72 is made up of selected individual basic aquifer simulation program options, previously described, so that it can simultaneously simulate the combination of aquifer conditions illustrated in figure 68. The only modification not previously described is the change in the method of calculating aquifer transmissivity values. When water levels are below the top of the aquifer, aquifer transmissivity values are computed as a function of the difference between the water level and the bottom of the aquifer. Where the water levels rise above the top of the aquifer, transmissivity values are computed as a function of the total thickness of the aquifer. This is accomplished in the computer program by comparing elevations of the top of the aquifer with those of the water level heads and selecting the smaller of the two to be used in transmissivity computations.

The job setup is made up of a combination of the setups previously described for the individual aquifer system conditions. A hypothetical aquifer situation shown in figure 70 illustrates the use of the composite program.

As indicated in figure 70b, a single well is pumping in the center of a finite aquifer near a river. Initial heads in both the aquifer and river are set equal to zero, the head reference level. It is assumed that the well is to be pumped at a constant rate of 1 mgd for 30 days and then increased to a rate of 2 mgd for an additional 30 days. The river is 100 feet wide and has streambed properties as indicated in figure 70a. The aquifer is being uniformly recharged at a constant rate of 0.005 gpd/ft<sup>2</sup> (140,000 gpd/mi<sup>2</sup>), and evapotranspiration takes place at a rate equal to the recharge rate when water levels are at the land surface. Evapotranspiration effects are assumed to cease when water levels fall 1 foot below the land surface or at an elevation of -1 foot. Overlying the aquifer is a confining layer with hydroau-

lic properties as given in figure 70a. Other aquifer properties also are listed in figure 70.

A square ( $x = y$ ) finite difference grid interval of 1000 feet is assumed, and the computer model then is 9 columns by 31 rows in size. Data from figure 70 indicate that the coordinates of the pumped well are  $i = 5, j = 16$ , and that the elevations of the top and bottom of the aquifer are -3 and -30 feet, respectively.

To change the pumping rate at 30 days requires first assuming that the 30 days can be simulated in five time increments and then computing an initial time increment DELTA from equation 46 and table 1 as

$$\begin{aligned} \text{DELTA} &= \text{TIME}/\delta = 30/7.44 \\ \text{DELTA} &= 4.032 \text{ days} \end{aligned}$$

The recharge factors for the streambed of the river are, by equation 57

$$\begin{aligned} R_{i,j} &= (P'/m') A_s = (0.75/1) 1000 \times 100 \\ R_{i,j} &= 75,000 \text{ gpd/ft} \end{aligned}$$

where  $A_s$  is the area of the streambed simulated at each node discretized into 100-foot widths by 1000-foot lengths along row 17 of the model. The elevations of the river surface and the bottom of the streambed are 0 and -3 feet, respectively (figure 70a).

The recharge factors for the evapotranspiration conditions are, by equation 59

$$\begin{aligned} R_{i,j} &= Q_{et(max)}/(RH_{i,j} - RD_{i,j}) \\ &= 5 \times 10^{-3} \times 1000 \times 1000/[0 - (-1)] \\ R_{i,j} &= 5000 \text{ gpd/ft} \end{aligned}$$

where  $Q_{et(max)}$  is computed on a per node basis. Since the recharge rate initially equals the rate of evapotranspiration, by the principle of superposition previously explained the  $Q$  array is set equal to zero.

The data deck setup for the above hypothetical test model is given in figure 71. A sample time-drawdown curve for an observation point located at grid coordinates  $i = 5, j = 18$  is given in figure 69.

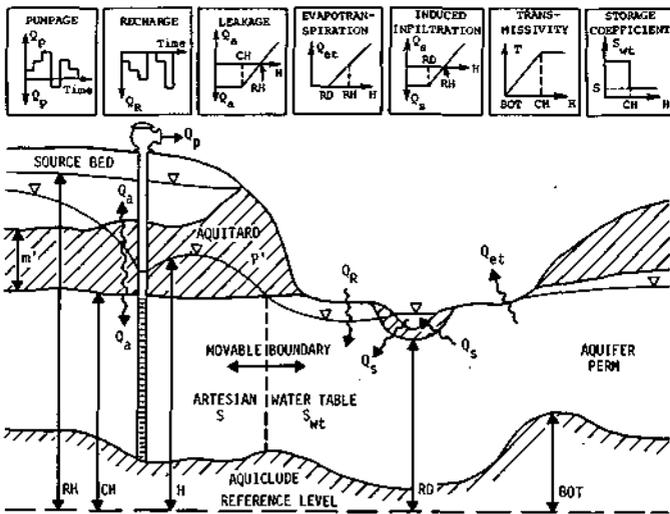


Figure 68. Combined aquifer simulation program parameters

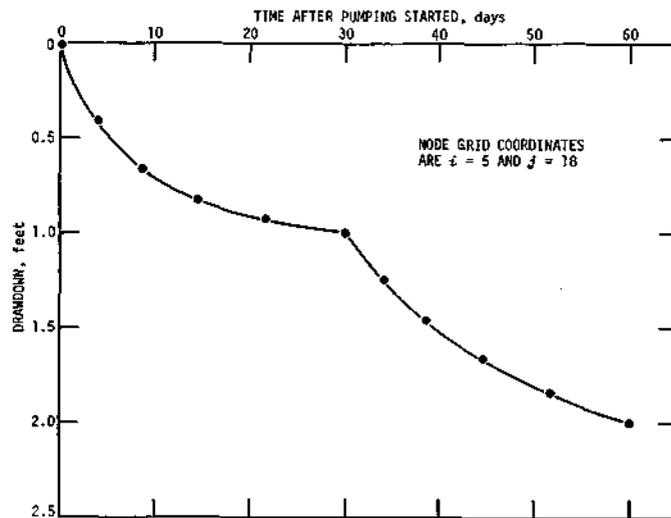


Figure 69. Time-drawdown curve for sample problem

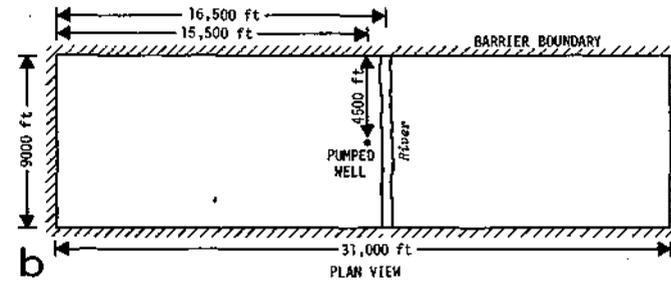
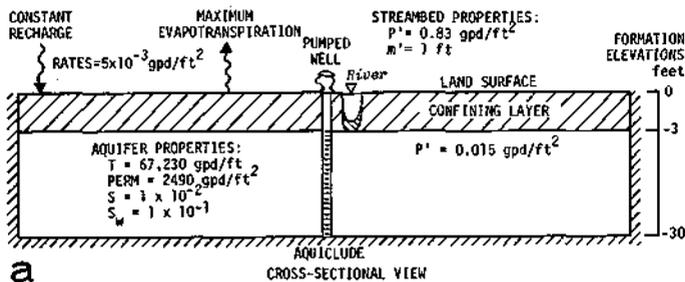


Figure 70. Example problem for combined aquifer simulation program

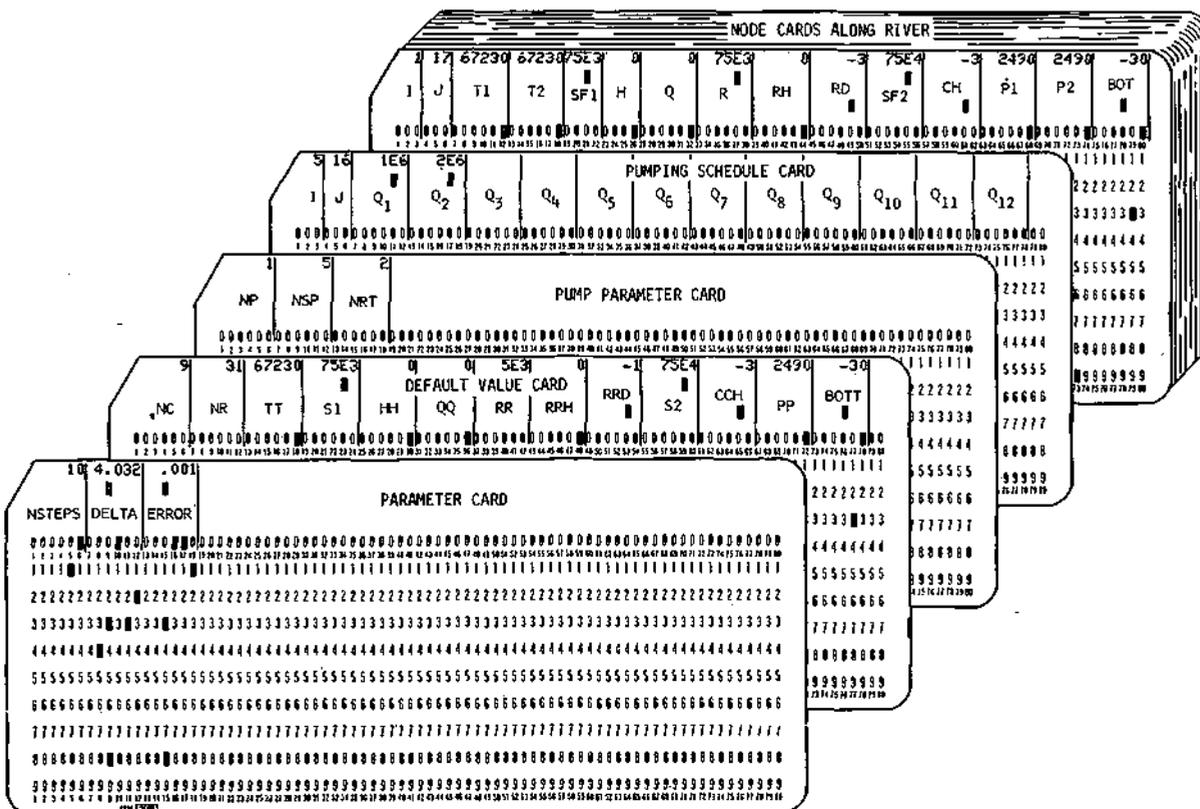


Figure 71. Sample data deck setup for composite aquifer simulation program

```

C ILLINOIS STATE WATER SURVEY
C BASIC AQUIFER SIMULATION PROGRAM WITH
C LEAKY ARTESIAN, INDUCED INFILTRATION,
C EVAPOTRANSPIRATION, ARTESIAN TO WATER-
C TABLE STORAGE COEFFICIENT CONVERSION,
C WATER TABLE CONDITIONS, AND VARIABLE NET
C WITHDRAWAL RATES
C
C DEFINITION OF VARIABLES
C
C HD(I,J)-----HEADS AT START OF TIME
C INCREMENT (I,J)
C H(I,J)-----HEADS AT END OF TIME
C INCREMENT (FT)
C SF1(I,J)-----STORAGE FACTOR FOR
C ARTESIAN CONDITIONS (GAL)
C SF2(I,J)-----STORAGE FACTOR FOR WATER-
C TABLE CONDITIONS (GAL/FT)
C Q(I,J)-----CONSTANT WITHDRAWAL
C RATES (GPD)
C PERM(I,J,1)-HYDRAULIC CONDUCTIVITY OF
C AQUIFER BETWEEN I,J AND I,J+1
C (GAL/DAY/SQ FT)
C PERM(I,J,2)-HYDRAULIC CONDUCTIVITY OF
C AQUIFER BETWEEN I,J, AND I+1,J
C (GAL/DAY/SQ FT)
C T(I,J,1)-----AQUIFER TRANSMISSIVITY
C BETWEEN I,J AND I,J+1
C (GAL/DAY*FT)
C T(I,J,2)-----AQUIFER TRANSMISSIVITY
C BETWEEN I,J AND I+1,J
C (GAL/DAY*FT)
C BOT(I,J)-----ELEVATION OF BOTTOM OF
C AQUIFER (FT)
C R(I,J)-----RECHARGE FACTOR
C (GAL/DAY/FT)
C RH(I,J)-----ELEVATION OF LAND OR
C STREAM SURFACE (FT)
C RD(I,J)-----ELEVATION OF BOTTOM OF
C STREAMBED OR ELEVATION
C BELOW WHICH EVAPOTRANS-
C PIRATION CEASES (FT)
C CH(I,J)-----ELEVATION OF TOP OF
C AQUIFER (FT)
C (GAL/DAY*FT)
C B-----PEACEMAN-RACHFORD
C B ARRAY
C G-----PEACEMAN-RACHFORD
C G ARRAY
C AA,BB,CC,DD-COEFFICIENTS IN WATER
C BALANCE EQUATIONS
C NR-----NO. OF ROWS IN MODEL
C NC-----NO. OF COLUMNS IN MODEL
C NSTEPS-----NO. OF TIME INCREMENTS
C DELTA-----TIME INCREMENTS (DAYS)
C HH,S1,QQ,TT-EXAMPLE MODEL
C DEFAULT VALUES
C I-----MODEL COLUMN NUMBER
C J-----MODEL ROW NUMBER
C IP-----I COORDINATE OF PUMP
C JP-----J COORDINATE OF PUMP
C P(K,KC)-----PUMPING RATE AT WELL
C K AT RATE KC (GPD)
C NP-----NUMBER OF PUMPS
C NSP-----NUMBER OF TIME INCREMENTS
C PER PUMPAGE CHANGE
C NRT-----NUMBER OF RATES IN
C PUMPING SCHEDULE
C
C DIMENSION H(50,50),HD(50,50),
C 1SF1(50,50),Q(50,50),T(50,50,2),
C 2B(50),G(50),R(50,50),DL(50,50)
C 3,IP(100),JP(100),P(100,12)
C 4,PERM(50,50,2),BDT(50,50),SF2(50,50)
C 5,RH(50,50),RD(50,50),CH(50,50)
C
C TURN OFF UNDERFLOW TRAP
C
C CALL ERRSET(208,256,-1,1)
C
C DEFINE INPUT AND OUTPUT DEVICE NUMBERS
C
C INTEGER OUT
C IN=5
C OUT=6
C
C READ PARAMETER CARD AND
C DEFAULT VALUE CARD
C
C READ(IN,10)NSTEPS,DELTA,ERROR,
C INC,NR,TT,S1,HH,QQ,RR,RRH,RRD,S2,CCH,PP,BOTT
C FORMAT(16,2F6.0/2I6,11F6.0)
C
C READ PUMP PARAMETER CARD
C
C READ(IN,15) NP,NSP,NRT
C FORMAT(3I6)
C
C READ PUMPING SCHEDULES
C
C DO 16 I=1,NP
C READ(IN,17) IP(I),JP(I),
C I(P(I),K),K=1,NRT)
C FORMAT(2I3,12F6.0)
C
C FILL ARRAYS WITH DEFAULT VALUES
C
C DO 20 I=1,NC
C DO 20 J=1,NR
C T(I,J,1)=TT
C T(I,J,2)=TT
C PERM(I,J,1)=PP
C PERM(I,J,2)=PP
C SF1(I,J)=S1
C SF2(I,J)=S2
C H(I,J)=HH
C HD(I,J)=HH
C R(I,J)=RR
C RH(I,J)=RRH
C RD(I,J)=RRD
C CH(I,J)=CCH
C BOT(I,J)=BOTT
C Q(I,J)=QQ
C
C READ NODE CARDS
C
C READ(IN,40,END=50)I,J,T(I,J,1), T(I,J,2),
C 1SF1(I,J),H(I,J),Q(I,J),R(I,J),RH(I,J),
C 2RO(I,J),SF2(I,J),CH(I,J),PERM(I,J,1),
C 3PERM(I,J,2),BOT(I,J)
C FORMAT(2I3,2F6.0,2F4.0,9F6.0)
C GO TO 30
C
C START OF SIMULATION
C
C 50 TIME=0
C DEL=DELTA
C KC=1
C DO 280 ISTEP=1,NSTEPS
C
C ENTER PUMPAGE SCHEDULES
C
C Z=(ISTEP-1.0)/NSP+1.0
C IF(Z-KC)49,46,49
C DO 47 K=1,NP
C I=IP(K)
C J=JP(K)
C Q(I,J)=P(K,KC)
C DELTA=DEL
C KC=KC+1
C
C 46
C
C 47
C
C (Continued on next page)

```

Figure 72. Composite aquifer simulation program listing

```

C   PREDICT HEADS FOR NEXT
C   TIME INCREMENT
C
49  DO 51 I=1,NC
    DO 51 J=1,NR
      D=H(I,J)-HO(I,J)
      HO(I,J)=H(I,J)
      F=1.0
      IF(DL(I,J).EQ.0.0)GO TO 48
      IF(ISTEP.GT.2)F=D/DL(I,J)
      IF(F.GT.5)F=5.0
      IF(F.LT.0.0)F=0.0
48  OL(I,J)=D
      H(I,J)=H(I,J)+D*F
51  IF(H(I,J).LE.BOT(I,J))H(I,J)=BOT(I,J)+0.01
C
C   REFINE HEADS
C
C   UPDATE TIME AND INITIALIZE ERROR
C   AND ITERATION COUNTER
C
      TIME=TIME+DELTA
      ITER=0
52  E=0.0
      ITER=ITER+1
C
C   COLUMN CALCULATIONS
C
      DO 160 I=1,NC
        DO 140 J=1,NR
C
C   CALCULATE PEACEMAN-RACHFORD
C   B AND G ARRAYS
C
          AA=0.0
          IF(H(I,J).LT.BOT(I,J))H(I,J)=BOT(I,J)
          IF(H(I,J).LT.RD(I,J))GO TO 53
          RE=0.0
          RB=1.0
          GO TO 54
53  RE=(RH(I,J)-RD(I,J))*R(I,J)
          RB=0.0
54  IF(H(I,J).LT.CH(I,J))GO TO 55
          S=SF1(I,J)
          GO TO 56
55  S=SF2(I,J)
56  BB=S/DELTA+R(I,J)*RB
          DD=HO(I,J)*S/DELTA-Q(I,J)+RE
          IF((HO(I,J)-CH(I,J))*H(I,J)-CH(I,J)).LT.0.0)DD=DD+
          1(HO(I,J)-CH(I,J))*(SF1(I,J)-SF2(I,J))/DELTA
          CC=0.0
          IF(I-1)60,70,60
          AA=-T(I,J-1,1)
          BB=BB+T(I,J-1,1)
          IF(J-NR)80,90,80
          IF(PERM(I,J,1).EQ.0.0)GO TO 85
          T(I,J,1)=PERM(I,J,1)*SQRT((AMIN1(H(I,J),CH(I,J))-
          1BOT(I,J))*(AMIN1(H(I,J+1),CH(I,J+1))-BOT(I,J+1)))
          CC=-T(I,J,1)
          BB=BB+T(I,J,1)
          IF(I-1)100,110,100
          BB=BB+T(I-1,J,2)
          DD=DD+H(I-1,J)*T(I-1,J,2)
          IF(I-NC)120,130,120
          IF(PERM(I,J,2).EQ.0.0)GO TO 125
          T(I,J,2)=PERM(I,J,2)*SQRT((AMIN1(H(I,J),CH(I,J))-
          1BOT(I,J))*(AMIN1(H(I+1,J),CH(I+1,J))-BOT(I+1,J)))
          BB=BB+T(I,J,2)
          DD=DD+H(I+1,J)*T(I,J,2)
          W=BB-AA*B(J-1)
          B(J)=CC/W
          G(J)=(DD-AA*G(J-1))/W
C
C   RE-ESTIMATE HEADS
C
          E=E+ABS(H(I,NR)-G(NR))
          H(I,NR)=G(NR)
          N=NR-1
          HA=G(N)-B(N)*H(I,N+1)
          E=E+ABS(HA-H(I,N))
          H(I,N)=HA
          N=N-1
          IF(N.GT.0)GO TO 150
          DO 160 N=1,NR
          IF(H(I,N).GT.BOT(I,N))GO TO 160
          E=E+BOT(I,N)+0.01-H(I,N)
          H(I,N)=BOT(I,N)+0.01
          160 CONTINUE
C
C   ROW CALCULATIONS
C
          DO 270 J=1,NR
          DO 250 I=1,NC
          AA=0.0
          IF(H(I,J).LT.BOT(I,J))H(I,J)=BOT(I,J)
          IF(H(I,J).LT.RD(I,J))GO TO 163
          RE=0.0
          RB=1.0
          GO TO 164
          163 RE=(RH(I,J)-RD(I,J))*R(I,J)
          RB=0.0
          164 IF(H(I,J).LT.CH(I,J))GO TO 165
          S=SF1(I,J)
          GO TO 166
          165 S=SF2(I,J)
          166 BB=S/DELTA+R(I,J)*RB
          DD=HO(I,J)*S/DELTA-Q(I,J)+RE
          IF((HO(I,J)-CH(I,J))*H(I,J)-CH(I,J)).LT.0.0)DD=DD+
          1(HO(I,J)-CH(I,J))*(SF1(I,J)-SF2(I,J))/DELTA
          CC=0.0
          IF(J-1)170,180,170
          BB=BB+T(I,J-1,1)
          DD=DD+H(I,J-1)*T(I,J-1,1)
          IF(J-NR)190,200,190
          IF(PERM(I,J,1).EQ.0.0)GO TO 195
          T(I,J,1)=PERM(I,J,1)*SQRT((AMIN1(H(I,J),CH(I,J))-
          1BOT(I,J))*(AMIN1(H(I,J+1),CH(I,J+1))-BOT(I,J+1)))
          195 DD=DD+H(I,J+1)*T(I,J,1)
          BB=BB+T(I,J,1)
          200 IF(I-1)210,220,210
          210 BB=BB+T(I-1,J,2)
          AA=-T(I-1,J,2)
          IF(I-NC)230,240,230
          IF(PERM(I,J,2).EQ.0.0)GO TO 235
          T(I,J,2)=PERM(I,J,2)*SQRT((AMIN1(H(I,J),CH(I,J))-
          1BOT(I,J))*(AMIN1(H(I+1,J),CH(I+1,J))-BOT(I+1,J)))
          235 BB=BB+T(I,J,2)
          CC=-T(I,J,2)
          W=BB-AA*B(I-1)
          B(I)=CC/W
          G(I)=(DD-AA*G(I-1))/W
C
C   RE-ESTIMATE HEADS
C
          E=E+ABS(H(NC,J)-G(NC))
          H(NC,J)=G(NC)
          N=NC-1
          HA=G(N)-B(N)*H(N+1,J)
          E=E+ABS(H(N,J)-HA)
          H(N,J)=HA
          N=N-1
          IF(N.GT.0)GO TO 260
          DO 270 N=1,NC
          IF(H(N,J).GT.BOT(N,J))GO TO 270
          E=E+BOT(N,J)+0.01-H(N,J)
          H(N,J)=BOT(N,J)+0.01
          270 CONTINUE
          IF(E.GT.ERROR) GO TO 52
C
C   PRINT RESULTS
C
          WRITE(OUT,300)TIME,E,ITER
          300 FORMAT(6H2TIME=,F6.2///,E20.7,15)
          DELTA=DELTA*1.2
          DO 280 J=1,NR
          280 WRITE(OUT,290)J,(H(I,J),I=1,NC)
          290 FORMAT(15,5X,10F10.4/(12X10F10.4))
C
          STOP
          END

```

Figure 72 (Concluded)

## DIGITAL MODEL FOR CAMBRIAN-ORDOVICIAN AQUIFER OF NORTHEASTERN ILLINOIS

A digital model designed for the Cambrian-Ordovician aquifer in the Chicago region of northeastern Illinois is an example of the actual use of various program sections of this report. Groundwater resources in this region of heavy pumpage had been studied extensively, and water level changes due to pumpage had been predicted by mathematical modeling. However, changes caused by greatly increased pumping in recent years required an improved method of prediction. Design and use of the digital model for this purpose are described below. The problem is illustrated by figures 73-79, and the program listing is given in figure 80.

As described in a detailed report for the Chicago region (Suter et al., 1959), the Cambrian-Ordovician aquifer consists in downward order of the Galena-Platteville dolomite, Glenwood-St. Peter sandstone, and Prairie du Chien Series of Ordovician age; and the Trempealeau dolomite, Franconia Formation, and Ironton-Galesville sandstone of Cambrian age. The Cambrian-Ordovician aquifer is underlain by shale beds of the Eau Claire Formation which have a very low permeability. Available data indicate that, on a regional basis, the entire sequence of strata from the top of the Galena-Platteville to the top of the Eau Claire essentially behaves hydraulically as one aquifer, has a fairly uniform thickness of about 1000 feet, and has a gentle dip to the east of about 13 feet per mile.

The Cambrian-Ordovician aquifer is under leaky artesian conditions and receives a small portion of recharge from Silurian age rocks or from glacial drift by vertical leakage of water through either the Maquoketa Formation or Pennsylvanian shales in areas of northeastern Illinois and southeastern Wisconsin where these units overlie the aquifer. The major portion of recharge occurs in areas to the west and northwest of the Chicago region where the Maquoketa and Pennsylvanian shales are missing and the Galena-Platteville dolomite, the uppermost unit of the aquifer, directly underlies the glacial drift.

According to Walton (1962), the leakage coefficient  $P'/m'$  of the Maquoketa Formation (see equation 52) is fairly uniform throughout northeastern Illinois and is about  $2.5 \times 10^{-7}$  gpd/ft<sup>3</sup>. Very little is known about the leakage coefficient of the Pennsylvanian deposits, but from available data, it is not unreasonable to assume that it is about the same as the leakage coefficient of the Maquoketa Formation.

According to Suter et al. (1959) and Foley et al. (1953), the transmissivity of the Cambrian-Ordovician aquifer is fairly uniform throughout most of northeastern Illinois and southeastern Wisconsin, and averages about 17,000 gpd/ft. On the basis of studies by Walton and Csallany (1962), the transmissivities of the individual units of the Cambrian-Ordovician aquifer would have the values depicted in figure 73. If water levels fall below the top of the aquifer, the transmissivity is assumed to decline according to figure

73. In addition there is evidence (Suter et al., 1959) suggesting that the transmissivity of the Cambrian-Ordovician aquifer decreases south and east of Chicago and that changes in the water-bearing properties are great enough to approximate the effect of two right-angle intersecting barrier boundaries occurring at distances of about 37 miles east and about 60 miles south of Chicago.

The artesian coefficient of storage of the Cambrian-Ordovician aquifer is also fairly uniform throughout the Chicago region and is estimated to average about 0.0005 (Suter et al., 1959; Walton, 1964). As groundwater development increases in the future, gravity drainage of upper units of the aquifer will occur and there will be a partial conversion from an artesian to a water table coefficient of storage. Since very few data are available to show water table storage coefficient changes when water levels fall below the top of the aquifer, the digital model was designed with a water table storage coefficient of 0.05 on the basis of studies by Walton (1964). However, further studies should be made in the future to verify use of the single water table condition storage coefficient.

Pumpage of groundwater from the Cambrian-Ordovician aquifer in the Chicago region increased from 200,000 gpd in 1864 to 52.3 mgd in 1960 (Sasman et al., 1961). Pumpage is concentrated in seven centers as shown in figure 75 for the period 1864 to 1960.

The purpose of designing a digital model for the Cambrian-Ordovician aquifer was to improve on the method previously used for predicting water level declines for projected future groundwater development. In the past, prediction of water level fluctuations due to changes in pumpage was accomplished with a simplified model aquifer and mathematical model (Walton, 1962). This type of modeling is linear in nature and had been highly successful until recently. In recent years extensive groundwater development has taken place to the extent that, in certain areas of the Chicago region, dewatering of the aquifer is beginning to occur. As this happens, the aquifer response to development becomes nonlinear. This is because a conversion from artesian to water table conditions takes place, a reduction of the saturated thickness of the aquifer occurs, the flow of water changes from two to three dimensions, and leakage rates across confining beds reach maximum values. Thus, as dewatering of the aquifer becomes more prevalent, the simplified linear model becomes less valid for predicting aquifer response.

The first step in designing the digital model was to discretize the aquifer and overlying confining layer as shown in figure 74. A 100 by 100 variable size finite difference grid was used to provide a detailed area of interest surrounding the seven pumping centers and to avoid the use of an excessive number of nodes. The grid interval in the area of detailed interest is 1 mile per node. The finite dif-

ference grid was positioned so that the last column and last row coincided with the eastern and southern barrier boundaries, respectively. The first column and row were positioned so that they would effectively be beyond the area of influence of the pumping centers.

The outcrop of the Galena-Platteville dolomite to the west and northwest was approximated with straight line boundaries falling along the 29th column and the 82nd row of the grid shown in figure 74.

The pumpage graphs of figure 75 were idealized in 5-year stepwise intervals, and these were programmed to change every 6th time increment (NSP = 6).

The computer program for the Cambrian-Ordovician aquifer model for the Chicago region is given in figure 80. A complete sample data deck, consisting mostly of values of pumping center pumpage rates, is given in figure 76. The computer program was written mainly by piecing together individual sections of the other programs of this report, but additional programming was included for various purposes as discussed below.

The main program features that have not been previously discussed are outlined by the letter-designated sections of figure 80. The programming of section A of figure 80 eliminates the need for node cards by adjusting the default values for the variable size grid used (see equation 7). Section B includes programming to incorporate a tape drive for saving head values at intervals of every 5 years. This is done so that adjustments in programming can be made periodically by stopping the simulation, evaluating partial results, and then continuing without the need to start from the beginning.

Section G of figure 80 was included to further speed convergence of head values in the first time increment after changes in pumping rates have occurred. An analogy explains this process. Let us say it is desired to fabricate a deep kettle from a flat sheet of copper with the use of a ball peen hammer. Many blows would be necessary to change the flat sheet into the kettle shape. A more efficient way of forming the kettle would be to start with a sledge hammer and, with a few blows, knock the sheet into an approximate kettle shape and then complete the process by a few finishing touches with the ball peen hammer. The sledge hammer is analogous to section C, and the ball peen hammer is analogous to the remaining iterative alternating direction programming of figure 80. Since the aquifer properties are locally relatively uniform, a guess can be made [by traditional formulas such as those given by Theis (1935) and Hantush (1956)] as to what the changes in water levels will be as a result of the incremental changes in pumpage. On the basis of the hydraulic properties of the aquifer, sec-

tion C of figure 80 was written to rough out the piezometric surface by operating on 100 nodes of the model adjacent to each pumping center after each change in pumping rate. The remainder of the program then converges rapidly for the heads at all nodes of the model.

Sections D and E of figure 80 adjust values of recharge and storage factors for the variable grid size (see equations 52 and 44). Section F computes new values of transmissivity when dewatering begins according to the data in figure 73. Section F further readjusts effective transmissivity values to fit the variable size grid of the model.

The digital computer model was checked for its validity with a past history comparison of its response with that measured in the field.

Figure 77 illustrates one of many such comparisons made between actual and digitally simulated water level decline maps. Further comparisons in water level declines versus time are shown in figure 78. In general, agreement between actual and digitally simulated aquifer response is good. However, since massive dewatering of the Cambrian-Ordovician aquifer has not yet occurred, the use of the model for predicting future water level declines is somewhat in question and should be further validated as more of the aquifer is dewatered.

Increased pumpage from the Cambrian-Ordovician aquifer was projected to the year 1995 on the basis of pumpage estimates made by Walton (1964). Total pumpage from the seven centers of pumping in 1995 was estimated to be about 145 mgd. Figure 79 illustrates total declines in water levels as computed with the digital model.

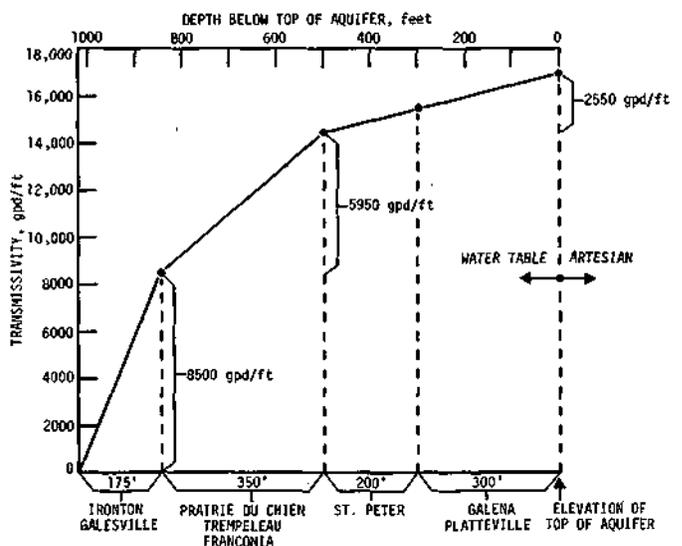


Figure 73. Transmissivity variation of units of Cambrian-Ordovician aquifer in Chicago region



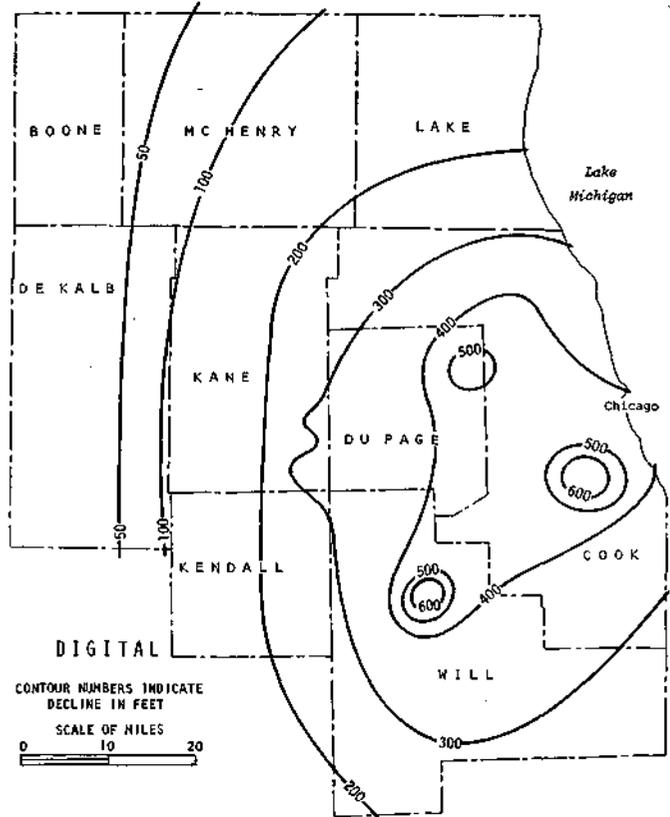
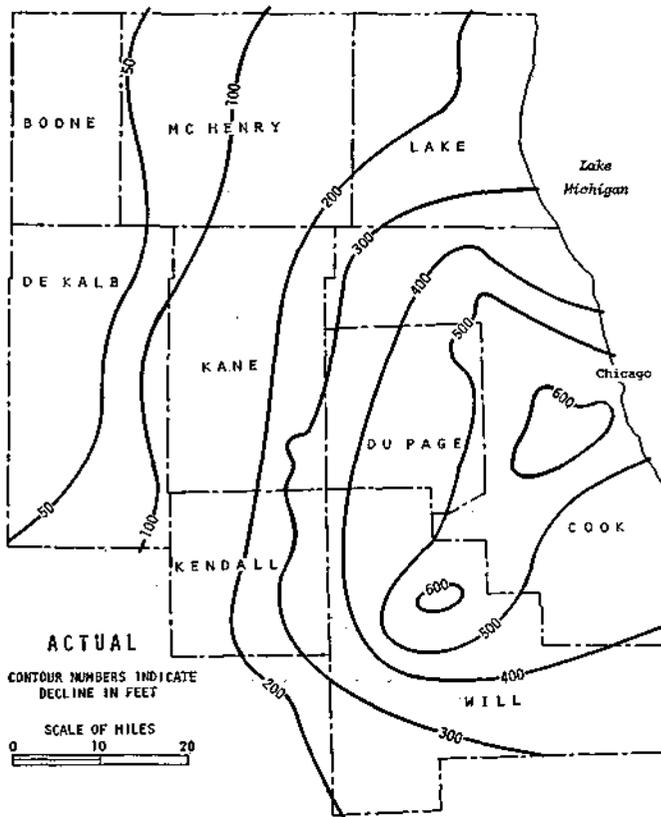


Figure 77. Comparison of actual and digital model declines in piezometric surface of Cambrian-Ordovician aquifer 1864-1958

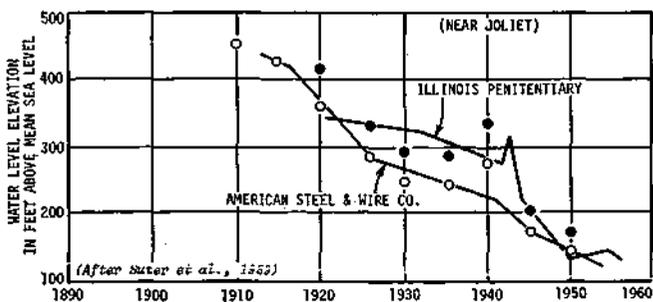
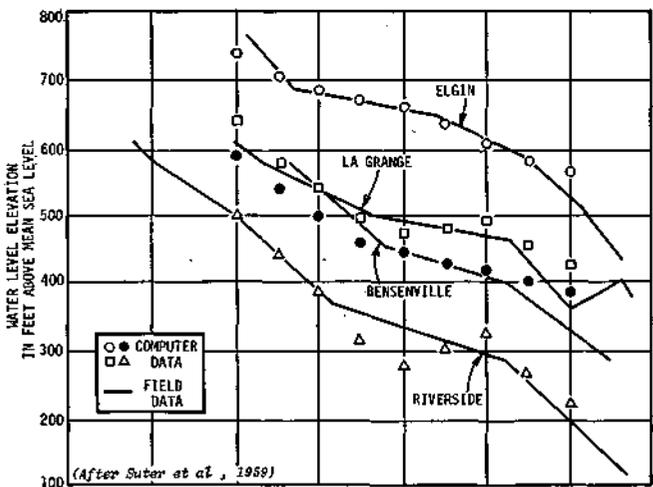


Figure 78. Decline of artesian pressure in selected deep wells in Chicago region

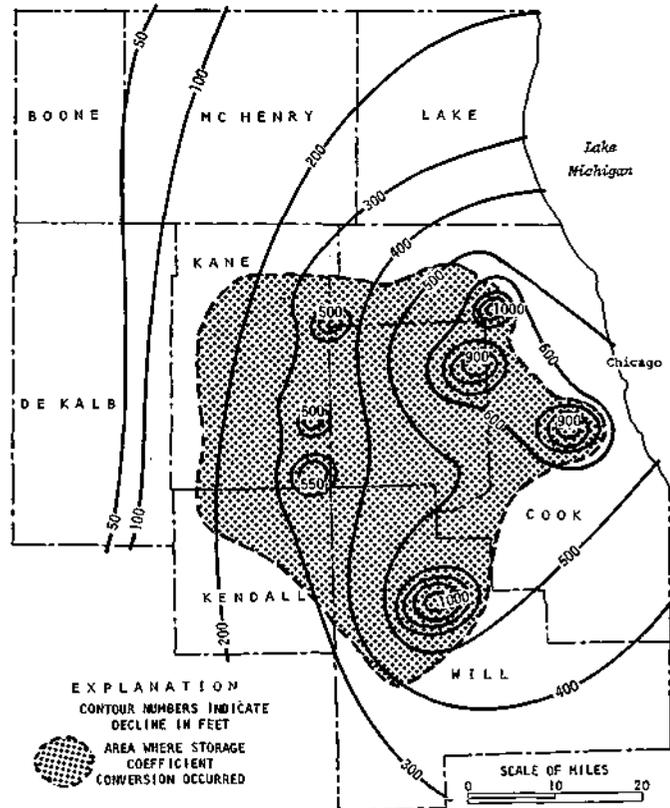


Figure 79. Model computed declines in piezometric surface of Cambrian-Ordovician aquifer 1864-1995

```

C      CHICAGO REGION DEEP AQUIFER SYSTEM MODEL,
C
C      COMBINED TWO-DIMENSIONAL AQUIFER
C      SIMULATION PROGRAM INCLUDING EFFECTS
C      OF INDUCED INFILTRATION, WATER-TABLE
C      CONDITIONS, NONLINEAR TRANSMISSIVITIES,
C      VARIABLE GRID SPACINGS,
C      CONVERTING FROM ARTESIAN TO WATER-
C      TABLE STORAGE COEFFICIENTS, AND
C      VARIABLE PUMPING RATE SCHEDULES.
C
C      DEFINITION OF VARIABLES
C
C      HO(I,J)-----HEADS AT START OF TIME
C                   INCREMENT (FT)
C      H(I,J)-----HEADS AT END OF TIME
C                   INCREMENT (FT)
C      S(I,J)-----STORAGE FACTOR (GAL/FT)
C      Q(I,J)-----CONSTANT WITHDRAWAL
C                   RATES (GPD)
C      T(I,J,1)----AQUIFER TRANSMISSIVITY
C                   BETWEEN I,J AND I+1,J
C                   (GAL/DAY*FT)
C      T(I,J,2)----AQUIFER TRANSMISSIVITY
C                   BETWEEN I,J AND I+1,J
C                   (GAL/DAY*FT)
C      R-----RECHARGE FACTORS
C                   (GAL/DAY/FT)
C      AA,BB,CC,DD--COEFFICIENTS IN WATER
C                   BALANCE EQUATIONS
C      NR-----NO. OF ROWS IN MODEL
C      NC-----NO. OF COLUMNS IN MODEL
C      NSTEPS-----NO. OF TIME INCREMENTS
C      DELTA-----TIME INCREMENTS (DAYS)
C      HH,S1,QQ,TT--EXAMPLE MODEL
C                   DEFAULT VALUES
C      I-----MODEL COLUMN NUMBER
C      J-----MODEL ROW NUMBER
C      IP-----I COORDINATE OF PUMP
C      JP-----J COORDINATE OF PUMP
C      P(K,KC)----PUMPING RATE AT WELL
C                   K AT RATE KC (GPD)
C      NP-----NUMBER OF PUMPS
C      MSP-----NUMBER OF TIME INCREMENTS
C                   PER PUMPAGE CHANGE
C      NRT-----NUMBER OF RATES IN
C                   PUMPING SCHEDULE
C      CH(I)-----DEPTH TO TOP OF
C                   GALENA-PLATTEVILLE
C                   FORMATION (FT BELOW
C                   ZERO REFERENCE LEVEL)
C
C      DIMENSION H(100,100),HO(100,100),
C      IQ(100,100),T(100,100,2),G(101),B(101),
C      2IP(100),JP(100),P(100,25),
C      3CH(100),DL(100,100),S(100,100)
C      CALL ERRSET(208,256,-1,1)
C
C      DEFINE SPACIAL MODAL INDICES
C
C      NR=100
C      NC=100
C
C      READ PARAMETER CARDS
C
C      READ(5,10)NSTEPS,DELTA,ERROR,ISTEPS
C      FORMAT(I6,2F6.0,I6)
C      READ(8)K,HO
C      IF(K.NE.1)GO TO 3
C      READ(5,11) NP,MSP,NRT
C      FORMAT(3I6)
C
C      READ PUMPING SCHEDULES
C
C      DO 12 I=1,NP
C      READ(5,13) IP(I),JP(I),
C      1(P(I,K),K=1,NRT)
C      FORMAT(2I3,12F6.0/13F6.0/13F6.0)
C
C      CALCULATE DEPTH TO TOP OF
C      GALENA PLATTEVILLE FORMATION
C
C      DO 15 I=1,100
C      CH(I)=MINO(-1,-26*(I-29)+13*(MINO(
C      1MAXO(35,I),85)-35))
C
C      FILL ARRAYS WITH DEFAULT VALUES
C
C      DO 20 I=1,100
C      DO 20 J=1,100
C      S(I,J)=4.2E5
C      T(I,J,1)=17000.0
C      T(I,J,2)=17000.0
C
C      ADJUST DEFAULT VALUES
C      FOR VARIABLE GRID INTERVALS
C
C      IF(I-34)765,750,755
C      T(I,J,2)=T(I,J,2)*1.33333
C      GO TO 765
C      755 IF(I-84)760,750,765
C      T(I,J,2)=T(I,J,2)*2.0
C      T(I,J,1)=T(I,J,1)*0.5
C      760 IF(J-34)790,770,780
C      T(I,J,1)=T(I,J,1)*1.33333
C      GO TO 790
C      770 IF(J-88)785,770,790
C      T(I,J,1)=T(I,J,1)*2.0
C      T(I,J,2)=T(I,J,2)*0.5
C      785 CONTINUE
C      790 IF(I.LT.35.OR.I.GT.85) GO TO 891
C      S(I,J)=S(I,J)*0.5
C      891 IF(J.LT.35.OR.J.GT.85) GO TO 892
C      S(I,J)=S(I,J)*0.5
C      892 Q(I,J)=0.0
C      DL(I,J)=0.0
C      H(I,J)=HO(I,J)
C
C      START OF SIMULATION
C
C      DEL=DELTA
C      KC=ISTEPS/6+1
C      TIME=(KC-1)*1825.
C      DO 320 ISTEP=ISTEPS,NSTEPS
C
C      ESTIMATE HEADS FOR
C      NEXT TIME INCREMENT
C
C      DO 70 I=1,NC
C      DO 70 J=1,NR
C      D=H(I,J)-HO(I,J)
C      HO(I,J)=H(I,J)
C      F=1.0
C      IF(DL(I,J).EQ.0.0)GO TO 60
C      IF(ISTEP.GT.1)F=D/DL(I,J)
C      IF(F.GT.2)F=2.0
C      IF(F.LT.0.0)F=0.0
C      DL(I,J)=D
C      H(I,J)=H(I,J)+D*F
C
C      ENTER PUMPING SCHEDULES
C
C      Z=(ISTEP-1.0)/MSP+1.0
C      IF(Z-KC)53,51,53
C      IF(ISTEP.EQ.1)STEP=GO TO 48
C      WRITE(8)ISTEP,HO
C      WRITE(6,49)ISTEP
C      49 FORMAT('RECORD OUT',15////)
C      48 SQ=0.0
C      IF(NSTEPS.GT.1)STEP GO TO 55
C
C      WRITE 5 YEAR HEADS
C      ON TAPE UNIT
C
C      WRITE(8)CH
C      DO 1 J=1,100
C
C      PRINT 5 YEAR HEADS
C      AT ALL NODES
C
C      1 WRITE(6,330)J,(HO(I,J),I=1,100)
C      CALL SYSTEM
C
C      ENTER PSEUDO
C      PIEZOMETRIC SURFACE
C
C      55 DO 52 K=1,NP
C      SQ=SQ+P(K,KC)
C      I=IP(K)
C      J=JP(K)
C      DQ=(P(K,KC)-Q(I,J))*1E-5
C      IF(DQ.EQ.0.0) GO TO 52
C      IS=I-5
C      IF=I+5
C      JS=J-5
C      JF=J+5
C      DO 54 II=IS,IF
C      DO 54 JJ=JS,JF
C      H(II,JJ)=H(II,JJ)-DQ
C      H(I,J)=H(I,J)+2.0*DQ
C      Q(I,J)=P(K,KC)
C      54
C      52
C      CALCULATE ERROR TERM
C
C      ERROR=SQ/3.0E5
C      DELTA=DEL
C      KC=KC+1
C
C      (Continued on next page)

```

Figure 80. Program listing for digital model of Cambrian-Ordovician aquifer

```

53  TIME=TIME+DELTA
ITER=0
80  E=0.0
ITER=ITER+1
C
C  REFINE ESTIMATES OF HEADS BY
C  IADI METHOD
C
C  COLUMN CALCULATIONS
C
DO 190 I=1,NC
I=I
IF(MOD(ISTEP+ITER,2).EQ.1)I=NC-I+1
C
C  SET UP RECHARGE AND STORAGE FACTOR
C  ADJUSTMENT COEFFICIENTS FOR VARIABLE
C  GRID SIZES
C
DO 170 J=1,NR
R=80000.0
IF(I.GE.29.OR.J.GE.82)R=6.6E-5*(I,J)
SS=S(I,J)/DELTA
C
C  RECHARGE AND STORAGE COEFFICIENT
C  CONTROL SECTION
C
IF(H(I,J).GT.CH(I)) GO TO 691
RE=-CH(I)*R
RB=0.0
SS=SS+100
GO TO 692
691  RE=0.0
RB=1.0
692  AA=0.0
BB=SS+R*RB
C
C  CALCULATE B AND G ARRAYS
C
CC=0.0
DD=HO(I,J)*SS-Q(I,J)+RE
IF((HO(I,J)-CH(I))*H(I,J)-CH(I)).LT.0.0)
100  IOD=DD-(HO(I,J)-CH(I))*99.0*S(I,J)/DELTA
IF(J-1)90,100,90
90  AA=-T(I,J-1,1)
BB=BB+T(I,J-1,1)
100  IF(J-NR)110,120,110
110  CC=-T(I,J,1)
BB=BB+T(I,J,1)
120  IF(I-1)130,140,130
130  BB=BB+T(I-1,J,2)
DD=DD+H(I-1,J)*T(I-1,J,2)
140  IF(I-NC)150,160,150
150  BB=BB+T(I,J,2)
DD=DD+H(I+1,J)*T(I,J,2)
160  W=BB-AA*B(I-1)
B(I,J)=CC/W
IF(W.EQ.0.)WRITE(6,998)AA,BB,B(I,J-1),J,CC,DD,HO(I,J),H(I,J)
170  G(I,J)=(DD-AA*G(I-1))/W
998  FORMAT(3F20.6,2I10,4E10.4)
C
C  RE-ESTIMATE HEADS
C
E=E+ABS(H(I,NR)-G(NR))
H(I,NR)=G(NR)
N=NR-1
180  HA=G(N)-B(N)*H(I,N+1)
E=E+ABS(HA-H(I,N))
H(I,N)=HA
N=N-1
IF(N)190,190,180
190  CONTINUE
C
C  ROW CALCULATIONS
C
DO 300 JJ=1,NR
J=JJ
IF(MOD(ISTEP+ITER,2).EQ.1)J=NR-J+1
C
C  SET UP RECHARGE AND STORAGE FACTOR
C  ADJUSTMENT COEFFICIENTS FOR VARIABLE
C  GRID SIZES
C
DO 280 I=1,NC
R=80000.0
IF(I.GE.29.OR.J.GE.82)R=6.6E-5*(I,J)
SS=S(I,J)/DELTA
C
C  RECHARGE AND STORAGE COEFFICIENT
C  CONTROL SECTION
C
IF(H(I,J).GT.CH(I)) GO TO 191
RE=-CH(I)*R
RB=0.0
SS=SS+100.0
GO TO 192
191  RE=0.0
RB=1.0
192  AA=0.0
BB=SS+R*RB
C
C  CALCULATE B AND G ARRAYS
C
CC=0.0
DD=HO(I,J)*SS-Q(I,J)+RE
IF((HO(I,J)-CH(I))*H(I,J)-CH(I)).LT.0.0)
200  IOD=DD-(HO(I,J)-CH(I))*99.0*S(I,J)/DELTA
IF(J-1)200,210,200
210  BB=BB+T(I,J-1,1)
DD=DD+H(I,J-1)*T(I,J-1,1)
220  IF(J-NR)220,230,220
DD=DD+H(I,J+1)*T(I,J,1)
BB=BB+T(I,J,1)
230  IF(I-1)240,250,240
240  BB=BB+T(I-1,J,2)
AA=-T(I-1,J,2)
250  IF(I-NC)260,270,260
260  BB=BB+T(I,J,2)
CC=-T(I,J,2)
270  W=BB-AA*B(I-1)
B(I,J)=CC/W
280  G(I)=(IOD-AA*G(I-1))/W
C
C  RE-ESTIMATE HEADS
C
E=E+ABS(H(1,NC)-G(1,NC))
H(1,NC)=G(1,NC)
N=NC-1
290  HA=G(N)-B(N)*H(N+1,J)
E=E+ABS(H(N,J)-HA)
H(N,J)=HA
I=N
IF(HA.GE.CH(I)) GO TO 890
C
C  COMPUTE TRANSMISSIVITIES AS A
C  FUNCTION OF HEADS AND MAKE ADJUSTMENTS
C  FOR VARIABLE GRID SIZES
C
T(I,J,1)=17000.0
T(I,J,2)=17000.0
IF(J.EQ.100) GO TO 820
H1=-SQRT((H(I,J)*H(I,J+1))-CH(I))
IF(H1.GE.0) GO TO 820
IF(H1.LT.-500) GO TO 810
T(I,J,1)=17000+.5.*H1
GO TO 820
810  T(I,J,1)=14450.+17.0*(H1+500)
820  IF(I.EQ.100) GO TO 840
H2=-SQRT((H(I,J)*H(I+1,J))-CH(I))
IF(H2.GE.0) GO TO 840
IF(H2.LT.-500) GO TO 830
T(I,J,2)=17000+.5.*H2
GO TO 840
830  T(I,J,2)=14450.+17.0*(H2+500)
840  IF(H1.LT.-850.0)T(I,J,1)=48.6*(H1+1025.0)
IF(H2.LT.-850.0)T(I,J,2)=48.6*(H1+1025.0)
IF(I-34)865,850,855
T(I,J,2)=T(I,J,2)*1.33333
GO TO 865
855  IF(I-84)860,850,865
860  T(I,J,2)=T(I,J,2)*2.0
T(I,J,1)=T(I,J,1)*0.5
865  IF(J-34)890,870,880
870  T(I,J,1)=T(I,J,1)*1.33333
GO TO 890
880  IF(J-88)885,870,890
885  T(I,J,1)=T(I,J,1)*2.0
T(I,J,2)=T(I,J,2)*0.5
890  CONTINUE
N=N-1
IF(N)300,300,290
300  CONTINUE
IF(E.GT.ERROR) GO TO 80
C
C  PRINT RESULTS FOR DETAILED
C  AREA OF INTEREST
C
T365=TIME/365.0+1860.
WRITE(6,310)T365,E,ITER
310  FORMAT(6H2YEAR=,F7.2///,E20.7,I5)
DELTA=DELTA*1.2
DO 320 J=35,85
320  WRITE(6,330)J,H(I,J),I=35,85)
330  FORMAT(I5,5X,10F10.4/12X10F10.4))
STOP
END

```

Figure 80 (Concluded)

## COMPARISON OF DIGITAL AND ELECTRIC ANALOG SIMULATIONS

A digital model was designed to study a complex aquifer situation previously described in an electric analog study by Walton and Prickett (1963). The complex aquifer situation is shown in figure 81. The aquifer properties shown were discretized with a square finite difference grid having an interval of 660 feet. The digital model, like the electric analog model, contained 19 columns and 30 rows. A single well was located at node coordinates  $i=12, j=16$  and was pumped at a constant rate of 1.11 mgd for a period of 180 days. The digital computer node card data for this model are shown in figure 83. Additional input data were an initial DELTA of 3.711 days, NSTEPS equal to 13, an ERROR value of 0.1 feet, and default values of transmissivity of zero and storage factors of 10,000. The category printout program of figure 28 was used to produce the digital computer output shown in figure 82.

The contour map of figure 82 was obtained from an electric analog constructed for the aquifer situation described above. A comparison of the digital and electric analog results indicates excellent agreement. The differences between the two results of figure 82 are due mainly to discretizing of

time in the digital model and circuit loading effects of the oscilloscope in the analog model.

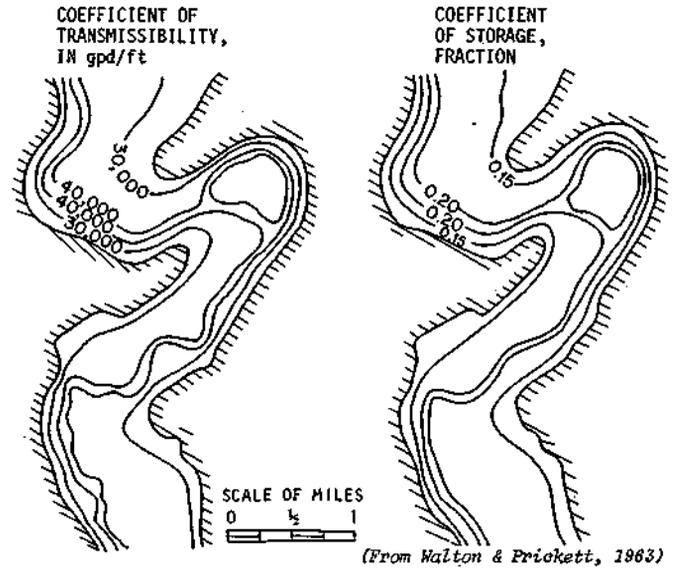
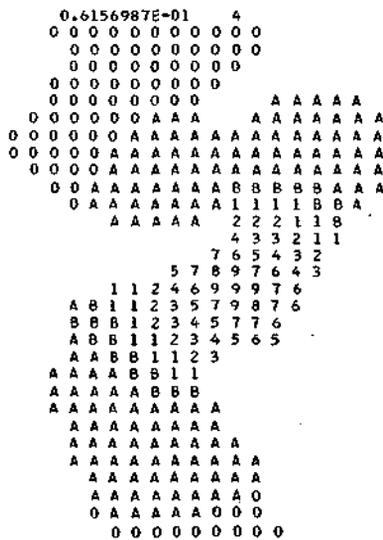


Figure 81. Diagrams of complex aquifer situation

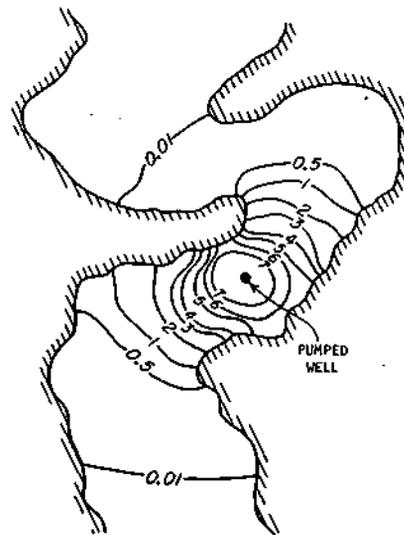
TIME=179.97



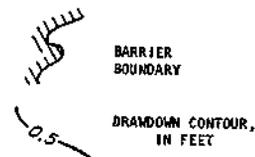
**DIGITAL MODEL EXPLANATION**

LETTER OR NUMBER	DRAWDOWN BETWEEN
BLANK	AQUIFER MISSING
0	0.0 AND 0.01 FEET
A	0.01 AND 0.50
B	0.5 AND 1
1	1 AND 2
2	2 AND 3
3	3 AND 4
4	4 AND 5
5	5 AND 6
6	6 AND 7
7	7 AND 8
8	8 AND 9
9	>9

**DIGITAL RESULTS**



**ELECTRIC ANALOG MODEL EXPLANATION**



**ELECTRIC ANALOG RESULTS**

Figure 82. Comparison of digital and analog water level contour maps

1	7	10E3	10E3	4E5	8	10	40E3	39E3	6E5	10	22	20E3	0E3	4E5	8	3	31E3	30E3	6E5
1	8	0E3	10E3	4E5	8	11	30E3	40E3	7E5	10	23	22E3	15E3	4E5	13	1	10E3	0E3	5E5
2	6	20E3	20E3	5E5	8	12	0E3	20E3	5E5	10	24	28E3	20E3	5E5	13	6	35E3	35E3	5E5
2	7	25E3	35E3	5E5	8	16	30E3	15E3	4E5	10	25	32E3	25E3	6E5	13	7	40E3	39E3	6E5
2	8	10E3	30E3	5E5	8	17	33E3	33E3	6E5	10	26	32E3	31E3	6E5	13	8	40E3	40E3	6E5
2	9	0E3	20E3	5E5	8	18	35E3	35E3	6E5	10	27	34E3	32E3	6E5	13	9	40E3	40E3	7E5
3	1	0E3	10E3	5E5	8	19	36E3	37E3	7E5	10	28	34E3	32E3	6E5	13	10	30E3	38E3	6E5
3	4	10E3	10E3	5E5	8	20	38E3	38E3	7E5	10	29	35E3	33E3	6E5	13	11	28E3	30E3	6E5
3	5	20E3	25E3	6E5	8	21	39E3	40E3	7E5	10	30	0E3	35E3	6E5	13	12	28E3	30E3	6E5
3	6	40E3	38E3	6E5	8	22	35E3	35E3	5E5	11	1	23E3	24E3	5E5	13	13	34E3	35E3	6E5
3	7	40E3	39E3	6E5	8	23	34E3	31E3	5E5	11	2	22E3	23E3	5E5	13	14	37E3	38E3	6E5
3	8	35E3	40E3	6E5	8	24	33E3	30E3	6E5	11	3	20E3	22E3	5E5	13	15	40E3	39E3	6E5
3	9	20E3	33E3	6E5	8	25	35E3	33E3	6E5	11	7	20E3	10E3	4E5	13	16	40E3	40E3	7E5
3	10	0E3	20E3	6E5	8	26	38E3	35E3	6E5	11	8	35E3	33E3	5E5	13	17	40E3	39E3	7E5
3	21	10E3	10E3	4E5	8	27	40E3	38E3	6E5	11	9	40E3	39E3	6E5	13	18	30E3	30E3	5E5
3	22	10E3	15E3	5E5	8	28	40E3	38E3	6E5	11	10	30E3	40E3	7E5	13	19	0E3	10E3	4E5
3	23	0E3	15E3	4E5	8	29	40E3	38E3	6E5	11	11	0E3	10E3	5E5	13	26	10E3	0E3	4E5
4	1	20E3	35E3	6E5	8	30	0E3	40E3	6E5	11	14	28E3	25E3	5E5	13	27	10E3	0E3	4E5
4	2	18E3	30E3	6E5	9	1	29E3	28E3	5E5	11	15	33E3	34E3	6E5	13	28	15E3	0E3	4E5
4	3	20E3	28E3	6E5	9	2	28E3	28E3	5E5	11	16	37E3	35E3	6E5	13	29	18E3	0E3	4E5
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6	8	40E3	37E3	6E5	6	27	35E3	38E3	6E5	7	28	40E3	40E3	6E5	19	7	10E3	0E3	5E5
6	9	40E3	39E3	7E5	6	28	30E3	35E3	6E5	7	29	38E3	40E3	6E5	19				

## PUMPED WELL CHARACTERISTICS

One of the main assumptions upon which the digital models of this report were based includes vector volume techniques requiring that the flow lines within each vector volume be parallel to one another (see figure 2). This assumption is a valid one with the exception of nodes near pumping wells. Near a pumped well node the flow lines converge on one another and the flow cross section continuously decreases toward the well.

Studies related to the effects of the converging flow problem around pumped well nodes of electric analog models were reported by Prickett (1967) and Rushton and Herbert (1966). These studies showed that if an analog model was constructed on the assumption of parallel flow lines within the vector volumes adjacent to a pumped well node, then the effective radius of the simulated fully penetrating pumped well can be computed from the following formula (Prickett, 1967).

$$r_w = a/4.81 \quad (63)$$

where

$r_w$  = effective radius of the simulated pumped well, in feet

$a$  = finite difference grid interval, in feet

Therefore, the drawdown or head measured at a pumped well node should be related to a well with a radius given by equation 63.

Similar studies were made while developing the digital models of this report, and the results were identical to those in the electric analog studies. Therefore, the drawdown or head calculated at a pumped well node of the digital model should also be related to a well having a radius given by equation 63.

Further, the following formula may be used to obtain the drawdown or head in a well which has a radius different from that given by equation 63.

$$s = 0.3665 (Q/T) \log (a/4.81r_w) \quad (64)$$

where

$s$  = additional drawdown or head decline to be added to calculated value from pumped well node of the digital model, in feet

$T$  = aquifer transmissivity in vicinity of pumped well, in gpd/ft

$Q$  = pumping rate of the well, in gpd

$\log$  = the logarithm with base 10

Equation 64 was derived directly from the Theim (1906) equation.

The effects of partial penetration of the production well, well losses, and gravel packs can also be digitally simulated, with modifications to the programs, by methods developed by Prickett (1967).

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