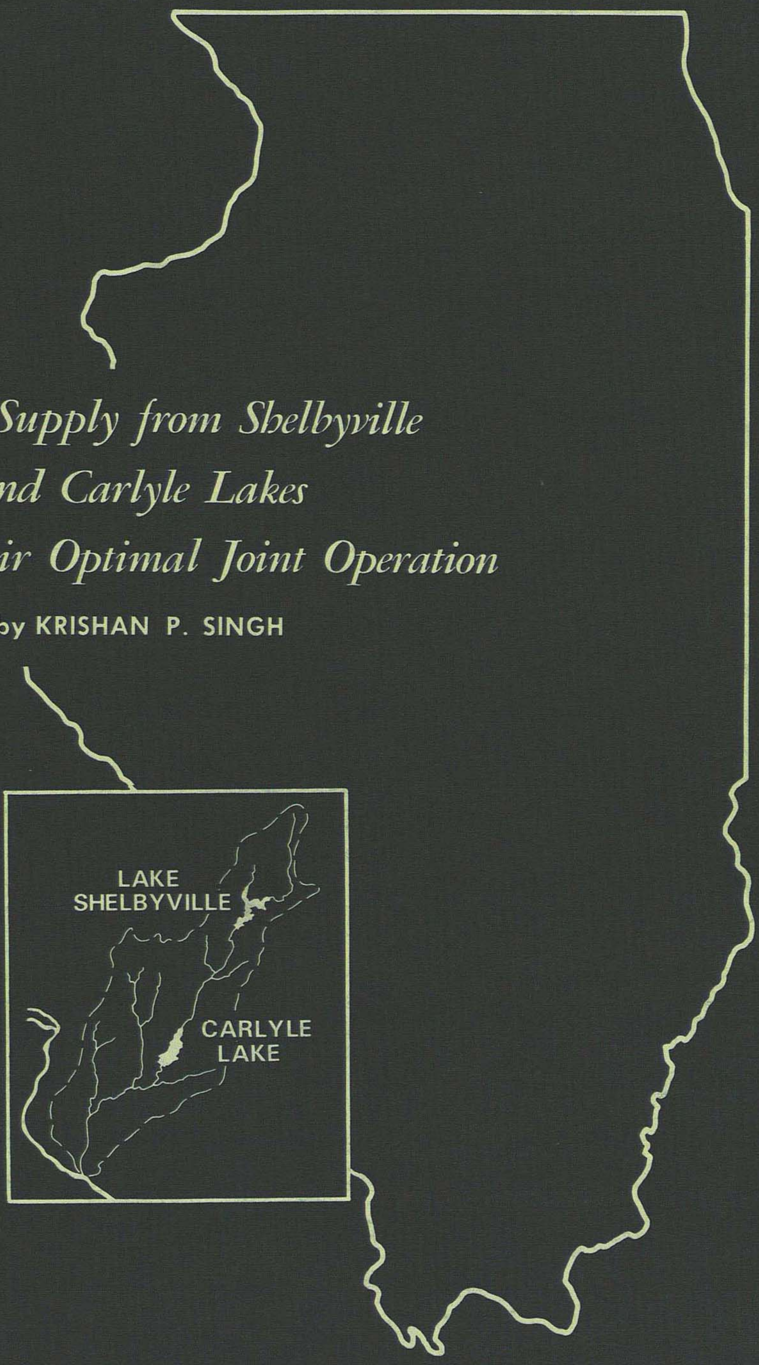
REPORT OF INVESTIGATION 84
STATE OF ILLINOIS
DEPARTMENT OF REGISTRATION AND EDUCATION

Water Supply from Shelbyville and Carlyle Lakes and Their Optimal Joint Operation

by KRISHAN P. SINGH



ILLINOIS STATE WATER SURVEY
URBANA
1977

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Water Supply from Shelbyville and Carlyle Lakes and Their Optimal Joint Operation

by **KRISHAN P. SINGH**

Title: Water Supply from Shelbyville and Carlyle Lakes and Their Optimal Joint Operation.

Abstract: The U.S. Army Corps of Engineers, St. Louis District, has been regulating Lake Shelbyville and Carlyle Lake on the Kaskaskia River in Illinois for recreation and flood control since completion of the dams in 1969 and 1967, respectively. These lakes will soon have to be regulated for the multiple purposes for which they were built, that is, recreation, flood control, water supply, and navigation. The state of Illinois has a reserve storage capacity of 25,000 ac-ft in Shelbyville and 33,000 ac-ft in Carlyle for meeting future water supply requirements. Optimal operating policies for the joint regulation of Shelbyville and Carlyle lakes for various levels of navigation and water supply requirements have been derived from more than 10,000 system simulation runs made with historical and synthetic data. The optimal operation rules indicate a rise in rule levels with increase in navigation and water supply requirements. The joint regulation can be updated with development of navigation and water supply to obtain the maximum benefits. When state and federal storages are not considered separate and distinct, and the total storage is regulated in the best interest of all purposes as done in deriving the optimal operating rules, the minimum lake levels reached are higher than the dead storage pool elevations even with 200 cfs (130 mgd) water supply withdrawal and full navigation.

Reference: Singh, Krishan P. Water Supply from Shelbyville and Carlyle Lakes and Their Optimal Joint Operation. Illinois State Water Survey, Urbana, Report of Investigation 84, 1977.

Indexing Terms: Carlyle Lake, computer model, evaporation, flood control, Lake Shelbyville, navigation, operations research, optimal joint operation, precipitation, recreation, reservoir operation, statistical methods, streamflow, synthetic hydrology, systems analysis, water supply.

STATE OF ILLINOIS
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DEPARTMENT OF REGISTRATION AND EDUCATION
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URBANA
1977

Printed by authority of the State of Illinois—Ch. 127, IRS, Par. 58.29
(6-77-2000)

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ABSTRACT

The U.S. Army Corps of Engineers, St. Louis District, has been regulating Lake Shelbyville and Carlyle Lake on the Kaskaskia River in Illinois since completion of the dams in 1969 and 1967, respectively. So far these lakes have been operated primarily for recreation and flood control. The navigation channel below Fayetteville to the Mississippi River has now been completed. The demand for water supply by the industries and towns in the Kaskaskia River basin below Shelbyville and Carlyle has been increasing during the last 2 years. The lakes will have to be regulated within a few years for the multiple purposes for which they were conceived and built, that is, recreation, flood control, water supply, and navigation.

In consideration of cost contributions by the state of Illinois to the Shelbyville and Carlyle projects and to operation, maintenance, and repair (OM&R) costs, the state was allocated a reserve storage capacity of 25,000 acre-feet (ac-ft) in Shelbyville and 33,000 ac-ft in Carlyle for meeting future water supply requirements. Analyses of the weekly flows, precipitation on and evaporation from the lakes, and navigation flow requirements for the 24-year (1942-1965) historical period and a 499-year period of synthetic data indicate that under the terms of the agreement between federal and state governments 100 cubic feet per second (cfs) or 65 million gallons per day (mgd) water supply withdrawal can be made from the system with an average deficit frequency of 1 in 50 years. A relationship has been established between the amount of water supply withdrawal and expected deficit frequency. A 100-cfs water supply withdrawal increases the recreation and agricultural damages (or benefits not realized) compared with no water supply. This increase is 0.118¢ per 1000 gallons of water supplied.

Optimal operating policies for the joint regulation of Shelbyville and Carlyle lakes, for various levels of navigation and water supply requirements, have been derived from more than 10,000 system simulation runs made with the historical and synthetic data. The generation of synthetic weekly flows at Shelbyville, Carlyle, and New Athens, precipitation on and evaporation from the two lakes, and navigation flow requirements for 499 years was a major task.

The optimal operation rules indicate a rise in rule levels with increase in navigation and water supply requirements. The joint regulation can be updated with development of navigation and water supply to obtain the maximum benefits from these lakes. The sensitivity of damages to departures from optimal rule levels is presented, together with major policy decisions affecting the desired operation of the lakes.

When state and federal storages are not considered separate and distinct, and the total storage is regulated in the best interest of all purposes as done in deriving the optimal operating rules, the minimum lake levels reached are higher than the dead storage pool elevations even with 200 cfs water supply withdrawal and full navigation. Therefore, a 200-cfs (130 mgd) water supply can be obtained from the two-lake system by following optimal operating policies.

INTRODUCTION

Dams were built by the U.S. Army Corps of Engineers on the Kaskaskia River at Shelbyville and Carlyle in Illinois in 1969 and 1967, respectively. The operation of Lake Shelbyville and Carlyle Lake has changed over the years because experience has shown the bankfull channel capacities downstream of the dams to be considerably lower than were adopted in the project designs. A previous study was conducted (Singh et al., 1975) to make an overall investigation of the regulation plans and to find if any improvement could be made to maximize the agricultural and recreational benefits to the state under the condition of no water supply and no navigation requirements. The intent was to derive an optimal policy for maximizing the overall benefits and to compare these benefits with those under the existing Corps policy for operating the lakes.

The optimal policy was based on results from a dynamic programming model and optimization of operating rules through a system simulation model structured for the physical nature of the system. The priorities for rule levels and releases as indicated by hydrology and economic tradeoffs between conflicting requirements were also considered. The optimal operation developed from 24 years of historical data cut down the expected annual damage, or benefit foregone because of adverse lake levels and flow releases, to 24 percent of the damage expected under the Corps operating policy. The optimal operation indicated significant changes in rule levels and flow releases for minimizing damages or maximizing benefits.

Objectives of This Study

The present study was undertaken to evaluate the levels of water supply demands or requirements that can be met from the state storage in the two lakes and the associated deficit frequencies, and to develop a matrix of optimal rule levels and release rules for varying navigation flow and water supply requirements.

Water Supply. The state of Illinois has storage allocations of 33,000 ac-ft in Carlyle Lake and 25,000 ac-ft in Lake Shelbyville. These storage allocations were made in return for the project costs shared by the state, and were reserved for meeting future requirements of water for municipal, industrial, and rural purposes. So far little use has been made of the water supply available from the two lakes, but substantial use is visualized during the next few years. The use of this supply is controlled by the Division of Water Resources, Illinois Department of Transportation.

The existing and presently governing low flow operating rules for these lakes (U.S. Army Corps of Engineers, 1964) were made on the basis of contracts between the federal government and the state of Illinois. Assuming a maximum duration of 2 years for a drought with zero net inflow to the lakes, steady withdrawals of 17 cfs at Shelbyville and 23 cfs at Carlyle would exhaust the state storages. Information was needed on various magnitudes of water supply withdrawals that can be made from Shelbyville and Carlyle lakes and the associated deficit frequencies. This information would aid not only in planning an efficient system of spaced water withdrawals from and returns to the river, but also in optimizing sales to different users depending on the variability of their requirements, consumptive use, and quality of return water.

An historical record of 24 years, 1942-1965, for 1) flows at Shelbyville, Carlyle, and New Athens; 2) net precipitation over the lake surfaces;and 3) navigation flow requirements can be used in low flow regulation. This record includes the drought of 1954, believed to have a recurrence interval from 75 to more than 100 years. It was necessary to generate synthetic hydrologic information of about 500 years with respect to flows, net precipitation, and navigation requirements in order to assess the deficit frequencies associated with different levels of water supply withdrawals.

Optimal Operating Rules. The Shelbyville and Carlyle projects were authorized under the Flood Control Acts of 1938 and 1958 for purposes of flood control, navigation releases, water supply, and recreation. They are presently being operated for flood control and recreation purposes. The navigation lock and dam at mile 0.8 of the Kaskaskia River and the navigation channel from the mouth of the river to Fayetteville in St. Clair County have been completed. The pool level in the navigation channel at the lock and dam is 368 feet above mean sea level.

The river traffic will grow over the years from zero at the present to the maximum envisaged in the navigation project. This growth period may be 10 to 20 years. Optimal operating rules for 0, 50, and 100 percent of the maximum navigation flow requirement were needed for flexibility in operation as navigation develops. The same holds true for water supply. Though the maximum water supply requirement has not been worked out, six levels of water supply (0, 40, 80, 120, 160, and 200 cfs) were used in deriving optimum operation rules. The combination of navigation and water supply requirements yields a total of 18 sets.

The available historical record is just a sample of hundreds of such samples that constitute the population. The optimal operating rules derived with the historical data for maximizing the overall benefit from recreation, agriculture, water supply, and navigation for each of the 18 sets had to be checked, verified, and substantiated with 10 synthetic sequences of 49 years each. The historical record does not contain flows high enough to raise the lakes to flood pools. The synthetic data filled that gap and indicated also the best operation in the range of high levels. The use of 10 synthetic sequences also underscored the variability of damages from one sequence to the other, dispelling the notions based on the historical data alone.

The optimal operation rules under varying navigation and water supply requirements provided the expected value of annual benefits from different uses. Sensitivity analyses were done to indicate the reduction in benefits or increase in damages with departures from the optimal rules.

Acknowledgments

This one-year study was jointly supported by the Division of Water Resources of the Illinois Department of Transportation, and the Illinois State Water Survey. Roger H. Smith and John K. Flowe of the Division of Water Resources served in a liaison capacity during the course of the study.

Douglas Noel, part-time graduate Research Assistant, helped greatly in data handling and processing, computer programs, and system simulations for the first nine months of the study. Stephanie Dean, an undergraduate student at the University of Illinois, helped later in making more simulation runs and tabulations. Many other Water Survey personnel contributed. Robert A. Sinclair, Systems Analyst, and Carl G. Lonnquist, Numerical Analyst, provided computer counseling when needed. Mrs. Patricia A. Motherway and Mrs. J. Loreena Ivens edited the report, and Mrs. Suzi O'Connor prepared the camera-ready copy. John W. Brother, Jr., supervised the preparation of illustrations.

The study was made under the general supervision of Dr. William C. Ackermann, Chief of the Water Survey, and John B. Stall, Head of the Hydrology Section.

WATER SUPPLY FROM LAKES SHELBYVILLE AND CARLYLE

The following are the main points of the contracts between the state and federal governments regarding storage in the Shelbyville and Carlyle lakes.

1) Water supply storage capacity of 25,000 ac-ft in Shelbyville and 33,000 ac-ft in Carlyle will be provided for the state of Illinois. Corresponding federal storage would be 155,000 and 200,000 ac-ft, respectively, making the joint-use storage 180,000 and 233,000 ac-ft.

	Lake Shelbyville	*Carlyle Lake*
Top of dead storage pool		
Elevation, *ft msl*	573.0	429.5
Storage, *ac-ft*	30,000	50,000
Top of joint-use pool		
Elevation, *ft msl*	599.7	445.0
Storage, *ac-ft*		
State	25,000	33,000
Federal	155,000	200,000
Total storage (includes dead storage)	210,000	283,000

2) The inflows to the lakes will be apportioned between state and federal storage as shown in figure 1 (U.S. Army Corps of Engineers, 1964). These inflows will be adjusted for rainfall over and evaporation from the lakes. A minimum low flow release of 10 cfs below Shelbyville and 50 cfs below Carlyle is guaranteed for downstream water quality control. Part of the navigation release required may be met from the Shelbyville federal storage. Any water apportioned to federal storage in excess of the allocated storage will be credited to the state's water supply account but not to exceed the state's allocated water supply storage.

3) The state of Illinois will be responsible for issuing permits and administering the water supply withdrawals from its account.

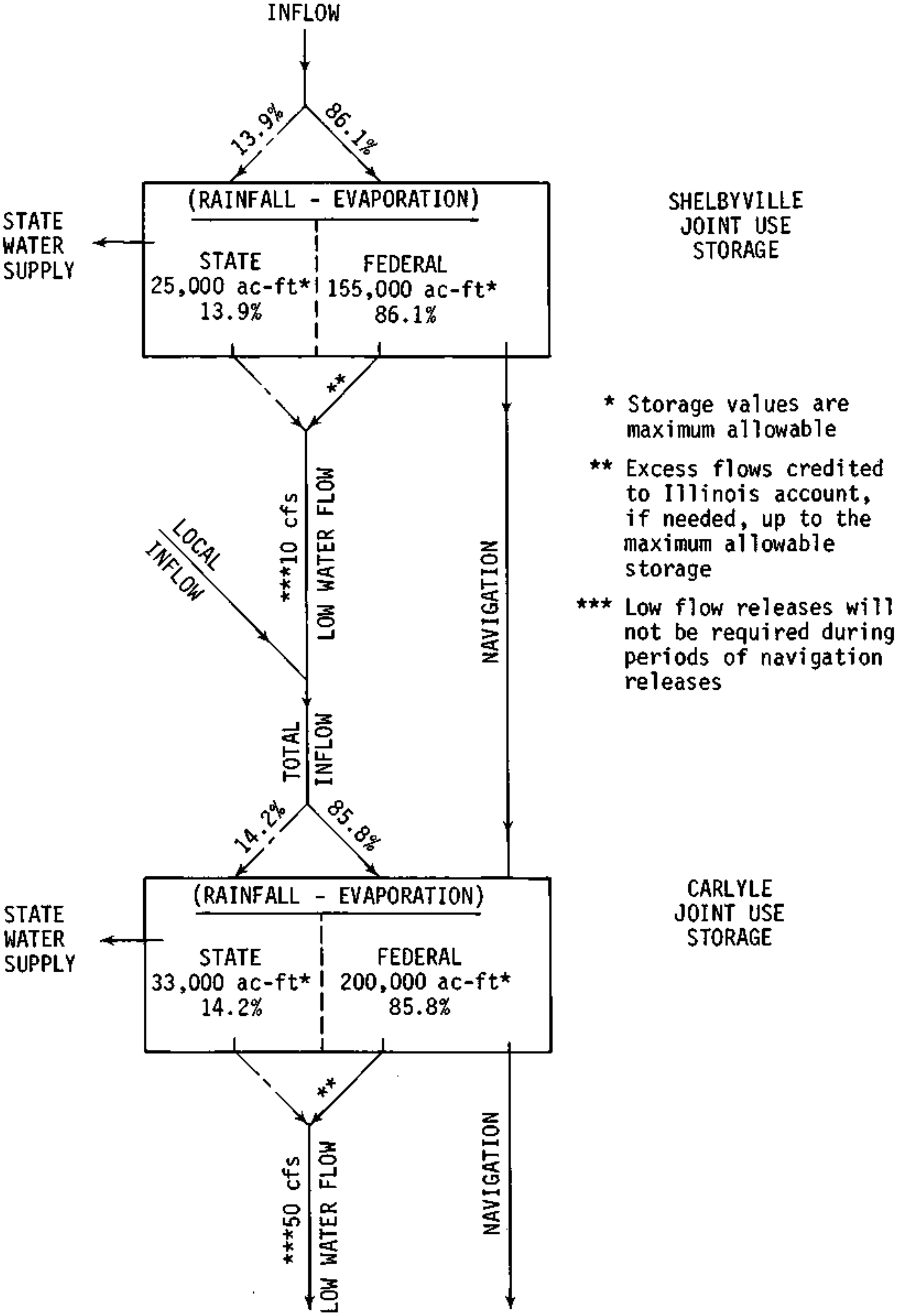
4) The two accounts will be debited for their proportionate share of low flow releases of 10 and 50 cfs.

As a part of this study a computer program incorporating the terms of the contracts has been developed to calculate the quantity of water supply that can be met from the state storage in the two lakes for various deficit frequencies.

Historical Record

The available weekly flows at Shelbyville, Carlyle, and New Athens gaging stations observed over a 24-year period from 1942 through 1965, and the concurrent weekly lockage requirements for navigation were used in making computer runs for various levels of supply and in determining any deficiencies associated with them. System water supply was divided in the ratio of the state storage in the two lakes, but a provision was made for meeting the total demand from one lake if the water supply storage in the other lake was exhausted. Navigation release in cfs from Shelbyville was taken as 155,000/(155,000 + 200,000) or 44 percent of the total release. Provision was made for the interaction of the storages if the federal storage in one lake was exhausted. The results of the computer analyses are given in table 1.

The deficit weeks, when water supply was unavailable, mostly occurred in the latter half of the 1954 calendar year (or over the last part of water year 1954 and the early part of water year 1955). The 1954-drought, though it occurred in the period 1942-1965, is believed to have



* Storage values are maximum allowable

** Excess flows credited to Illinois account, if needed, up to the maximum allowable storage

*** Low flow releases will not be required during periods of navigation releases

Figure 1. Apportionment of inflows to state and federal storages in Shelbyville and Carlyle Lakes

Table 1. Historical Water Supply Data from Shelbyville and Carlyle Lakes

Water supply (cfs)	Deficits Weeks (1954–1955)	(ac-ft)	Minimum federal storage (ac-ft)
50	0	(+1,004)	243,850
75	26	27,066	244,600
100	40	55,520	244,850
125	48	83,280	244,950
150	60*	124,920	245,000

*Another 3 deficit weeks in 1965

a recurrence interval of 75 to more than 100 years (Hudson and Roberts, 1955). It is of interest to note that the federal reserve of 355,000 ac-ft never fell below 243,850 ac-ft. It is quite conceivable, therefore, that a water supply of 100 cfs can be easily obtained by using about 20 percent of the unused federal storage.

Daily flows are available for the years 1915 through 1975, i.e., 61 years, at the Vandalia gaging station between Shelbyville and Carlyle on the Kaskaskia River. For a drainage area of 1980 square miles at this station, the six lowest mean annual flows are 52.6, 123, 365, 400, 416, and 446 cfs for the water years 1954, 1931, 1941, 1934, 1955, and 1940, respectively. Further analysis has shown that the low flow periods starting and ending with months having monthly mean flow less than 200 cfs, are:

1) August 1953 through December 1954: 17 months mean monthly flow for the period = 48.2 cfs
2) July 1930 through March 1931: 9 months mean monthly flow for the period = 50.6 cfs
3) July 1940 through March 1941: 9 months mean monthly flow for the period = 72.5 cfs

It is evident that the 1954 drought flow was not only the lowest in the 61 years of the flow record but also the drought duration was double that of the second and third droughts. Thus it stands to reason that this drought may have a frequency of 75 to more than 100 years.

Synthetic Record

In order to obtain information on the deficits and associated frequencies, the 499-year synthetic weekly flows at Shelbyville, Carlyle, and New Athens and corresponding weekly navigation releases at the lock and dam were used in making computer runs for different levels of water supply. The results are given in table 2, in which n is the number of times the deficit occurs in 499 years, w is the average number of weeks in each occurrence, and subscripts 0, 3, and 6 denote values of n and w neglecting occurrences with deficit durations of 0, 3, and 6 weeks. Maximum durations of deficit for supplies of 75, 100, 125, and 150 cfs were 8, 20, 31, and 35 weeks, respectively. Minimum total federal storage was never less than 221,000 ac-ft during the water supply deficit periods.

Table 2. Synthetic Water Supply Data from Shelbyville and Carlyle Lakes

Water supply (cfs)	Deficits a* n_0	 a* w_0	 b* n_3	 b* w_3	 c* n_6	 c* w_6
50	0	0	0	0	0	0
75	4	5	2	7	1	8
100	10	13	8	15	7	16
125	25	12	21	13	16	16
150	51	11	39	13	25	17

*See figure 2

Meeting Water Supply Demands

Water for Municipal and Industrial Demands. The results from the analyses of historical (24 years) and synthetic (499 years) weekly flow data are shown in figure 2. Assigning a recurrence interval of 75 to 100 years for the 1954 drought, a water supply of 100 cfs (shown by star in figure 2) with the expectation of a deficit of once in 50 years can be sustained from the

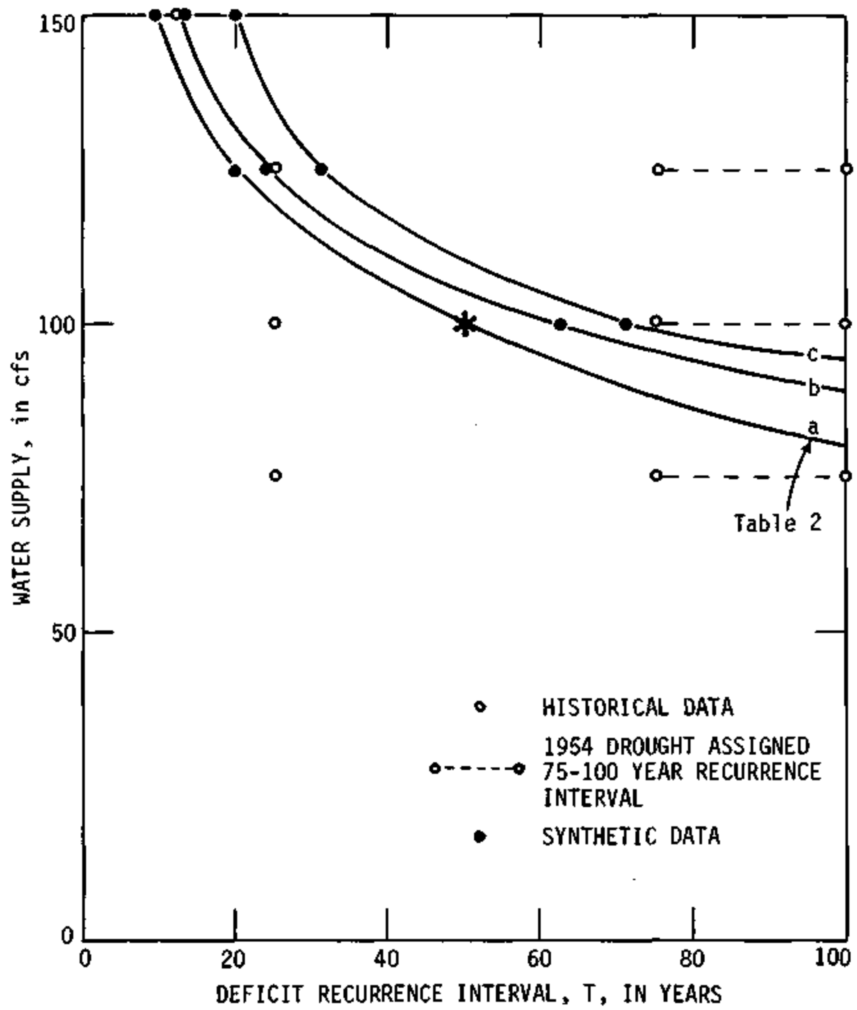


Figure 2. Water supply from state storages in Shelbyville and Carlyle Lakes

state storage in the two lakes in accordance with conditions of the contract and the stipulated dead storage and joint-use storage pool levels.

A water supply of W cfs can be used to meet a municipal demand of M cfs and an industrial demand of I cfs, taking into consideration the consumptive use of the two demands and suitable return flows to the Kaskaskia River after proper treatment. In the expression

$$W = MC_m + IC_i$$

C_m denotes the average ratio of consumptive use to demand for municipalities, C_i represents the average ratio of consumptive use to demand for industries, and individual municipal and industrial demands are considered small and numerous. Thus, M and I are limit values. The actual values will be somewhat smaller. As an example, with M = 100 cfs, $C_m = 0.2$, I = 200 cfs, and $C_i = 0.4$, the required water supply from the state storage is 100 cfs. The lower the values of C_m and C_i the higher will be the values of M and I for a given value of W; in other words, much higher municipal and industrial demands can be met by suitable withdrawals and returns to the river in order to make the best use of water supply W.

Tributary Inflow below Carlyle to New Athens. A minimum of 30 cfs daily flow (Curtis, 1969) can be assumed to enter the Kaskaskia River between Carlyle and New Athens. Part of this flow may be used for water supply and returned to the river after use.

Meeting Lockage Requirements from the Mississippi River. It is possible to use federal storage, which sustains navigation flow releases, for water supply if needed for strategic industries or other preferred users. Cost of pumping water from the Mississippi River into the lock will include operation, maintenance, and repair costs and the amortized value of the pump and conveyance facility. If these costs are less than the revenue from the water supply under consideration, the pumping alternative to further increase water supply may become a viable solution.

The cost of pumping should not exceed the difference in overall damages with full navigation and with no navigation. The damages are given in table 3 for optimal operation based on the historical data of 24 years.

Table 3. Damages, in Thousand Dollars, over 24 Years

W (cfs)	*Full navigation*	*No navigation*	*Difference*	*Annual difference*
0	1668.9	1619.0	49.9	2.08
40	1855.8	1688.0	167.8	6.99
80	2164.9	1887.2	277.7	11.57
120	2553.8	2066.0	487.8	20.33
160	3028.1	2319.7	708.4	29.52
200	3581.6	2733.9	847.7	35.32

The capacity of the pumping facility is determined by the maximum difference in level between 368 ft and the Mississippi River stage. It is 28 ft, and the lockage requirement, in cfs, for continuous pumping throughout the day is 28 x 11.2 - (50 + 30), or 234; 50 cfs is the minimum low flow release below Carlyle, 30 cfs is assumed to be the minimum contribution from the drainage area between Carlyle and New Athens, and 11.2 cfs is the navigation flow required for a 1-foot difference in water levels between the navigation channel and the Mississippi River (Singh et al., 1975). Cost of the pumping station (Singh, 1971) is estimated at $369,000 with the cost index in 1976 double that in 1966. For a 25-year pump life and 8 percent discount rate, the annual cost of the pumping station alone is calculated as $34,600. The capacity of the pumps will be approximately 3 times the demand or more, to restrict the filling time to one-third of the day or less. Thus, the annual cost of a pumping station will exceed $100,000. Pipeline cost, pumping cost, and operation, maintenance, and repair costs would be additional. Therefore, the pumping alternative is not justifiable considering only the economics of pumping costs and reduction in damages.

HISTORICAL DATA

Data for the Years 1942-1965

In the previous study (Singh et al., 1975) joint operation rules, with no water supply and with either no or full navigation flow requirement, were developed for the Shelbyville and Carlyle lakes with the use of hydrologic and other data for the water years 1942-1965. The same base period is utilized in this study for deriving a matrix of rule levels and operation rules for various levels of navigation and water supply requirements. The data inputs used in this study are described here, together with any variations from those used in the previous study.

Streamflows. The daily flows at four gaging stations — Shelbyville, Vandalia, Carlyle, and New Athens (figure 3) — were used from the concurrent flow record of 24 years (1942-1965).

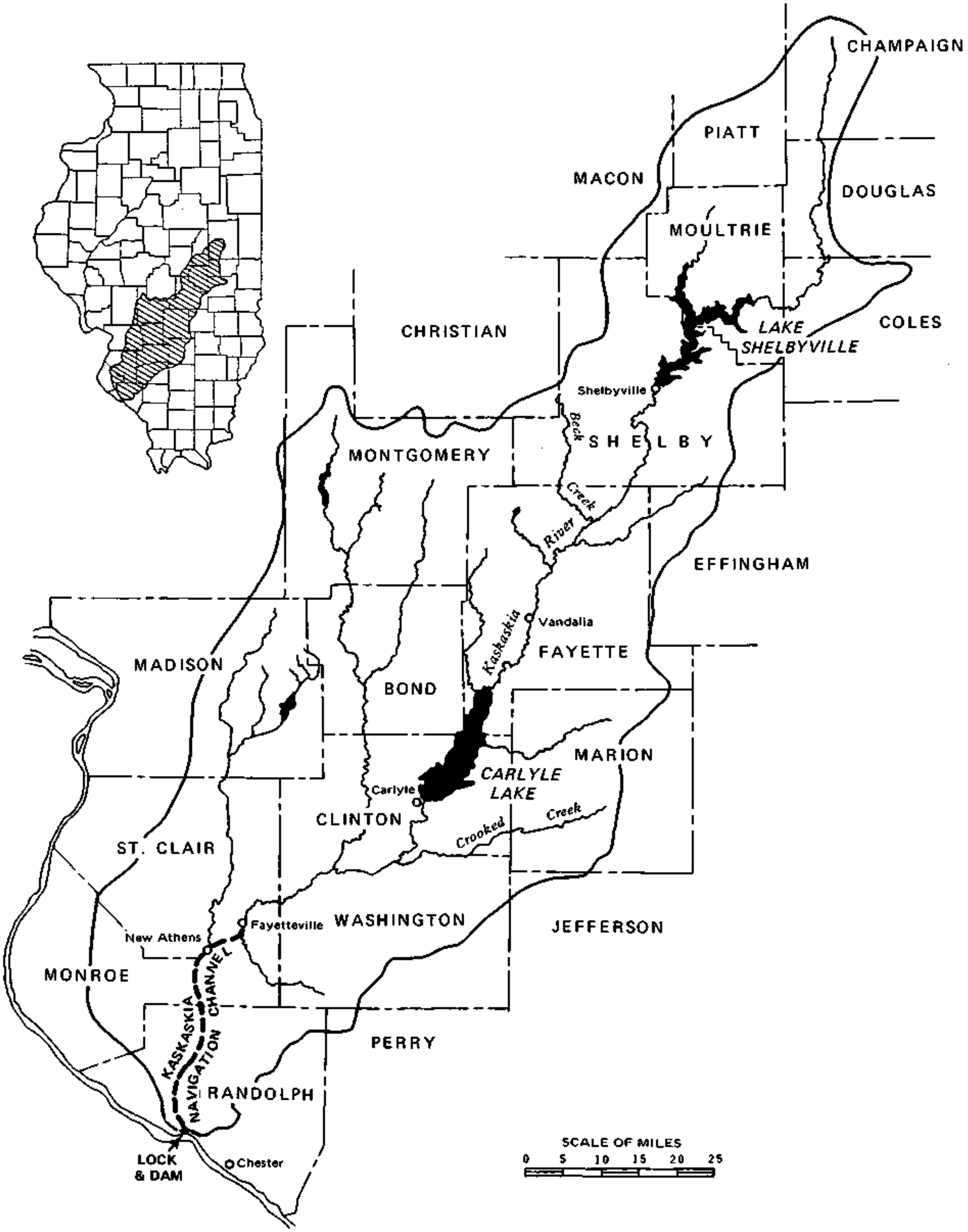


Figure 3. Location map showing Lake Shelbyville and Carlyle Lake

This 24-year period excludes years when streamflows were modified either by construction activities or by regulation of the lakes (the first of the two dams was at Carlyle and it was completed in 1967). A statistical analysis of daily flows indicated a suitable lag or travel time of 2 days from Shelbyville to Carlyle and 1 day from Carlyle to New Athens. The daily flows were aggregated to weekly flows, considering 52 weeks in a year and allowing proper lags for travel times. The weekly flows at Shelbyville and Carlyle have been used in defining the inflows to the lakes, and the flows at Carlyle and New Athens have been used in calculating any flow releases needed from the lakes for meeting navigation flow requirements.

Navigation Flows. In the previous study, the Mississippi River stage observed at Chester was assumed to apply at the confluence with the Kaskaskia River 7.6 miles upstream. In this study, the river stage at the confluence is interpolated between those at St. Louis and Chester and is roughly 4 feet higher than that at Chester. This has reduced the navigation flow requirements.

The navigation lock at 0.8 mile of the Kaskaskia River is 600 x 84 feet with a normal pool of 368 feet above mean sea level (msl) during low flow conditions (U.S. Corps of Engineers, 1964). Ten synchronous and six other lockages were assumed for an average day when the river transportation is fully developed. The Mississippi River stage data are available at Chester (7.6 miles downstream of the Kaskaskia River) and at St. Louis (62.5 miles upstream).

	Mississippi River at Chester	*Mississippi River at St. Louis*
USGS station number	7-0205	7-0100
Above Ohio River, *mile*	109.9	180.0
Gage datum, *ft above msl*	341.05	379.94

The Mississippi River stage at the confluence with the Kaskaskia River (at mile 117.5) was calculated from

$$\mathrm{WSLD} = \mathrm{WSC} + (117.5 - 109.9)\,(\mathrm{WSS} - \mathrm{WSC})/(180.0 - 109.9)$$

$$\mathrm{WSS} = 379.94 + \mathrm{SS}$$

$$\mathrm{WSC} = 341.05 + \mathrm{SC}$$

in which WSLD, WSS, and WSC denote water surface level, in ft above msl, at the downstream of Kaskaskia lock and dam, at St. Louis, and at Chester, respectively; and SS and SC are the Mississippi stages observed at St. Louis and Chester. Concurrent stage records were available for the water years 1943 through 1965. Stage data for 1942 were not available at Chester, but values of WSLD were derived for that year by the use of stage data at St. Louis and information from 23 years of concurrent data.

Daily navigation flow requirement, cfs

$$= (16 \times 600 \times 84 \times h \times 1.2)/86400$$

$$= 11.2\,h \quad \text{for } h > \text{zero}$$

in which h = 368.0 -WSLD, in feet; and the multiplier 1.2 allows for a 20 percent increase to account for leakage and evaporation loss. The daily flow requirements are aggregated to weekly flow requirements, which are translated to the weekly flow release from Carlyle, Q_{cn}.

$$Q_{cn} = 11.2h - (Q_{na} - Q_{ch}) \qquad \text{but} \nless \text{min flow release}$$

In the above expression, Q_{na} denotes the weekly flow recorded at the New Athens gage and Q_{ch} is the weekly flow at the Carlyle gage, the week starting 1 and 2 days earlier than that for calculating 11.2h. The analysis of available data shows that during low flow periods, the use of weeks instead of days is satisfactory.

Precipitation on Lakes. Precipitation records at Shelbyville and Carlyle were used to determine the contribution to lake inflows by direct precipitation over the lake surfaces. In the previous study, the average monthly precipitation was used instead of the historically observed values which vary from month to month and for the same month in different years.

For the 24-year period (1942-1965), precipitation data at Carlyle are available from *Climatological Data,* published by the U.S. Department of Commerce. The raingage station at Shelbyville was discontinued in the early 1940s, making it necessary to generate a simulated 24-year precipitation record at Shelbyville.

A simple linear model was used to compute monthly precipitation at Shelbyville, P_s, from monthly precipitation P_c and P_u at Carlyle and Urbana, respectively. Value of P_s was obtained from

$$P_s = (3P_c + 4P_u)/7$$

and monthly precipitation at Vandalia, P_v, was checked by

$$P_v = (2.7P_s + 5.3P_c)/8$$

considering the distances between the towns. An average annual precipitation of 38.33 inches at Vandalia was obtained with the above expressions and it compared very favorably with the historical value of 38.26 inches for the same period. Computed and observed monthly precipitation also showed similar fits. The linear model was therefore considered satisfactory for generating monthly precipitation at Shelbyville from those at Carlyle and Urbana.

The monthly precipitation was converted to weekly values, assuming uniform distribution within each month and allowing a 2-day lag for the start of the week at Carlyle. Precipitation over the lake significantly affects only the low inflows to the lakes; for example, 1 inch of precipitation in a week on Lake Shelbyville, when it is at a level of 595 ft, contributes 56.7 cfs. For extended low flow periods many weeks are involved in the lowering of lake levels, and minor flow variations in contiguous weeks are not of much consequence when the monthly contribution is matched by the sum of weekly contributions.

Evaporation from Lakes. Evaporation from the lake surface reduces the storage in the reservoir. In the previous study, average values of monthly evaporation from Shelbyville and Carlyle lakes were derived from the data at four stations in the general vicinity of the lakes. These stations were located at Washington University (St. Louis, Missouri) and at Springfield, Carbondale, and Urbana, Illinois.

The available meteorological data contain the pan evaporation at Carlyle for the years 1963 through 1974, but the data for 1972 through 1974 are incomplete. Pan evaporation data are not available at Shelbyville, but are available at Urbana. The relevant concurrent data at Carlyle and Urbana are given in table 4 for the months April through October. No values are available for the months November through March.

Pan-to-lake coefficients for converting pan evaporation to lake evaporation for all months (Roberts and Stall, 1967) are available for 10 towns in Illinois. The coefficients for Carlyle were computed from the Springfield and Carbondale coefficients, weighted 2 to 1. The Shelbyville coefficients were obtained from the Urbana and Carlyle coefficients, with a slightly higher weight given to Urbana. The coefficients for Shelbyville, Carlyle, and Urbana are given in table 5.

The monthly pan evaporation observed at Carlyle and Urbana for the concurrent record of 1963 through 1974 were multiplied with the applicable pan-to-lake coefficients. Correlations were developed between the computed monthly lake evaporation at Carlyle and Urbana to derive

Table 4. Observed Monthly Pan Evaporation at Carlyle and Urbana*

Year	Station	Monthly pan evaporation (inches) Apr	May	Jun	Jul	Aug	Sep	Oct
1963	Carlyle	6.34	7.22	9.29	8.82	7.42	7.02	6.90
	Urbana	5.57	7.02	9.48	8.00	5.37	5.44	5.67
1964	Carlyle	5.63	8.05	8.39	8.20	8.95	6.35	4.93
	Urbana	4.51	7.62	8.39	7.15	7.86	5.56	4.18
1965	Carlyle	5.65	7.76	8.70	8.24	8.65	5.05	4.13
	Urbana	4.08	7.71	7.97	7.83	6.60	4.91	3.65
1966	Carlyle	4.75	6.87	8.84	10.73	7.42	5.10	4.26
	Urbana	4.00	6.84	9.07	10.17	7.36	5.61	3.99
1967	Carlyle	6.12	7.04	7.58	7.50	6.86	5.60	4.07
	Urbana	4.40	6.61	7.18	8.22	6.66	5.35	2.92
1968	Carlyle	6.30	5.72	8.41	8.80	8.04	5.41	4.73
	Urbana	5.13	6.01	8.02	8.73	6.95	4.56	3.78
1969	Carlyle	5.13	5.91	8.59	7.74	7.57	4.94	3.46
	Urbana	4.34	6.53	7.65	7.77	7.35	5.47	3.85
1970	Carlyle	5.75	8.04	7.19	8.95	7.27	6.01	3.48
	Urbana	5.10	7.76	7.92	10.14	7.69	5.34	3.51
1971	Carlyle	6.51	6.56	9.19	7.99	7.56	6.30	4.53
	Urbana	6.08	7.39	10.22	8.44	7.00	4.70	4.45
1972	Carlyle	4.68	7.73	10.07	8.84	6.88		3.52
	Urbana	3.81	6.64	7.53	7.64	6.34	5.19	2.58
1973	Carlyle	4.44	6.85	8.15	8.96	8.05	4.80	
	Urbana		6.25	7.89	8.10	7.73		
1974	Carlyle	6.22	5.96	7.73	10.44		4.85	4.16
	Urbana		3.70	5.49	7.49	4.96	3.43	2.64

**from Illinois Meteorological Data, Annual Summaries (1963-1974)*

Table 5. Average Monthly Pan-to-Lake Coefficients
for Shelbyville, Carlyle, and Urbana

	Average monthly pan-to-lake coefficients Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Shelbyville	0.65	0.75	0.79	0.78	0.79	0.78	0.77	0.76	0.74	0.69	0.65	0.60
Carlyle	0.63	0.71	0.74	0.75	0.76	0.74	0.74	0.73	0.71	0.67	0.62	0.60
Urbana	0.66	0.78	0.82	0.81	0.81	0.81	0.79	0.79	0.76	0.71	0.67	0.60

monthly evaporation at Carlyle for the period 1942-1965 with the use of monthly lake evaporation data available at Urbana (Roberts and Stall, 1967). The regression equation is of the form

$$E_c = a + b\,E_u$$

in which E_c and E_u are monthly lake evaporations at Carlyle and Urbana, in inches, for the month under consideration; and a and b are the intercept and the coefficient. Values of a are plotted in figure 4 for the months April through October. The plot indicates that a is zero for the months November through March.

Carlyle and St. Louis are at about the same latitude, implying a greater similarity in evaporation characteristics at these two stations during the winter months. Therefore, b for any month in the period November through March was taken as $\overline{E}_{st}/\overline{E}_u$, in which $\overline{E}_{st}$ is the long-term monthly lake evaporation at St. Louis, and $\overline{E}_u$ is the long-term monthly lake evaporation at Urbana for the same month. November through March values of $\overline{E}_{st}$ are 1.32,0.72,0.70, 1.05, and 2.00 inches and $\overline{E}_u$ values are 0.82, 0.33, 0.31, 0.61, and 1.39 inches (Roberts and Stall, 1967). The values of b for all months are shown in figure 4.

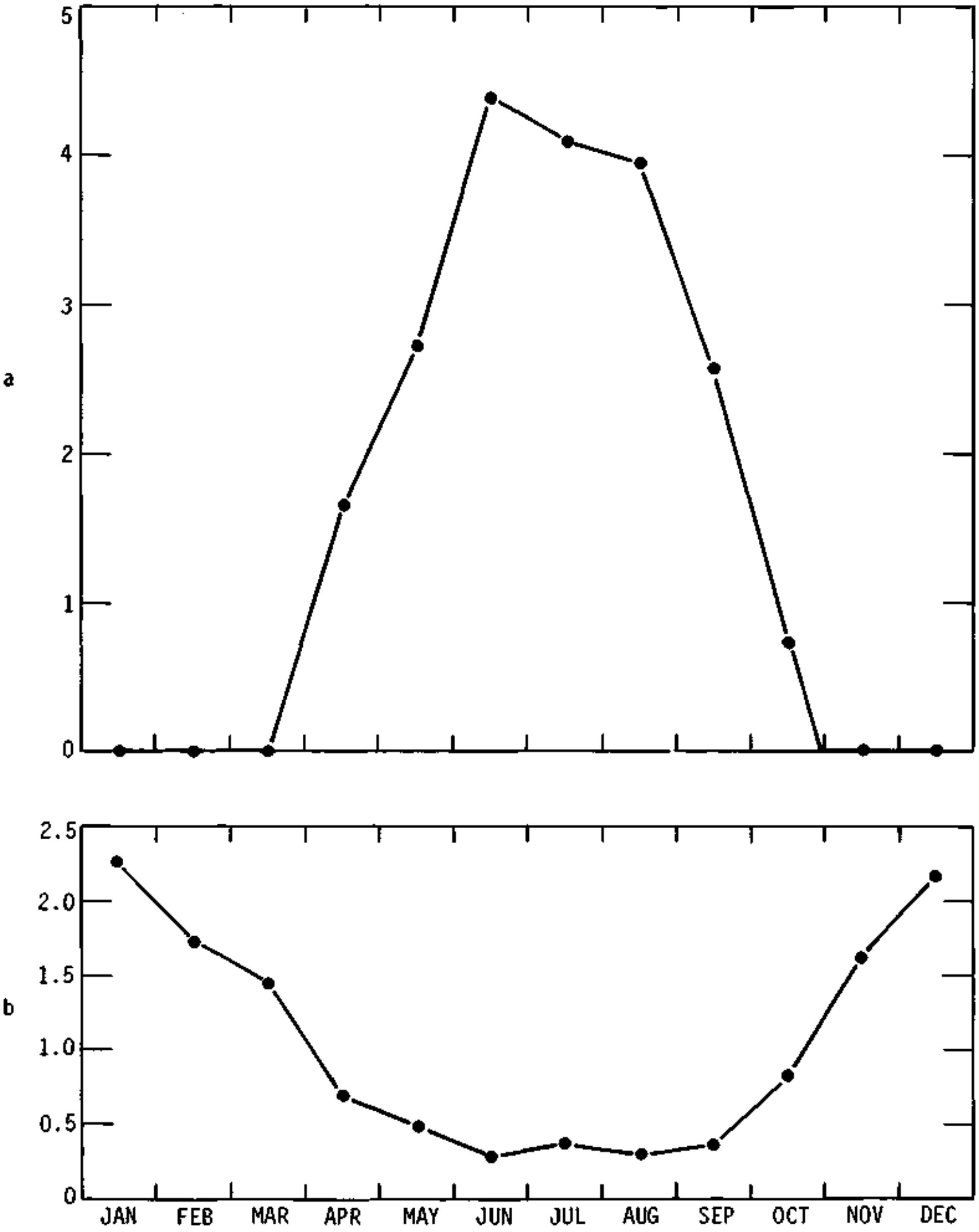


Figure 4. Intercept a and coefficient b in evaporation regression equation

For generating monthly lake evaporation at Shelbyville, a linear model similar to that used for generating monthly precipitation at Shelbyville, was employed. In the expression

$$E_s = (3E_c + 4E_u)/7$$

E_s is the lake evaporation expected at Shelbyville for a given month in a year, when E_c and E_u are the lake evaporation at Carlyle and Urbana for the same month and year. The weekly values of evaporation were derived in a manner similar to that for the weekly precipitation values. Subtraction of evaporation from precipitation for a given week at Shelbyville or Carlyle yielded the net gain or loss in inches of water from the lake surface.

Physical, Agricultural, and Recreational Data

Pertinent physical, agricultural, and recreational data were discussed in detail in the previous study by Singh et al. (1975). To minimize reference to that study, a brief description of these data is included in this report.

Physical Data. Carlyle Lake and Lake Shelbyville were completed by the Corps of Engineers in 1967 and 1969 by building dams across the Kaskaskia River at river mile 106 and 222, respectively, upstream of its confluence with the Mississippi River. Physical data pertaining to these lakes and their operation are given in table 6.

Agricultural Data. Areas under cultivation can be flooded both in the lake areas because of high lake levels and along the river downstream because of high releases that cause overbank flows. The severity and frequency of these damages depend on the efficiency of regulation, though damages cannot be totally avoided because of the rather moderate flood storage capacity of the lakes and the low nondamaging flow capacity of the river channel below the dams.

For agricultural damage assessment the requisite data inputs are: the value of crops, the average crop yields for the respective damage areas, the percent area under different crops, time distribution of various farming operations for each crop, unit monetary values of these operations, direct production investment estimates, drying periods for possible restarting of farming operations after flooding, loss of crops by flooding during growing and harvesting periods, etc.

Main crops in the damage area are corn, soybeans, and wheat. Net prices, per bushel, for ready-to-harvest crops in the fields are taken as $1.50 for corn, $4.00 for soybeans, and $2.10 for wheat. These prices are derived from normalized prices, after subtracting harvesting, transportation to elevators, and drying charges. The crop yields in the areas subject to flooding in the four damage reaches, their dollar yields, and percent areas for various crops are given in table 7. The yield in dollars per typical acre in a damage reach is obtained by multiplying component fractions of an acre under different crops with respective yields in bushel per acre and net price, and summing the products.

Farming operations for a crop are plowing, disking and harrowing, seeding and planting, fertilizing and spraying, cultivating, and harvesting. The normal schedules of these operations were determined by field visits and talks with district officers of the U.S. Soil Conservation Service. For the purpose of an efficient damage assessment program, the direct production investment (DPI) is considered to include investment in farming operations which may have to be repeated after flooding if replanting the same crop or planting a substitute crop is still possible. The DPI costs vary with the week of plowing, usually weeks 31 through 39 for corn and substitution crops, weeks 34 through 39 for soybeans, and weeks 1 through 4 for wheat (week 1 starts October 1). These weeks are termed the *state* weeks, because not only the DPI but also the final dollar yield depend on the state week. For example, a late planted crop yields less because the full land area is not available for planting due to incomplete drainage after flooding, reduction in yield due to insufficient time for optimum crop development, and lack of suitable moisture and climatic conditions for maximum crop growth.

A methodology (Singh, 1976) was developed to compute agricultural damages (or benefits not realized) that would accrue from individual and recurrent flooding, considering the data on 1) land use and crop yields, 2) sequence of farming operations and their unit costs, 3) loss of direct production investment because of flooding during farming operations, 4) the periods suitable for planting or replanting of crops and substitute crops, 5) crop loss due to flooding

Table 6. Pertinent Physical Data for Shelbyville and Carlyle Lakes

	Shelbyville	*Carlyle*
A. Lakes		
Top of dead storage pool		
Elevation, *ft msl*	573.0	429.5
Area, *ac*	3,000	6,700
Storage, *ac-ft*	30,000	50,000
Top of joint-use pool		
Elevation, *ft msl*	599.7	445.0
Area, *ac*	11,000	24,600
Storage, *ac-ft*	210,000	283,000
Top of flood control pool		
Elevation, *ft msl*	626.5	462.5
Area, *ac*	25,300	58,400
Storage, *ac-ft*	682,000	983,000
Top of surcharge pool, *ft msl*	638.2	467.2
Minimum release, *cfs*	10	50
Nondamaging flow release, *cfs*	1,800	4,000
B. Lake surface area and storage curves (figure 5)		
C. Level, *ft msl*, above which upstream damage occurs	610	450

D. Downstream damage reaches

Below Shelbyville		*Below Carlyle*	
Flow release (cfs)	*Area flooded (ac)*	*Flow release (cfs)*	*Area flooded (ac)*
1,800	0	4,000	0
2,000	750	4,500	3,500
2,200	1,270	5,000	6,300
2,400	1,830	5,500	8,250
2,600	2,280	6,000	9,400
2,800	2,720	6,500	10,100
3,000	3,170	7,000	10,600
3,200	3,530	7,500	10,900
3,400	3,900	8,000	11,200
3,800	4,700	8,500	11,500
4,200	5,630	9,000	11,750
4,600	6,310	9,500	12,000
5,000	6,580	10,000	12,200

$$A = 2.58 + 0.975Q - 0.035Q^2 \qquad 5 < Q \leqslant 7$$

$$A = 4.245 + 0.620Q - 0.0182Q^2 \qquad 7 < Q \leqslant 14$$

$$A = 8.2 + 0.40Q \qquad 10 < Q \leqslant 15$$

$$A = 5.5 + 0.73Q - 0.01Q^2 \qquad 15 < Q \leqslant 25$$

A = area flooded in thousand acres
Q = flow release in thousand cfs

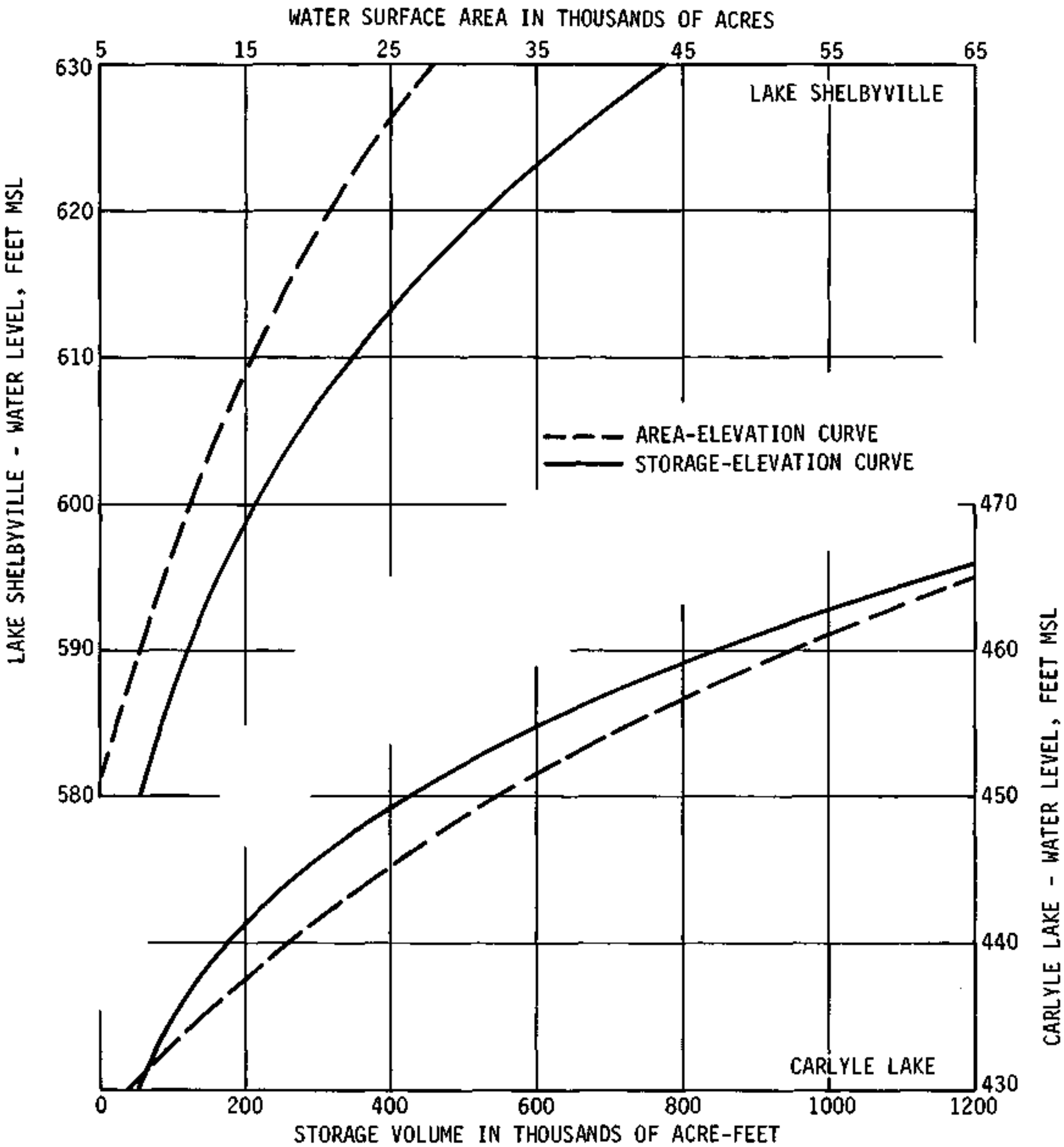


Figure 5. Water surface area and storage capacity curves for Lake Shelbyville and Carlyle Lake

during the growing season, and 6) partial crop loss due to flooding during the harvesting season. The data inputs were transformed into concise matrices. The methodology, written as an efficient computer subroutine, formed an integral part of the operation system program for the lakes.

Recreational Data. The main recreational activities in the lake areas and immediate areas downstream are: camping, picnicking, swimming, boating, water skiing, fishing, and hunting. The number of annual visitors for those activities at each of the two lakes was estimated from the 1971-1974 visitor data available at the U.S. Army Corps of Engineers' field offices at Shelbyville and Carlyle, and from consultation with the Corps and the Illinois Department of Conservation. The number of visitors is shown in table 8, together with the dollar per recreation day values (Water Resources Council, 1973).

Table 7. Pertinent Agricultural Data for the Four Damage Reaches

	Damage reaches			
	Above Shelbyville	*Below Shelbyville*	*Above Carlyle*	*Below Carlyle*
Crop yields, *bu/ac*				
Corn	108	98	85	93
Soybeans	37	35	27	32
Wheat	44	44	44	44
Percent area distribution				
Corn	15	60	21	45
Soybeans	7	30	25	25
Wheat	3	0	15	0
Timber, etc.	75	10	39	30
Crop yields per typical acre, *dollars*				
Corn	24.30	88.20	26.78	62.78
Soybeans	10.36	42.00	27.00	32.00
Wheat	2.77		13.86	
Total	37.43	130.20	67.64	94.78

In order to calculate visitors for weeks 1 through 52, starting October 1, weekly multipliers were developed with the use of the Corps field data for the years 1972-1974. The multipliers are graphed in figure 6 for camping, picnicking, swimming, skiing, boating, and fishing at Shelbyville and Carlyle. Weekly multipliers for hunting are 0.036, 0.123, 0.170, 0.139, 0.119, 0.131, 0.135, 0.104, and 0.043 for weeks 3 to 11 (mid-October to mid-December) at the two lakes. Number of visitors in a week equals the number for the whole year multiplied by the applicable weekly multiplier.

Table 8. Number of Visitors and Values for Seven Recreational Activities

	Annual visitors		
Activity	*Lake Shelbyville*	*Carlyle Lake*	*$/recreation day*
Camping	365,000	110,000	1.50
Picnicking	130,000	190,000	1.50
Swimming	400,000	100,000	1.50
Boating	300,000	200,000	1.50
Skiing	60,000	40,000	1.50
Fishing	350,000	250,000	3.00
Hunting	35,000	20,000	3.00

Information on water levels above and below which the usual number of visitors cannot fully participate in a particular recreational activity and the percentage reduction in the activity with rise and fall in lake levels are important for evolving a good regulation scheme. The levels and percent damage figures finally determined as a result of talks with the Corps and the Illinois Department of Conservation are given in table 9.

A computer subroutine, similar to the one used by the Corps, was developed to compute recreational loss, or benefit foregone, because of high or low water levels in Shelbyville and Carlyle lakes. The number of visitors lost in each activity was multiplied by the dollar value assigned to that activity with the total loss being the recreation damage. The net recreation benefit equals the potential (or with no damages) recreation benefit minus the damages accrued over the year. The program allows for a dry-out period of duration specified by the user. The Corps documentation of the procedure indicates a 5-day drying period. For the weekly flows compiled for this study, the dry-out period was taken as one week.

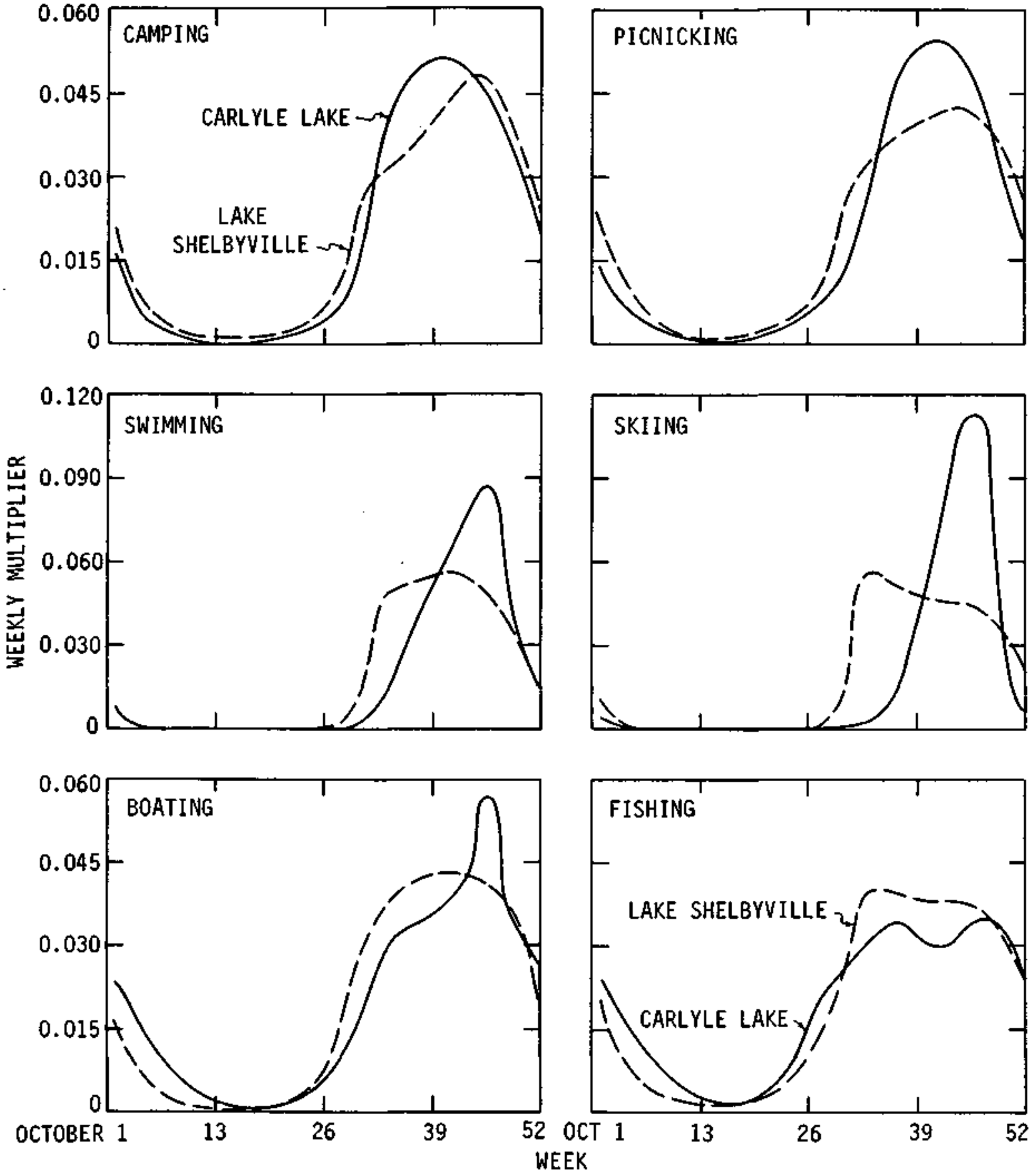


Figure 6. Distribution of visitors for various recreational activities
(Number of visitors per week equals the product of that week's multiplier
and total of visitors over the year for the recreation activity under consideration)

Table 9. Percent Recreational Loss per Foot of Change in Lake Level

Activity	*Low level**	*Percent loss*	*Max loss*	*High level†*	*Percent loss*	*Max loss*
Shelbyville						
Camping		0			0	
Picnicking		0			0	
Swimming	589	5.0	50	603	8.3	70
Boating	585	25	100	610	8.3	39
Skiing	585	25	100	610	8.3	39
Fishing	585	15	75	610	5.0	75
Hunting	589	10	100	602	3.5	95
Carlyle						
Camping		0		450	10	100
Picnicking		0		450	5.0	50
Swimming	438	25	100	450	10	100
Boating	438	25	100	450	10	100
Skiing	438	25	100	450	10	100
Fishing	438	15	75	450	6.0	72
Hunting	443	15	100	450	10	60

*Lowest lake levels below which recreational loss begins
†Highest lake levels above which recreational loss begins

SIMULATION MODEL STRUCTURE

There are two definite, if not distinct, phases to building a model to study a complex situation (McLean and Shepherd, 1976). First, there is the stage of identifying the problem and specifying it in terms of a structure, this structure usually being a set of mathematical equations which describe the system under study. In the second stage this model structure is optimized and used to derive the expected behavior of the system by using complex mathematical techniques.

The physical system consists of Lake Shelbyville, Carlyle Lake, and the Navigation Lock and Dam at Kaskaskia River mile 222, 107, and 0.8, respectively. The physical data relating to the lakes and project purposes have already been discussed briefly. There are four areas or reaches which are affected by the regulation of the lakes: areas along the two lakes affected by high water levels, and bottomlands (affected by high flows) along the river below each of the lakes.

High lake levels and high flow releases damage crops; the intensity and magnitude of agricultural damage varies throughout the year. There is a certain range of lake levels for realizing maximum recreation potential; both higher and lower levels beyond this range restrict recreational use. The problem is to derive a matrix of operating rules to maximize recreational, agricultural, and water supply benefits under varying navigation flow requirements and water supply withdrawals. For deriving such rules, a model must be structured from a comprehensive, rational analysis of various uses affected by the regulation, as well as economic tradeoffs involved because of their conflicting goals and requirements.

Agricultural Use

The main crops, corn and soybeans, cover a period of May through mid-November, and the wheat crop grown in lake areas above 610 ft at Shelbyville and above 450 ft at Carlyle covers a period of October through mid-July. Crop yields in dollars per typical acre are:

	Corn and soybeans	*Wheat*
Area above Shelbyville	$ 34.66	$ 2.77
Area below Shelbyville	130.20	
Area above Carlyle	53.78	13.86
Area below Carlyle	94.78	

It is evident that corn and soybean crops bring in the most money and need top priority for safety against flooding. Wheat yields in the area above Carlyle are five times greater than those above Shelbyville, a fact that calls for lower lake levels in Carlyle during October to mid-July.

It is not always possible to restrict both lake levels and flow releases during the corn-soybean period, May through mid-November, and so priorities need to be spelled out to allow more flexibility in the operation of the lakes. An analysis of weekly flows for the period 1942-1965 indicates that the average weekly flows exceeded the nondamaging flow below Shelbyville and Carlyle for 20 and 24 percent of the time during the months January through June and about 5 percent of the time during the months July through October. The high flows in January-June are much higher than those in July-October. The most critical period for corn and soybeans is May and June as far as high lake levels and high flow releases are concerned.

Corn and soybeans can be planted or replanted if started by about the middle of June. Any flooding of crops during July, August, and September causes a total crop loss. About 95 percent or more of the crop is harvested by the end of October. The chance of flooding during July through October should be reduced to a minimum to insure no major crop loss. The priorities and requirements in the interest of agriculture are shown in table 10.

Recreational Use

Lake Shelbyville provides 63.2 percent, or $3.0375 million of the annual potential recreational benefit of $4.8075 million. Out of the seven recreational activities of camping, picnicking, swimming, boating, skiing, fishing, and hunting, fishing accounts for 37.4 percent of the benefits. The distribution of potential recreation benefits from month to month at the two lakes (table 11) sets the priorities for maintaining the lake levels within the no-damage range of levels.

Table 10. Agricultural Priorities and Requirements

Priority	*Subpriority*	*Period*	*Requirements*
I Corn and soybeans			
	1	Weeks 39-4 (June 24-October 28)	Nondamaging flow releases and nondamaging lake levels
	2	Weeks 37-38, and 5-6 (June 10-23, and October 29-November 11)	Nondamaging flow releases and nondamaging lake levels
	3	Weeks 31-36 (April 29-June 9)	Nondamaging flow releases and nondamaging lake levels
II Wheat			
	1	Weeks 1-41 (October 1-July 14)	Carlyle Lake level not to exceed 450 ft
	2	Weeks 1-41 (October 1-July 14)	Lake Shelbyville level not to exceed 610 ft

Considering the economics of recreation benefits only, the priorities for holding the lake levels within the no-damage range of levels are given below.

Priority	*Period*
I	May through September (weeks 31–52)
II	April and October (weeks 27–30 and 1–4)
III	March, November, and December (weeks 23–26 and 5–13)
IV	January and February (weeks 14–22)

Table 11. Distribution of Potential Recreation Benefits

	Percent of recreation benefit	
Month	*Shelbyville*	*Carlyle*
Jan	0.4	0.3
Feb	0.5	0.6
Mar	1.8	3.1
Apr	5.2	5.7
May	17.2	11.0
Jun	17.2	16.2
Jul	18.3	17.5
Aug	17.7	20.5
Sep	12.1	11.6
Oct	5.4	6.6
Nov	2.7	5.0
Dec	1.5	1.9

Historical Flow and Economic Tradeoffs

Historical Flow Data. The percent chance that a weekly flow may exceed the nondamaging flow at Shelbyville and Carlyle during the 24-year record is given in table 12.

The crucial period of high inflows spans weeks 14 through 39 (or 40), or January through June, though the magnitude and frequency of high inflows in June are comparatively less than for May or April. High flows in May and June can flood croplands in the lake areas if flow releases are restricted, or they can flood bottomlands below Shelbyville and Carlyle if nondamaging lake levels are maintained, or they can flood both lake areas and bottomlands. Restricting flow releases raises lake levels, and later high inflows, though occurring less frequently, can either raise lake levels or lead to increased flow releases causing loss of crops which cannot then be replanted. High lake levels in May and June can adversely affect recreational benefits, and any further increase in levels in July or August will make the situation worse. Therefore, some lake storage cushion is needed for absorbing the high inflows, and some high flow releases may have to be allowed (say once in 5 to 10 years) in May and early June. Such occasional high flow releases would not only keep lake levels within the no-damage range, but also would insure farmers a crop most of the years at the expense of some inconvenience of either delaying or repeating operations such as disking and harrowing, and seeding and planting.

Table 12. Percent Chance of Exceeding Nondamaging Flows

Period	*Number of weeks*	*Percent chance*	*High flow range (thousand cfs)*
Lake Shelbyville			
47 through 2	8	0	(one week 1841 cfs)
3 through 13	11	4.5	1.8–4.3
14 through 39	26	20.4	1.8–11.5
40 through 42	3	6.9	1.8–8.2
43 through 46	4	4.1	1.8–6.9
Carlyle Lake			
47 through 2	8	0	
3 through 13	11	5.7	4–9
14 through 40	27	23.6	4–34
41 through 42	2	12.5	4–10
43 through 46	4	4.1	4–9

Table 13. Rule Levels for Various Periods

Period	*Rule levels*	
	Lake Shelbyville	*Carlyle Lake*
Weeks 37 through 11	**ESH**	**ECH**
Weeks 12 through 30	**ESL**	**ECL**
Weeks 31 through 36	**ESL or ESH**	**ECL, ECLL, or ECH**

ESL = low rule level (elevation) in Lake Shelbyville
ESH = high rule level in Lake Shelbyville
ECL = low rule level in Carlyle Lake
ECLL = lower than ECL rule level in Carlyle Lake
ECH = high rule level in Carlyle Lake

The weekly flows during the months July through October exceed nondamaging flow releases 3 percent of the time and these flows can be easily absorbed by the lakes with a moderate rise in lake levels, causing very little damage to recreation and agriculture.

Economics. The economics of relative agricultural and recreational damages, various economic tradeoffs, and complex interrelationships between the damages in lake areas and downstream reaches have been explained in the previous report (Singh et al., 1975). In formulating the model structure, various combinations of lake levels, flow conditions, stages of crop development, time distribution of recreational activity and benefits, etc., were considered in assessing the relative disutility of water from week to week.

Model Structure

The model structure is briefly explained here, stressing only the features of particular interest to water resource planners and decision makers entrusted with the job of operating a lake system as typified by Shelbyville and Carlyle lakes.

1) Rule Levels. On the basis of the previous study and priorities for agricultural and recreational use, as well as information from historical flow data and economic tradeoffs, the rule levels (or the most desirable lake levels) for various periods are as defined in table 13. The period spanning weeks 31 through 36 is crucial for agriculture and is also a period of relatively high inflows. In the previous study, ESL and ECLL were kept as rule levels in this period.

2) Release Rules. In 1969, the U.S. Army Corps of Engineers scaled down the nondamaging flow releases below Shelbyville and Carlyle from 4500 and 7000 cfs (U.S. Army Corps of Engineers, 1964) to 1800 and 4000 cfs, respectively, based on their field experience. For all practical purposes, the flow releases up to these new limits will not flood bottomlands along the river in the two damage reaches below Shelbyville and Carlyle. Corn and soybean crops are grown in the bottomlands and the overall activity period for the crops covers May through mid-November. For the rest of the period, mid-November through April, the flooding of bottomlands causes no damage to crops but there is minor damage to farmsteads, fences, etc. Since 1969, the Corps has been following a policy of allowing flow releases up to 4500 and 10,000 cfs below Shelbyville and Carlyle in this 'dump' period. During very high inflows when the lake levels exceed the flood pool level of 626.5 ft at Shelbyville and 462.5 ft at Carlyle, the flow releases are drastically increased to restrict further rise in lake levels. This is designated as flood regulation. Because the lake levels with the 24-year historical data did not exceed the flood pool levels, the flood regulation as defined by the Corps was not tested.

Table 14. Release Rules for Various Periods

Period	*Lake Shelbyville*	*Carlyle Lake*
Weeks 37 through 6	$OSMX = 1800$ cfs if $E_s \leq ESF$ $= 1800 + SF(E_s - ESF)$ if $E_s > ESF$	$OCMX = 4000$ cfs if $E_c \leq ECF$ $= 4000 + CF(E_c - ECF)$ if $E_c > ECF$
Weeks 7 through 30	$OSMX = 4500$ cfs For conditional flow releases from Lake Shelbyville to obtain storage interaction with Carlyle Lake during low and high inflows, refer to the previous report p. 46-47.	$OCMX = 10000$ cfs
Weeks 31 through 36	For conditional flow releases exceeding 1800 and 4000 cfs from Shelbyville and Carlyle during high inflow and high lake level conditions, see the previous report p. 46-47.	
Weeks 1 through 52	$OSMN = 10 + WS_s$ Flood regulation if $E_s > 626.5$	$OCMN = 50 + WS_c$ Flood regulation if $E_c > 462.5$

OSMN = Minimum allowable flow release below Shelbyville, in cfs
OSMX = Maximum allowable flow release below Shelbyville, in cfs
OCMN = Minimum allowable flow release below Carlyle, in cfs
OCMX = Maximum allowable flow release below Carlyle, in cfs
E_s = Lake Shelbyville water level at the beginning of the week
E_c = Carlyle Lake water level at the beginning of the week
ESF = Lake Shelbyville level above which flow release increases linearly up to the flood pool level during weeks 37 through 6
ECF = Carlyle Lake level above which flow release increases linearly up to the flood pool level during weeks 37 through 6
SF = Shelbyville flow release increase rate per foot of rise in lake level
CF = Carlyle flow release increase rate per foot of rise in lake level
WS_s = Water supply withdrawal in cfs from Lake Shelbyville
WS_c = Water supply withdrawal in cfs from Carlyle Lake

From consideration of avoiding major crop losses, holding lake levels within the no-damage range to maximize recreation and providing flexibility in operation, some higher than nondamaging flow releases may have to be allowed during the weeks 31 through 36. The release rules formulated are given in table 14.

3) Structure of the Model. The rule levels and release rules together with the interlinking system equations and constraints helped to build a rational structure for the model. The structure was translated to a system simulation program with 10 parameters to be optimized with the use of 24 years of weekly flows at Shelbyville, Carlyle, and New Athens, navigation flow requirements, and precipitation on and evaporation from the two lakes, for different levels of navigation development and water supply withdrawals. The 10 parameters are: ESL, ESH, ECL, ECLL, ECH, ESF, SF, ECF, CF, and A; A is the fraction of navigation flow demand met from Lake Shelbyville. Other parameters involved in the system equations governing conditional flow releases for effecting interaction of lake storages under low and high inflow conditions during weeks 37 through 6, did not need any change in values from those given in the previous study.

OPTIMAL OPERATING RULES WITH HISTORICAL DATA

The simulation model, written as an interactive program for use on the time-sharing computer facilities of the University of Illinois, was used to derive optimal operation strategies with varying navigation and water supply requirements. Some questions and alternatives regarding the system operation were reviewed and investigated. More than 6000 simulation runs were made to derive 'optimal' operation plans. The sensitivity of overall damages (or benefits foregone) to change in rule levels was studied to highlight such effects. Finally, the expected values of annual benefits were tabulated for various water supply withdrawal rates.

The simulation model is described in terms of data inputs, recreation and agricultural damage assessment subroutines, and a simulation program which uses data inputs, rule levels and release rules, and system equations.

Data Inputs. There are six hydrologic data inputs to the model. These are: weekly flows at Shelbyville, Carlyle, and New Athens, properly lagged for travel time from one station to the other; weekly values of h, or 368.0 minus the Mississippi River level below the lock and dam, set to zero if negative; and weekly values of precipitation on and evaporation from the two lake surfaces, in inches.

The recreation data comprise number of visitor days per week for each of the 52 weeks and for the seven categories of recreation activities at each of the two lakes. The data on dollars per recreation day for each activity, the no-damage range of lake levels, and the functions specifying reduction in recreation activity per foot change in adverse lake levels, provide necessary information for recreation damage or benefit assessment.

The agricultural data cover land use and crop yields; direct production investment matrices (Singh, 1976) based on sequence of farming operations and their unit costs, and the beginning week of operations for the area under consideration; and the crop losses due to flooding during farming operations, crop growth, and harvesting. The crop areas affected by flooding are above 610 ft elevation in Lake Shelbyville and 450 ft elevation in Carlyle Lake. In the bottomlands downstream, the area flooded versus flow release information is stored in tabular form in the computer. The lake areas and storages together with associated lake levels are also stored in the computer.

Recreation Damage Subroutine. The purpose of this subroutine is to compute recreation loss, if any, given the lake levels at the beginning of the week (or at the end of the previous week). The number of visitors lost in each activity because of adverse high or low water level is multiplied by the dollar value assigned to that activity and the total loss for all the activities of the two lakes gives the recreation damage during the week. The weekly damage values are summed for the 52 weeks in the year. Net annual recreation benefit equals the potential (or with no damages) recreation benefit minus the damages accrued over the year. During falling lake levels, the program allows for a dry-out period of one week.

Agricultural Damage Subroutine. The methodology developed for evaluating flood damages or benefits as an integral part of the overall system operation are described elsewhere (Singh, 1976) and in the previous report. The developed methodology computes flood control benefits accruing from reduction in individual and recurrent floodings considering the data on land use and crop yields, sequences of farming operations and their unit costs, loss of direct production investment because of flooding during the growing season, and partial crop loss during the harvesting season. The data inputs are transformed into concise matrices. The methodology is written as an efficient computer subroutine.

Simulation Program. This program simulates the system operation given the lake levels at the beginning of the week; the weekly inflows at Shelbyville, Carlyle, and New Athens; the value of h at the lock and dam; and rainfall minus evaporation for the two lake surfaces. The weekly operation conforms to the provision of rule levels, release rules, system equations, constraints, and conditional flow releases.

The weekly flow at Shelbyville is adjusted for net precipitation over the lake. Flow release is calculated knowing the lake level at the beginning of the week and the operation rules. The storage equation yields the lake level at the end of the week and this is the beginning-of-the week level for the next week. The inflow to Carlyle consists of historic flow at Carlyle minus that at Shelbyville, plus flow release from Shelbyville. The inflow is adjusted for net precipitation over the lake. Flow release is calculated in the same manner as for Shelbyville. The recreational and agricultural damage assessment subroutines compute damages (or benefits foregone) because of any adverse lake levels and flow releases from week to week.

Operational Considerations

Some pertinent considerations affect the operation of the lakes and the optimized rule levels and release rules. These are discussed briefly before presenting the optimization results.

Navigation and Water Supply Withdrawals. The number of lockages, and hence the flow at the navigation lock and dam needed to meet the lockage requirements, depends on the volume of barge and other traffic using the lock. The navigation use will develop from zero at the present time to the maximum envisaged in the project over a number of years. It will, therefore, be desirable to derive the optimal operating rules for different levels of navigation demand, not only to assess the relative impact on overall benefits or damages but also to provide suitable data for decision makers to increase the flexibility of the operation.

Demand for water supply is expected to grow gradually. Water is needed by towns and newly developing industries. Different levels of water supply demands and associated optimal operating rules and benefits should provide sufficient data for establishing suitable withdrawals.

The following matrix of navigation and water supply withdrawals was used in deriving optimal rules. This gives 18 sets of data.

Percent navigation	*Water supply withdrawal (cfs)*					
0	0	40	80	120	160	200
50	0	40	80	120	160	200
100	0	40	80	120	160	200

Location of Water Supply Withdrawals. Water supply withdrawals from Lake Shelbyville and Carlyle Lake are proportional to the respective state storages, or 17/40 and 23/40 times the total water supply. These withdrawals can be considered either from the lakes directly, or from the Kaskaskia River below the respective damage reaches, i.e., below the town of Cowden and below the confluence with Crooked Creek. Optimizations were carried out for the 18 combinations of navigation and water supply requirements with water supply withdrawals directly from the lakes or from the river below the damage reaches. The overall damages (or benefits not realized) are higher when these withdrawals are made from the river. Over the historical period of 24 years, these damages exceed those with direct water supply from the lakes by an average of 16, 50, 85, 139, and 178 thousand dollars for water supply rates of 40, 80, 120, 160, and 200 cfs, respectively. Because most of the water supply will be needed below the damage reaches and because the cheapest means of transporting this water from the lakes to the users is by way of the river, the final optimization results will be presented with the assumption that the water supply withdrawals are from the river below the damage reaches.

Rule Level in Lake Shelbyville, Weeks 31-36. In the previous report, the rule level in Lake Shelbyville during weeks 31 through 36 was kept at 590 ft, the same as for the period covering weeks 12 through 30, for the condition of no water supply and navigation requirement. To assess the effect of water supply withdrawals and navigation flow requirements on the desirability of keeping the rule level at ESL (low rule level) or ESH (high rule level) optimizations were carried out assuming water supply withdrawals from the river and the rule level in weeks 31-36 at ESL or ESH. The results are generalized in figure 7. It is evident that keeping the rule level at ESH leads to minor decreases in benefits for WS = 0 and N = 0 and 50 percent, and WS = 40 and 80 cfs and N = 0. For 50 or 100 percent navigation demand and WS ≥ 40 cfs, keeping the rule level at ESH greatly increases benefits as the water supply withdrawals and navigation flow requirements increase. Hence, the final optimization results will be presented with the 31-36 week rule level as ESH in Lake Shelbyville. This lake accounts for about two-thirds of the recreation benefit, and allowing the higher level during May and June not only will assure better recreational activity but also will quiet the fears of the recreation interests regarding the possibility of unduly low lake levels in dry years.

Parameters SF, ESF, CF, and ECF. The Corps of Engineers' present operating rules for the corn and soybean crop period provide a linear increase in flow release from 1800 to 4500 cfs when the lake level rises from 610.0 to 626.5 ft in Shelbyville, and from 4000 cfs to 10,000 cfs for rise in level from 450.0 to 462.5 ft in Carlyle. Values of ESF and SF are 610 ft and 163.6 cfs/ft and those for ECF and CF are 450 ft and 480 cfs/ft.

Optimizations with the 24 years of data indicate that ESF and ECF in the ranges of 615 to 618 ft and 452.5 to 453.5 ft do not cause any flow releases exceeding the nondamaging flows of 1800 and 4000 cfs. The operation governed by those parameters applies to weeks 37 through 6, the growth and harvesting period of corn and soybeans. Any flow releases above the nondamaging flows cause excessive damages to crops because the area flooded versus flow release curves are steep in the beginning. Therefore, the optimal values of ESF ane ECF are set at the lowest levels minimizing overall damages (or benefits not realized). Values of SF and CF cannot be obtained from such optimizations; the damages are minimum with SF and CF equal to zero. The weekly flows in the 24-year period were not large enough to raise lake levels above 618 to 620 ft in Shelbyville and 452.5 to 454.5 ft in Carlyle for different combinations of navigation and water supply requirements, with the derived optimum operation rules. An attempt is made later to evaluate SF and CF using the 10 synthetic sequences of 49 years each.

Optimization Results

Numerous system simulations were made for each of the 18 sets of navigation and water supply requirements to find the optimal values of the parameters ESL, ESH, ECL, ECLL, ECH,

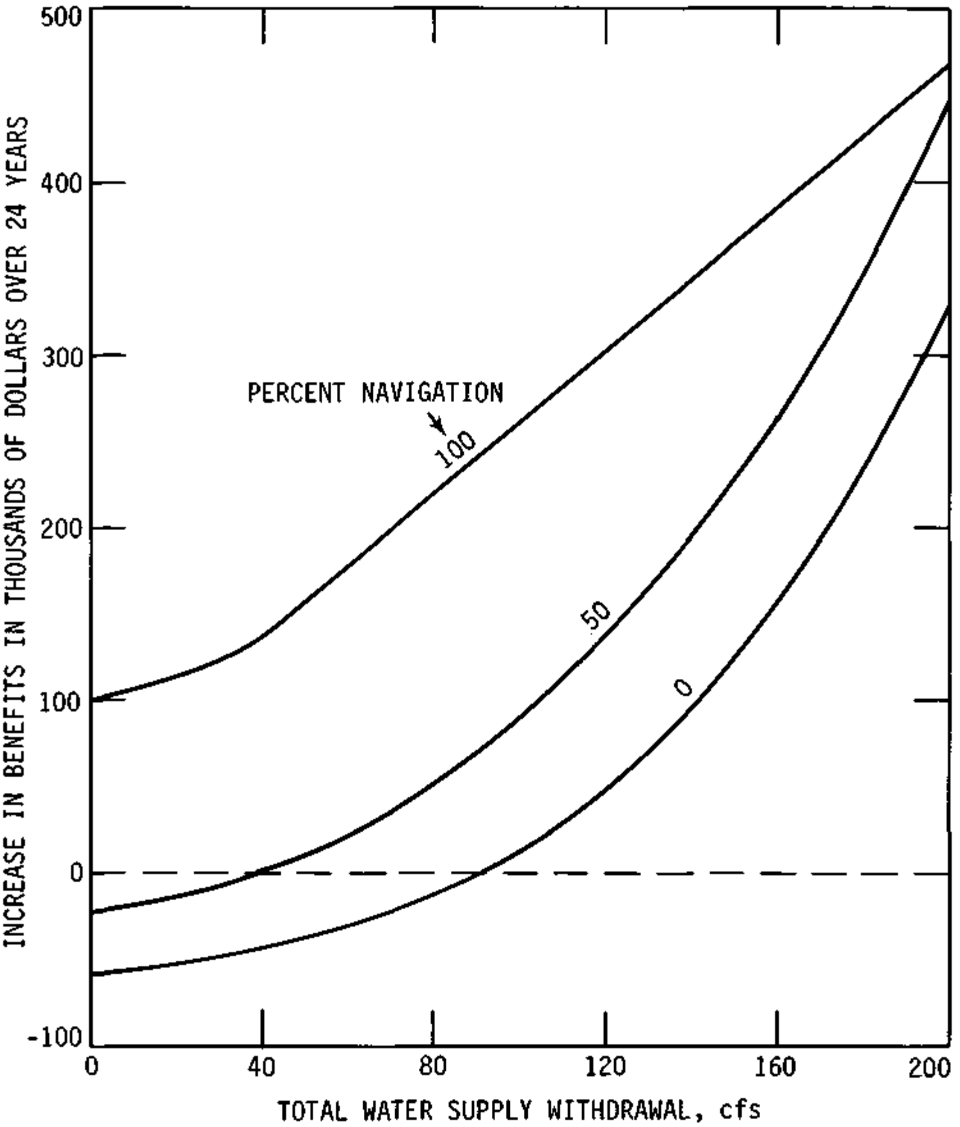


Figure 7. Increase in benefits (recreation and agriculture) over 24 years with the rule level for weeks 31-36 in Lake Shelbyville at ESH instead of ESL

ESF, ECF, and A. Table 15 contains the optimal values of these parameters as well as information on recreation, agricultural (and property), and total damages (or benefits not realized) when 24 years of historical data (1942-1965) are used. With the exception of ECLL and A, the remaining 6 parameters were optimized to 0.5 ft, but ECLL was optimized to 0.1 ft and A to 0.05. The recreation and agricultural damages (or benefits not realized) are shown in figure 8 for water supply withdrawals varying from 0 to 200 cfs and navigation flow requirements, N, varying from 0 to 100 percent.

For developing a 100 cfs water supply with the condition of full navigation flow requirement, the increase in damage to recreation and agriculture over that for no water supply is $667,000 over 24 years, or 0.118¢ per 1000 gallons of water supplies. The optimal solutions indicate that with the 24-year data, even with 200 cfs water supply and full navigation flow requirement, the lowest levels reached in Shelbyville and Carlyle are 579.32 and 432.04 ft; the tops of the respective dead storage pools are 573.0 and 429.5 ft, respectively.

Table 15. Parameters for Optimal Operation and Minimum Damages (Benefits Foregone) over 24 Years

WS	*N*	*Parameters*								*Damages (in thousands of dollars)*		
(cfs)	*(%)*	*ESL*	*ESH*	*ECL*	*ECLL*	*ECH*	*ESF*	*ECF*	*A*	*Rec*	*Agr*	*Total*
0	0	590.0	593.5	440.5	438.7	443.0	615.0	452.5	.0	684.9	934.1	1619.0
	50	590.0	593.5	440.5	438.7	443.0	615.0	452.5	.0	692.0	933.6	1625.6
	100	590.5	594.0	441.0	438.9	443.0	615.5	452.5	.0	727.1	941.8	1668.9
40	0	590.0	593.5	441.5	439.1	443.0	615.0	452.5	.0	732.4	955.6	1688.0
	50	590.5	594.0	441.5	439.1	443.0	615.5	452.5	.0	759.1	957.0	1716.1
	100	591.0	594.5	441.5	439.1	443.0	615.5	452.5	.25	898.9	956.9	1855.8
80	0	592.0	595.0	441.5	439.1	443.0	616.0	452.5	.0	918.6	968.6	1887.2
	50	592.0	595.5	441.5	439.1	443.0	616.0	452.5	.0	942.8	970.3	1913.1
	100	592.5	596.0	442.0	439.4	444.0	616.5	452.5	.20	1141.3	1023.6	2164.9
120	0	592.5	596.0	442.0	439.4	443.0	616.5	452.5	.0	1039.0	1027.0	2066.0
	50	593.0	596.5	442.0	439.4	443.0	616.5	452.5	.0	1093.0	1017.1	2110.2
	100	593.5	597.0	442.5	440.2	444.0	617.0	453.0	.10	1397.9	1155.9	2553.8
160	0	593.0	596.5	442.0	440.3	443.0	618.0	453.0	.0	1170.0	1149.7	2319.7
	50	593.5	596.5	442.5	440.7	443.0	618.0	453.0	.0	1237.1	1218.0	2455.1
	100	594.0	597.0	443.0	441.2	444.0	618.0	453.0	.0	1698.4	1329.7	3028.1
200	0	594.0	597.0	443.0	441.0	443.0	618.0	453.0	.0	1451.1	1282.8	2733.9
	50	594.0	597.0	443.0	441.0	443.0	618.0	453.0	.0	1652.8	1269.1	2921.9
	100	594.0	597.0	444.0	442.2	444.0	618.0	453.5	.0	2089.9	1491.7	3581.6

Rule Level in Carlyle, Weeks 31-36. The optimization studies indicate a lower rule level in this period than in weeks 12 through 30. There are different reasons for this anomalous behavior. Agricultural damage in the Lake Shelbyville area starts when the water level exceeds 610 ft and the damage per acre flooded is about one-half of that in the Carlyle Lake area. Values of optimum ESL lie in the range of 590 to 594 ft, giving an average 18-ft rise to 610 ft. Agricultural damage in the Carlyle Lake area starts when the level exceeds 450 ft. Values of optimum ECL lie in the range of 440.5 to 444.0 ft, giving an average 8-ft rise to 450 ft. Storage capacity for the 18-ft rise in Shelbyville is 200,410 ac-ft and for the 8-ft rise in Carlyle is 209,720 ac-ft. The drainage area at Carlyle is 2680 square miles compared with 1030 square miles at Shelbyville. Even allowing for reduction in flows below Shelbyville because of the lake, the storage capacity in Carlyle needs to be about two times that presently available if a higher rule level is desired during the weeks 31-36.

System simulations were run keeping the rule level in weeks 31 through 36 the same as ECL in table 15, for different combinations of navigation and water supply requirements. The resulting recreational and agricultural damages are given in table 16. These damages could be reduced by reoptimizing ESF and ECF for the new rule level in weeks 31-36. The optimal results are also included in table 16. The total damages over the 24-year period for full navigation demand and ECLL = EC L are graphed in figure 8, with ESF and ECF as in table 15 and with their optimized values for the new conditions as in table 16.

Percentage increases in the total damages for the 18 combinations of navigation and water supply requirements when ECLL equals ECL are:

	Percent increase in damages		
	Recreation	*Agriculture*	*Total*
ECLL = ECL	3.5	81.7	41.9
ECLL = ECL and reoptimized ESF and ECF	6.1	35.9	20.7

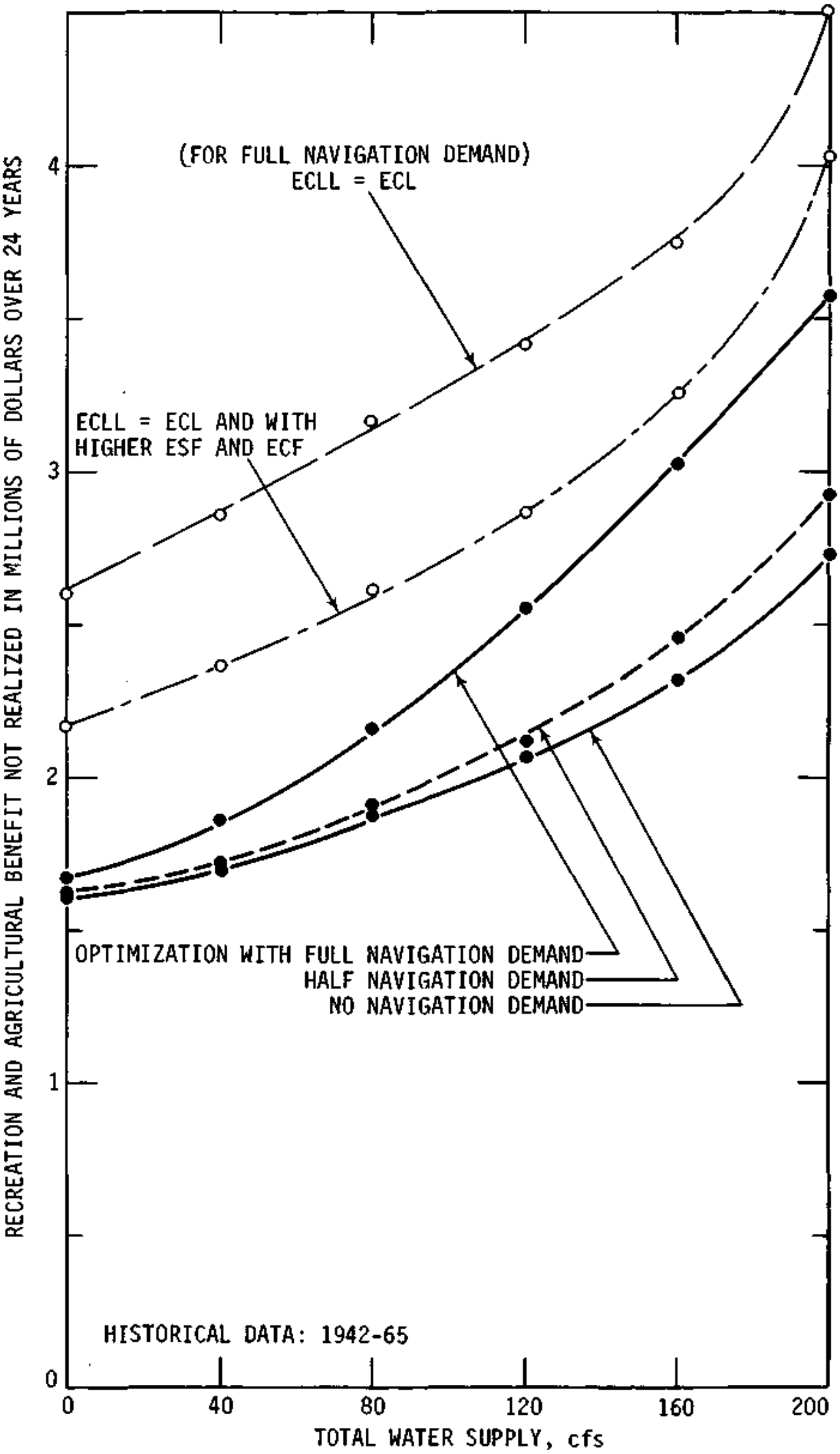


Figure 8. Recreation and agricultural benefits foregone as a function of total water supply and navigation demands

Table 16. Recreation and Agricultural Damages with ECLL = ECL

WS (cfs)	N (%)	*Damages (in thousands of dollars) when*							
		ESF and ECF as in table 15			*ESF and ECF reoptimized*				
		Rec	*Agr*	*Total*	*ESF*	*ECF*	*Rec*	*Agr*	*Total*
0	0	786.5	1630.2	2416.7	616.5	453.5	802.5	1245.0	2047.5
	50	792.9	1629.6	2422.5	616.5	453.5	808.8	1244.5	2053.3
	100	851.9	1736.0	2587.9	617.0	454.0	849.6	1313.7	2163.3
40	0	810.3	1908.4	2718.7	616.5	454.0	837.7	1398.4	2236.1
	50	842.2	1889.4	2731.6	616.5	454.0	868.8	1404.6	2273.4
	100	933.2	1928.0	2861.2	617.0	454.0	960.6	1410.9	2371.5
80	0	924.0	1912.4	2836.4	617.5	454.0	950.5	1416.3	2366.8
	50	950.8	1956.7	2907.5	617.5	454.0	980.6	1425.4	2406.0
	100	1096.6	2079.0	3175.6	617.5	454.5	1135.1	1484.1	2619.2
120	0	1043.9	2091.3	3135.2	618.0	454.5	1083.3	1483.7	2567.0
	50	1087.2	2103.1	3190.3	618.0	454.5	1127.7	1490.6	2618.3
	100	1273.6	2136.7	3410.3	618.5	455.0	1308.2	1551.7	2859.9
160	0	1188.2	1927.4	3115.6	618.0	454.5	1213.3	1489.0	2702.3
	50	1313.5	2003.1	3316.6	618.0	454.5	1342.6	1536.0	2878.6
	100	1595.1	2139.2	3734.3	618.5	455.0	1633.4	1620.9	3254.3
200	0	1555.0	2127.1	3682.1	618.5	455.0	1593.7	1614.0	3207.7
	50	1757.2	2127.1	3884.3	619.0	455.0	1795.9	1713.9	3509.8
	100	2245.0	2261.0	4506.0	619.0	455.0	2270.4	1769.5	4039.9

Not only are the agricultural damages increased tremendously, but also the reoptimized ESF and ECF are 1 to 2 feet higher than in table 15. Therefore, a lower rule level as determined from optimizations for minimizing damages is needed for the weeks 31 through 36 in Carlyle Lake.

Optimum Rule Levels. The rule levels for Lake Shelbyville and Carlyle Lake for the six water supply withdrawals and zero and full navigation flow requirements are shown in figure 9. The pattern of increase in rule levels with increase in water supply and/or navigation requirements is reasonable and rational. There are no fluctuations and oscillations, and the continuity of the curves suggests satisfactory optimizations.

Distribution of Annual Damages. The annual damages (or benefits not realized) with the 24-year data, full navigation demand, and water supply withdrawals of 0, 40, 80, 120, 160, and 200 cfs, ranked low to high, are given in table 17. Damages in the range of $1000 to $10,000 occur 15, 15, 13, 12, 11, and 12 years out of the 24 years for the six water supply withdrawals. Damages in this range are mostly damages to property because of high releases during the dump period, weeks 7 through 30. Annual damages are less than $100,000 for 21, 20, 19, 19, 18, and 18 of the 24 years. At a water supply withdrawal of 100 cfs and with full navigation demand, annual damage may exceed $100,000 on an average of once in 6 years. The historical data show only 1 or 2 years of very high damage.

Components of High Damages. Table 18 shows the recreation and agricultural components of the annual damages exceeding $100,000 for the condition of no and full navigation demand and the six water supply withdrawal rates. With no navigation demand, average agricultural damage exceeds recreation damage for the water supply withdrawals considered. With full navigation, average agricultural damage exceeds recreation damages for water supply withdrawals of 0, 40, and 80 cfs. For higher withdrawal rates, the average recreation damage exceeds agricultural damage during the years when total annual damage exceeds $100,000.

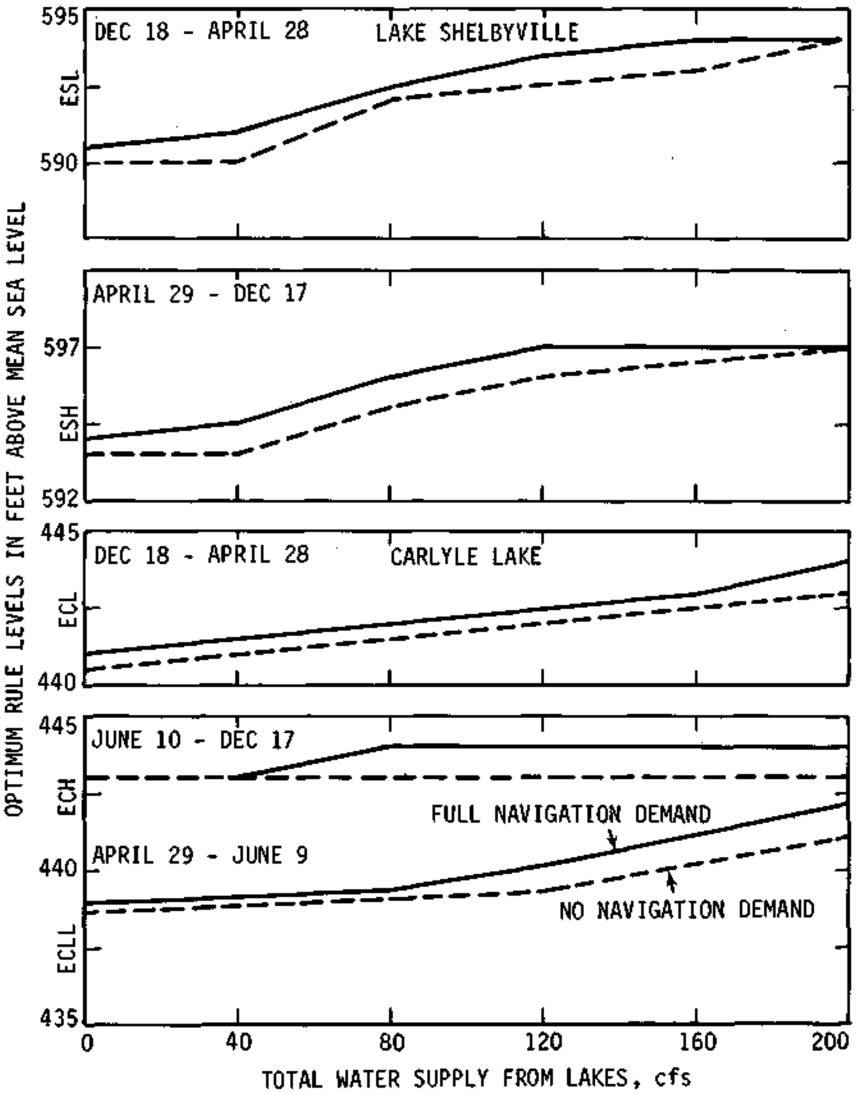


Figure 9. Optimal seasonal rule levels for Lake Shelbyville and Carlyle Lake with the 24-year historical data

Annual Minimum and Maximum Levels. For the sake of clarity, the distribution of annual minimum and maximum levels in Lake Shelbyville and Carlyle Lake with the optimal operation rules is shown in figure 10 for the condition of full navigation demand and water supply withdrawal rates of 0, 80, and 160 cfs. The two lowest levels occur during the drought of 1954-1955 and the two highest levels occur during the wet years of 1950 and 1957. Other information of interest follows.

Annual minima at Lake Shelbyville:

a) For practically 40 percent of the years, the annual minimum equals ESL, the rule level during weeks 12 through 30
b) The median value of annual minimum is about 590 ft for the three water supply withdrawal rates
c) For about 40 percent of the years minimum weekly levels are below 590 ft and are affected by water supply withdrawals

Table 17. Distribution of Annual Damages (Benefits not Realized) for 24 Years of Data, 1942-1965

		Annual damages (in thousands of dollars) with full navigation demand and water supply withdrawals (in cfs) of					
Rank	*Percent time*	*0*	*40*	*80*	*120*	*160*	*200*
1	4	1.05	1.05	1.05	1.05	1.05	1.05
2	8	4.20	3.94	3.71	3.22	2.94	2.90
3	12	4.77	4.80	4.88	4.92	4.68	4.67
4	16	5.17	5.19	5.53	5.53	5.85	5.64
5	20	5.54	5.54	5.86	5.86	6.25	5.84
6	24	5.87	5.87	6.25	6.02	6.28	6.25
7	28	6.25	6.25	6.98	6.25	6.98	6.98
8	32	6.54	6.67	6.98	6.98	6.98	6.98
9	36	6.77	6.98	6.98	6.98	6.98	6.98
10	40	6.98	6.98	6.98	6.98	6.98	6.98
11	44	6.98	6.98	7.19	6.98	7.32	6.98
12	48	6.98	6.98	7.59	7.58	15.38	7.59
13	52	6.98	6.98	8.30	13.68	23.63	18.95
14	56	6.98	8.70	23.73	28.02	23.67	35.95
15	60	7.38	9.78	23.87	36.21	35.95	75.45
16	64	17.98	32.44	32.16	56.82	80.42	81.77
17	68	23.34	33.83	47.21	68.67	82.05	87.43
18	72	29.79	41.03	54.99	84.38	88.68	88.38
19	76	32.71	60.11	74.77	87.88	161.06	161.37
20	80	43.58	90.23	144.79	161.32	200.95	231.75
21	84	56.01	104.68	157.81	197.53	283.90	417.27
22	88	154.01	155.20	174.14	262.26	373.85	519.27
23	92	320.41	333.68	381.24	420.02	458.32	589.46
24	96	902.63	911.87	971.91	1068.63	1138.00	1205.67

Annual maxima at Lake Shelbyville:

a) For about 20 percent of the years, annual maximum equals ESH, the rule level during weeks 31 through 11
b) The median values are 596.8, 598.6, and 600.4 ft for water supply withdrawals of 0, 80, and 160 cfs, respectively
c) For about 15 percent of the years, annual maxima exceed the level of 610 ft

Annual minima at Carlyle Lake:

a) For about 70 percent of the years, annual minimum is at ECLL, the rule level during weeks 31 through 36
b) The median value of annual minimum equals ECLL
c) For about 16 percent of the years, annual minimum exceeds ECLL

Annual maxima at Carlyle Lake:

a) For about 35 percent of the years, annual maximum equals ECH or the rule level during weeks 37 through 11
b) The median value of annual maximum level is 447 ft
c) For about 17 percent of the years, annual maximum exceeds the level of 450 ft

The lowest annual levels (during the drought of 1954-1955, believed to have a drought frequency of 75 to 100 years or more) in Lake Shelbyville are 588.9, 586.0, 583.7, 583.5, 582.1, and 579.3 for water supply withdrawals of 0, 40, 80, 120, 160, and 200 cfs, respectively, when full navigation flow requirements are met. The top of the dead storage pool is at 573.0 ft. Similarly, the lowest annual levels in Carlyle Lake are 437.8, 437.2, 436.5, 434.8, 433.3, and 432.0 for the six withdrawal rates. The top of the dead storage pool is at 429.5 ft. Hence, with

Table 18. Components of Annual Damages Exceeding $100,000

(Damages in thousand dollars)

WS	*No navigation demand*			*Full navigation demand*		
(cfs)	*Total*	*Rec*	*Agr*	*Total*	*Rec*	*Agr*
0	152.6	0	152.6	154.0	0	154.0
	303.8	132.2	171.6	320.4	141.0	179.4
	894.1	404.0	490.1	902.6	412.0	490.6
40				104.7*	104.7	0
	155.2	0	155.2	155.2	0	155.2
	316.0	133.5	182.5	333.7	149.8	183.9
	888.9	401.5	487.4	911.9	419.3	492.6
80				144.8*	144.8	0
				157.8	2.1	155.7
	155.4	0.4	155.0	174.1*	174.1	0
	350.7	167.0	183.7	381.2	180.0	201.2
	924.4	429.1	495.3	971.9	453.7	518.2
120				161.3	6.3	155.0
	112.6*	112.6	0	197.5*	197.5	0
	157.3	2.0	155.3	262.3*	262.3	0
	378.7	179.2	199.5	420.0	200.9	219.1
	982.0	456.4	525.6	1068.6	495.7	572.9
160				161.1	8.7	152.4
				201.0	32.9	168.1
	145.5*	145.5	0	283.9*	283.9	0
	154.3	3.6	150.7	373.8*	373.8	0
	411.2	192.0	219.2	458.3	216.7	241.6
	1090.3	516.9	573.4	1138.0	538.4	599.6
200	161.9	8.6	153.3	161.4	8.6	152.8
	175.6	28.8	146.8	231.7	46.1	185.6
	213.8*	213.8	0	417.3*	417.3	0
	249.0*	249.0	0	519.3	225.6	293.7
	453.5	214.3	239.2	589.5*	589.5	0
	1104.9	526.0	578.9	1205.7	562.0	643.7

*Damage to recreation only in water years 1954 and 1955 because of low lake levels in that drought period.

the navigation and water supply requirements considered, the minimum lake levels remained quite high above the dead storage pools.

Sensitivity Analysis

Sensitivity of the damages (or benefits not realized) with the optimal operation rules (table 15) must be analyzed with respect to increase in ESL, ESH, and ECH because these rule levels are lower than those laid down in the lake project reports. The effect of varying the recreation and/or agricultural damage rate on the optimal rules also needs investigation. The results of sensitivity analyses are given here.

Sensitivity of Damages to ESL. For the 18 combinations of navigation and water supply requirements, the system simulations were made with ESL equal to 591, 592, 593, and 594 ft when these levels exceeded ESL in table 15, keeping the other parameters unchanged. The ESF

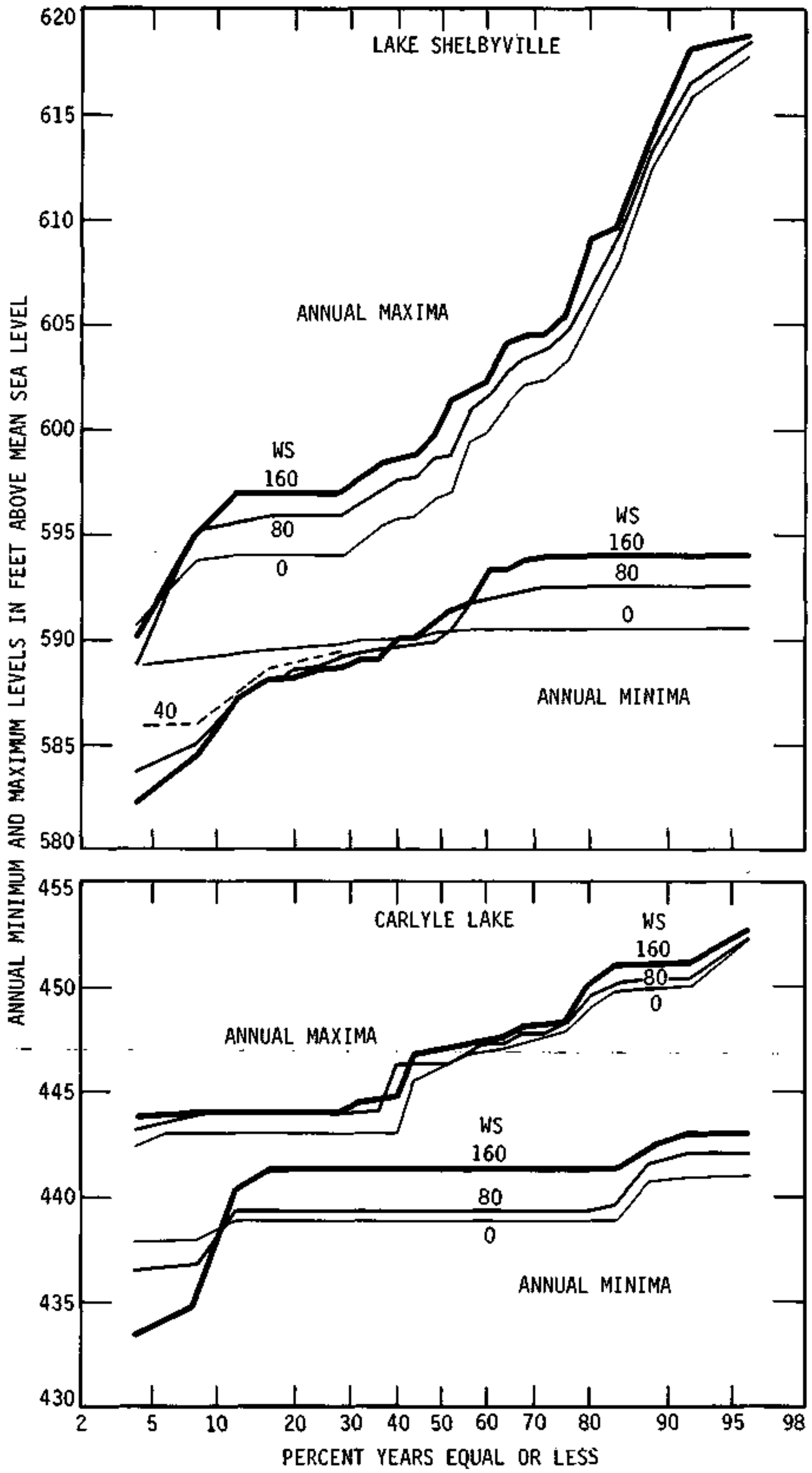


Figure 10. Annual minimum and maximum levels in Lake Shelbyville and Carlyle Lake for varying water supply and full navigation demand

Table 19. Increase in Damages with Increase in ESL

(Damage figures in thousand dollars over 24-year period)

WS (cfs)	N (%)	*Table 15*		*Increase in damage with ESL equal to*			
		Damage	*ESL*	*591*	*592*	*593*	*594*
0	0	1619.0	590.0	90.5	197.6	284.2	356.1
	50	1625.6	590.0	90.1	197.8	284.3	356.3
	100	1668.9	590.5	25.5	137.3	241.3	325.3
40	0	1688.0	590.0	116.4	224.9	315.8	404.6
	50	1716.1	590.5	43.5	150.4	253.6	350.3
	100	1855.8	591.0		116.1	233.3	330.1
80	0	1887.2	592.0			105.0	212.3
	50	1913.1	592.0			70.6	199.3
	100	2164.9	592.5			50.9	157.3
120	0	2066.0	592.5			30.0	130.9
	50	2110.2	593.0				93.8
	100	2553.8	593.5				4.9
160	0	2319.7	593.0				49.2
	50	2455.1	593.5				51.6
	100	3028.1	594.0				
200	0	2733.9	594.0				
	50	2921.9	594.0				
	100	3581.6	594.0				

and ECF from table 15 were found to be optimal with the new values of ESL also. The results of these simulations are given in table 19. The percent increase in damages is about 2 for 0.5 ft increase, 5 for 1.0 ft increase, and 12 for 2.0 ft increase in optimum ESL values. Recreation and agricultural damages contribute about 36 and 64 percent, respectively, of the increase in damages.

Sensitivity of Damages to ESH. The system simulations were made with ESH equal to 594, 595, 596, and 597 ft when these levels exceeded ESH in table 15, keeping the other parameters unchanged. The ESF and ECF in table 15 were found to be optimal with the new values of ESH also. The results of these simulations are given in table 20. The percent increase in damages is about 2.5 for 0.5 ft increase, 5.2 for 1.0 ft increase, and 19.1 for 2.0 ft increase in optimum ESH values. Recreation and agricultural damages contribute about 31 and 69 percent, respectively, of the increase in damages.

Sensitivity of Damages to ECH. Simulation runs were made with ECH equal to 444 and 445 ft when these levels exceeded ECH in table 15, keeping the other parameters unchanged. The ESF and ECF in table 15 were found to be optimal with the new values of ECH. The simulation results are given in table 21, together with the component increases in recreation and agricultural damages. With full navigation, ECH can be raised from 443 to 444 at a very minor increase in damage when water supply is zero, and at 1.6 percent increase in damages with 40 cfs water supply. The damage increase is 2.0, 3.2, 3.0, and 6.7 percent for water supply withdrawals of 80, 120, 160, and 200 cfs, respectively, considering full navigation and a 1 foot increase in ECH in table 15, i.e., from 444 to 445 ft. For a 1.0 ft increase in ECH (with the exception of zero water supply) practically all of the damage increase is sustained by recreation.

Recreational and Agricultural Damage Rates. System optimizations were done for full navigation and six water supply withdrawal rates and ECLL as shown in table 15 for two conditions: one with a double recreation damage rate and normal agricultural damage rate, and the second

Table 20. Increase in Damages with Increase in ESH

(Damage figures in thousand dollars over 24-year period)

WS (cfs)	*N (%)*	*Optimum damage*	*ESH*	*Increase in damage with ESH equal to*			
				594	*595*	*596*	*597*
0	0	1619.0	593.5	61.0	245.3	468.3	701.0
	50	1625.6	593.5	60.3	244.2	467.3	699.9
	100	1668.9	594.0		103.7	328.5	568.0
40	0	1688.0	593.5	30.5	182.0	399.0	625.7
	50	1716.1	594.0		84.4	388.4	557.2
	100	1855.8	594.5		45.3	223.0	455.2
80	0	1887.2	595.0			75.7	285.3
	50	1913.1	595.5			62.5	272.1
	100	2164.9	596.0				120.7
120	0	2066.0	596.0				157.1
	50	2110.2	596.5				67.6
	100	2553.8	597.0				
160	0	2319.7	596.5				25.4
	50	2455.1	596.5				10.2
	100	3028.1	597.0				
200	0	2733.9	597.0				
	50	2921.9	597.0				
	100	3581.6	597.0				

Table 21. Increase in Damages with Increase in ECH

(Damage figures are in thousand dollars over 24-year period)

WS (cfs)	*N (%)*	*Optimum damage*	*ECH*	*Increase in damage with ECH equal to*					
				444			*445*		
				Rec	*Agr*	*Total*	*Rec*	*Agr*	*Total*
0	0	1619.0	443	1.7	3.2	4.9	19.2	13.2	32.4
	100	1668.9	443	0.2	3.1	3.3	18.4	14.5	32.9
40	0	1688.0	443	4.9	0.4	5.3	22.2	5.1	27.3
	100	1855.8	443	30.2	0.3	30.5	48.9	5.4	54.3
80	0	1887.2	443	33.9	0.4	34.3	53.0	5.0	58.0
	100	2164.9	444				39.6	3.4	43.0
120	0	2066.0	443	75.0	0.2	75.2	124.6	4.4	129.0
	100	2553.8	444				74.7	7.1	81.8
160	0	2319.7	443	195.5	0.3	195.8	551.0	8.0	559.0
	100	3028.1	444				86.7	4.8	91.5
200	0	2733.9	443	333.3	0.2	333.5	772.9	5.1	778.0
	100	3581.6	444				229.6	11.0	240.6

with a normal recreation damage rate and double agricultural and property damage rate. The derived optimum values of ESL and ESH matched those in table 15; ECL was 0.5 ft lower with double agricultural damage rate for WS = 80, 120, and 160 cfs; ECH was 0.5 ft lower in the two cases; and ESF and ECF matched the values in table 15. When the recreation damage rate was double the normal rate, the average of new recreation damage divided by optimal recreation damage (table 15) ratios was 1.96, not much different from 2.0. Similarly, with double agricultural damage rate, the corresponding average of the damage ratios was 1.92. Because of practically no change in rule levels and comparative damages (damage/damage rate), the system is insensitive to the range of damage rates tested.

Recommendations. The results of various sensitivity analyses show that the following changes can be made in the rule levels, if deemed necessary to provide a little higher lake levels (so often requested by the recreation interests) without significantly affecting the overall damages.

1) ESL can be raised from 590.0 to 590.5 for WS = 0 and navigation = 0 and 50%, and WS = 40 and navigation = 0.

2) ECH can be raised from 443 to 444, causing some damage to recreation for WS = 40 and N = 100%, WS = 80 and N = 0%, and progressively higher damage for WS = 120, 160, and 200 cfs with N = 0.

Expected Value of Annual Benefits

The potential or maximum annual recreation benefit is $4.8075 million if the lake levels in Shelbyville and Carlyle are within the no-damage range of levels for the various recreation activities. The expected annual value of agricultural damage for the two river reaches below Shelbyville and Carlyle for the 24-year data is $0.6596 million as given in the previous report. Because the damages in the two lake areas for natural flow conditions cannot be assessed directly, the damage in the two river reaches is considered the base. Thus, the expected value of the potential, or maximum agricultural benefit is $0.6596 million if there were no damages to the crops with the operation of the two lakes. The revenue from water supply sales may be taken at 20¢ per 1000 gallons, or $1.8884 million per 40 cfs per year. The benefits for providing water for navigation, enhancement of water quality at low flow in the Kaskaskia River below Shelbyville to the confluence with the Mississippi River, and any reduction in flooding along the middle and lower Mississippi River because of the flood control effect of the two lakes on the Kaskaskia River have not been evaluated and may be taken as X million dollars per year.

The distribution of expected annual benefits and damages (benefits not realized) attributable to reduction in recreation and flooding of croplands because of adverse lake level and flow release conditions are given in table 22 for full navigation requirement and six levels of water supply withdrawals. The benefits and damages are plotted with respect to water supply in figure 11. The expected annual value of damages (or benefits not realized) varies from $0.0695 to $0.1492 million, compared with the annual potential benefit of $5.4671 for recreation and agriculture. Benefits from navigation, water quality improvement, and damage reduction along the Mississippi River are additional. Benefits from water supply sales are directly proportional to the withdrawals.

Table 22. Expected Value of Annual Benefits

(In millions of dollars)

	Water supply (cfs)					
	0	*40*	*80*	*120*	*160*	*200*
Recreation	4.8075	4.8075	4.8075	4.8075	4.8075	4.8075
Agriculture	0.6596	0.6596	0.6596	0.6596	0.6596	0.6596
Water supply		1.8884	3.7768	5.6652	7.5536	9.4420
Total	5.4671	7.3555	9.2439	11.1323	13.0207	14.9091
Damages	0.0695	0.0773	0.0902	0.1064	0.1262	0.1492
Net	5.3976	7.2782	9.1537	11.0259	12.8945	14.7599

Note: X million dollars per year may be added to the net expected value of annual benefit for the navigation and water quality enhancement benefits.

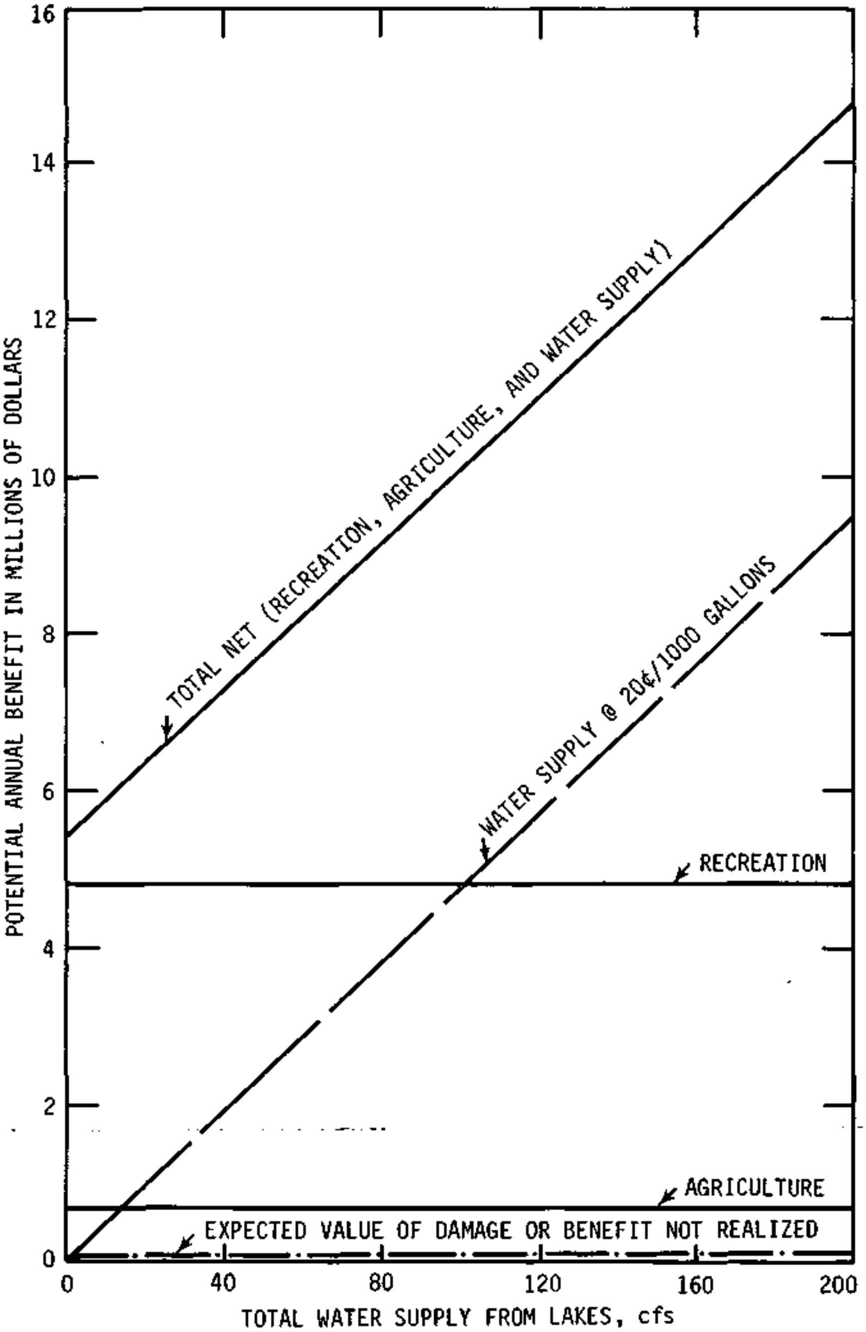


Figure 11. Expected annual benefits and damages with full navigation demand and 0 to 200 cfs of water supply

Water supply requirements equivalent to 2 to 3 or more times the net water supply withdrawal can be met by making withdrawals and returns at suitable locations along the river and by efficient management of return flows. The reuse of water (when requirements exceed allowable net withdrawal) can result in a reduction of water supply revenue rate to meet the annual fixed and operation, maintenance, and repair (OM&R) costs incurred by the state for the operation of the two lakes.

SYNTHETIC DATA

Synthetic Monthly Flows

The 24-year historical record does not contain river flows high enough to raise the lakes to even 620 ft at Shelbyville and 455 ft at Carlyle, though the top of flood pools are 626.5 and 462.5 ft, respectively. The water supply withdrawals and associated deficit frequencies can be inferred but they cannot be spelled out without a very long flow record.

An historical hydrologic sequence or data series is a sample that may never be repeated in the same way. The system response obtained with this sequence is unlikely to be a satisfactory representation of the various future responses of the system under consideration. A set of system responses can be obtained with simulated hydrologic sequences, called synthetic traces (Fiering, 1966). A number of these sequences of any desired length can be sequentially generated to aid in decision making with respect to system design, operation, and performance. These synthetic sequences must resemble the historical sequence in terms of certain properties that characterize the historical sequence (Matalas, 1967).

Stochastic models of streamflow generation are becoming more popular as a tool in hydrologic planning and design. A stochastic model is predicated to preserve meaningful statistical properties of an historical record in equally likely generated sequences that can be used to give a broad spectrum of response of the system under consideration.

Selection of the Underlying Distribution. No prior knowledge exists to justify the assumption that a specified distribution (Matalas and Wallis, 1973) is indeed the underlying distribution of streamflows. The most commonly used distributions for fitting monthly streamflows are the normal and the Pearson type 3 applied to observed flows or their logarithms. It has been shown (Singh and Sinclair, 1972; Singh, 1974; and Singh and Lonnquist, 1974) that for the monthly flows and flood distributions, the two-distribution gives a much better fit than the Pearson type 3, though the latter gives a better fit than the normal distribution. Two-distribution and Pearson type 3 distribution fitted to the November flows at Carlyle are shown in figure 12. The versatility of the two-distribution in fitting observed distributions of various shapes is evident from this figure.

Monthly flows were generated with both Pearson type 3 and two-distribution models. The generated sequences showed that the two-distribution sequences not only were more in accord with the historical data but also generated fewer high flows exceeding that physically possible.

Two-Distribution Model. A two-distribution model assumes that the observed hydrologic distributions can be fitted and simulated as a mixture of two normal or lognormal distributions. A brief mathematical background is included here.

A two-distribution can be written mathematically as

$$p\{x\} = a p_1\{x\} + (1-a) p_2\{x\} \qquad 0 \leqslant a \leqslant 1 \tag{1}$$

$$p_1\{x\} = \frac{1}{\sigma_1 (2\pi)^{1/2}} \int_{-\infty}^{x} \exp\left[-\frac{(x' - \mu_1)^2}{2\sigma_1^{\,2}}\right] dx' \tag{2}$$

$$p_2\{x\} = \frac{1}{\sigma_2 (2\pi)^{1/2}} \int_{-\infty}^{x} \exp\left[-\frac{(x' - \mu_2)^2}{2\sigma_2^{\,2}}\right] dx' \tag{3}$$

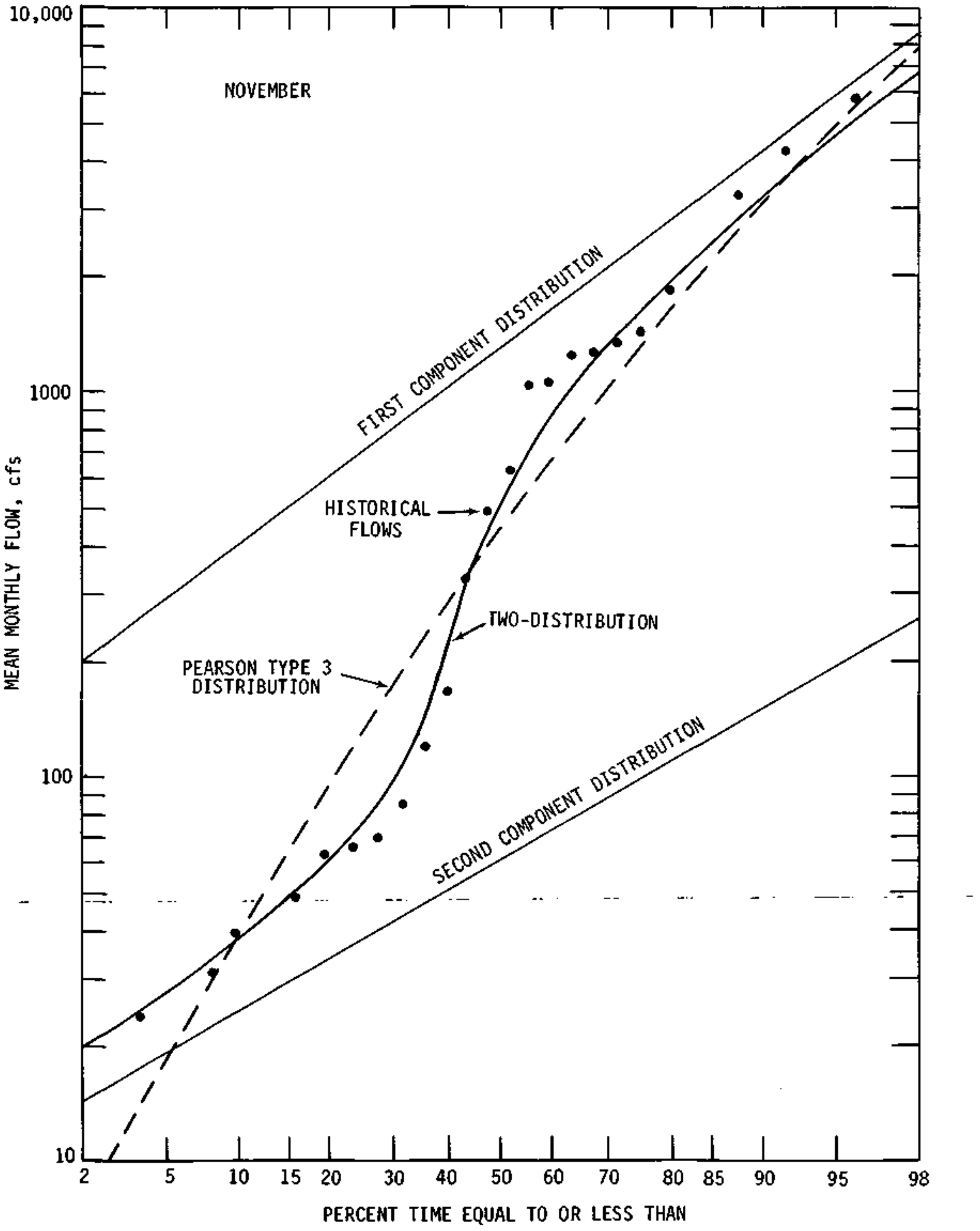


Figure 12. Two-distribution and Pearson type 3 fitted to observed November flows at Carlyle

in which p is the probability of being equal to or less than x; μ_1, μ_2, σ_1, and σ_2 are the means and square roots of variances for the two component distributions; subscripts 1 and 2 refer to the first and second component distribution; and a is the relative weight of the first component distribution. The two-distribution parameters are linked to mean $\bar{x}$ and standard deviation s of the given distribution according to the following equations (Cohen, 1967).

$$\bar{x} = a\mu_1 + (1 - a)\mu_2 \tag{4}$$

$$s^2 = a\sigma_1{}^2 + (1 - a)\sigma_2{}^2 + a(1 - a)(\mu_2 - \mu_1)^2 \tag{5}$$

The optimization of the five parameters $\mu_1, \mu_2, \sigma_1, \sigma_2$, and a for the logarithm of any month's observed flows, x, is achieved via a computer program developed for this purpose.

Persistence and Dependence. The four gaging stations, Shelbyville, Vandalia, Carlyle, and New Athens, are at mile 222, 161, 106, and 41 of the Kaskaskia River. The monthly flow records at the four stations indicate persistence — a tendency for high flows to follow high flows and low flows to follow low flows at a station, and dependence — a tendency for flows at a downstream station to be directly correlated with the flow at the upstream station. The persistence and dependence need to be maintained in the synthetically generated flow sequences. Persistence is described by the correlations between normalized flows for various months of the year at a station, and dependence is described by the correlations between normalized flows at downstream and upstream stations. In order to calculate these correlation coefficients, the population of variables must be bivariate or multivariate normal. The normalizing of flow logarithms is achieved by $(x - \bar{x})/s$ in the case of normal distribution, by the Wilson and Hilferty (1931) transformation if it is Pearson type 3, and by the following procedure in the case of the two-distribution.

1) First compute y_1 and y_2

$$y_1 = (x - \mu_1)/\sigma_1 \text{ and } y_2 = (x - \mu_2)/\sigma_2 \tag{6}$$

2) Convert y_1 and y_2 to p_1 and p_2 with the normal probability table
3) Compute $p = ap_1 + (1 - a)p_2$
4) Convert p to standard deviate Y using the normal probability table

The methodology was written as a subroutine in the two-distribution program.

Multiple regression analyses with the Y data (12 months x 24 years) at each of the four stations yielded regression coefficients which showed either random positive or negative values when Y at the upstream station was regressed with those at the downstream stations. The monthly regression coefficients showed orderly variation when the Y at the downstream station was regressed with those at the upstream station or stations. The results of these regressions are shown in table 23 for the following two equations:

$$(Y_{na})_{i,j} = a(Y_s)_{i,j} + b(Y_v)_{i,j} + c(Y_c)_{i,j} + d(Y_{na})_{i-1,j} + \epsilon_{i,j} \tag{7}$$

$$(Y_{na})_{i,j} = a(Y_c)_{i,j} + b(Y_{na})_{i-1,j} + \epsilon_{i,j} \tag{8}$$

in which Y is the two-distribution normalized standard deviate; i is the month under consideration, 1 through 12; j is the number of the year; subscripts s, v, c, and na refer to Shelbyville, Vandalia, Carlyle, and New Athens; a, b, c, and d are regression (beta) coefficients; and e is the random error. When i = 1, i - 1 equals 12 and j equals j - 1 in equations 7 and 8. Not only are the values of a and b orderly and stable as obtained from equation 8, but also the multiple correlation coefficients (though a little less than from equation 7) are more significant because of a larger number of degrees of freedom. In the case of monthly flows at Shelbyville equation 8 becomes

Table 23. Beta Coefficients and R^2 for New Athens Monthly Two-Distribution Normalized Standard Deviates Regressed with Shelbyville, Vandalia, and Carlyle Deviates

	For equation 7 *Beta coefficients*					*For equation 8* *Beta coefficients*		
Month	*a*	*b*	*c*	*d*	*R^2*	*a*	*b*	*R^2*
Oct	0.1250	−0.5496	1.3403	0.1041	0.9689	0.8940	0.1303	0.9649
Nov	0.0094	−0.5838	1.4760	0.1145	0.9638	0.9181	0.0982	0.9519
Dec	0.1811	−1.7782	2.4095	0.1874	0.9675	0.8592	0.1254	0.9312
Jan	−0.3950	−0.2035	1.5333	0.0536	0.9903	0.9700	0.0307	0.9821
Feb	−0.1552	−0.0452	1.1337	0.0766	0.9939	0.9265	0.0902	0.9927
Mar	−0.5196	0.4506	0.9517	0.1021	0.9543	0.9087	0.0967	0.9324
Apr	−0.3835	0.2403	1.0775	0.0813	0.9825	0.9441	0.1050	0.9755
May	−0.1892	0.4510	0.6898	0.0302	0.9379	0.9710	0.0327	0.9294
Jun	−0.4835	0.5838	0.7910	0.0748	0.9499	0.9307	0.0595	0.9318
Jul	−0.3162	0.3392	0.9430	0.0000	0.9593	0.9663	0.0126	0.9510
Aug	−0.0562	−0.4928	1.4041	0.1042	0.9062	0.9021	0.0571	0.8940
Sep	−0.1328	−0.5150	1.3351	−0.1012	0.8358	0.7259	−0.0857	0.7818

$$(Y_s)_{i,j} = b(Y_s)_{i-1,j} + \epsilon_{i,j} \tag{9}$$

The distribution of e as obtained from regression analyses was found to approximate the normal distribution. Therefore, ϵ is assumed to be normally distributed with zero mean and unit standard deviation, or as N(0,1), and simulated by SE $t_{i,j}$ in which SE is the standard error or $(1 - R^2)^{0.5}$ s, R being the multiple correlation coefficient, and t is the normal standard deviate.

Model Equations. The following model equations were derived for generating two-distribution normalized standard deviates at each of the four stations.

Shelbyville

$$(Y_s)_{i,j} = b(Y_s)_{i-1,j} + (SE_s)_i\, t_{i,j} \tag{10}$$

Vandalia

$$(Y_v)_{i,j} = a(Y_s)_{i,j} + b(Y_v)_{i-1,j} + (SE_v)_i\, t_{i,j} \tag{11}$$

Carlyle

$$(Y_c)_{i,j} = a(Y_v)_{i,j} + b(Y_c)_{i-1,j} + (SE_c)_i\, t_{i,j} \tag{12}$$

New Athens

$$(Y_{na})_{i,j} = a(Y_c)_{i,j} + b(Y_{na})_{i-1,j} + (SE_{na})_i\, t_{i,j} \tag{13}$$

Values of a, b, and SE at the four stations for each of the 12 months are given in table 24. A computer program was developed and used for generating 499 years of synthetic Y with the above equations.

Conversion of Deviates to Flows. A short computer subroutine (Singh and Lonnquist, 1974) was used to convert 499 years of $Y_{i,j}$ to $x_{i,j}$ and thence to $Q_{i,j}$, the mean monthly flows. The distribution of monthly runoff with the Pearson type 3 model matched the historical distribution poorly or not as well as the synthetic runoff with the two-distribution model. The distribution of 499 years of generated monthly flows for November and the historical flows are shown in figure 13. When the observed flow distributions did not exhibit marked reverse curvature, the fit of the Pearson model was not as poor as for months with marked reverse curvature.

The subroutine for transformation of generated Y to Q requires values of Y and the five two-distribution parameters, a, μ_1, μ_2, σ_1, and σ_2, given in table 25 for all four stations. For conversion of Y generated with the Pearson type 3 model, an inverse transform is applied:

Table 24. Model Equation Parameters for Generating
Two-Distribution Normalized Standard Deviates

Month	*a*	*b*	*SE*	*a*	*b*	*SE*
		Shelbyville			*Vandalia*	
Oct		0.717	0.704	0.892	0.102	0.312
Nov		0.742	0.603	1.040	-0.071	0.260
Dec		0.793	0.569	0.923	0.066	0.192
Jan		0.798	0.517	0.987	0.021	0.218
Feb		0.867	0.557	0.932	0.081	0.138
Mar		0.460	0.894	0.941	0.021	0.277
Apr		0.286	0.945	0.980	0.027	0.174
May		0.233	0.960	0.942	-0.030	0.413
Jun		0.532	0.844	0.928	0.067	0.299
Jul		0.701	0.713	0.821	0.194	0.260
Aug		0.482	0.878	0.789	0.127	0.520
Sep		0.609	0.803	0.804	0.090	0.506
		Carlyle			*New Athens*	
Oct	1.071	-0.101	0.159	0.894	0.130	0.190
Nov	0.967	0.009	0.187	0.918	0.098	0.216
Dec	1.032	-0.053	0.103	0.859	0.125	0.247
Jan	0.963	0.031	0.104	0.970	0.031	0.132
Feb	0.930	0.076	0.089	0.927	0.090	0.086
Mar	0.936	0.058	0.243	0.909	0.097	0.263
Apr	0.945	0.113	0.214	0.944	0.105	0.160
May	0.962	-0.032	0.233	0.971	0.033	0.271
Jun	0.924	0.118	0.234	0.931	0.060	0.271
Jul	0.910	0.097	0.260	0.966	0.013	0.229
Aug	1.007	-0.036	0.209	0.902	0.057	0.331
Sep	0.955	0.037	0.285	0.726	-0.086	0.376

$$x_{i,j} = \bar{x}_i + s_i \left[\frac{2}{g_i}\left(1 + \frac{Y_{i,j}g_i}{6} - \frac{g_i^2}{36}\right)^3 - 1 \right] \qquad (14)$$

The distribution of synthetic flows was plotted and compared with the historical flow distribution. These comparisons indicated that some 'uncharacteristic' values in the historical record should be adjusted and that some limiting device needs to be added to the generation program to filter out unrealistic high flows. Flow adjustments were made for 9 monthly flows out of the 1248 historical flows at Shelbyville. In order to obtain better fitting of the historical and generated flow sequences, minor adjustments were made in the two-distribution parameters for two or three months. This made the annual distributions obtained from historical and generated sequence fit better. Table 25 contains the final parameter values.

Outliers in Synthetic Data. Estimates of probable maximum precipitation, PMP, were used to define outliers in the 499 years of synthetic monthly flows. The PMP values are available for storms of 6, 12, 24, and 48 hours duration and areas from 10 to 1000 square miles (Riedel et al., 1956) for the 12 months. The depth-area-duration relations were extrapolated to 2000 square miles. Values of 2-day PMP for the drainage area above Shelbyville and Vandalia for the months October through September were obtained by multiplying the 24-hour 200 square mile PMP value at the center of the drainage area with the factor from depth-area-duration curves for 2-day duration and 1030 and 1980 square miles, respectively, for each of the 12 months. In the case of the drainage area above Shelbyville, the monthly PMP was obtained by adding the

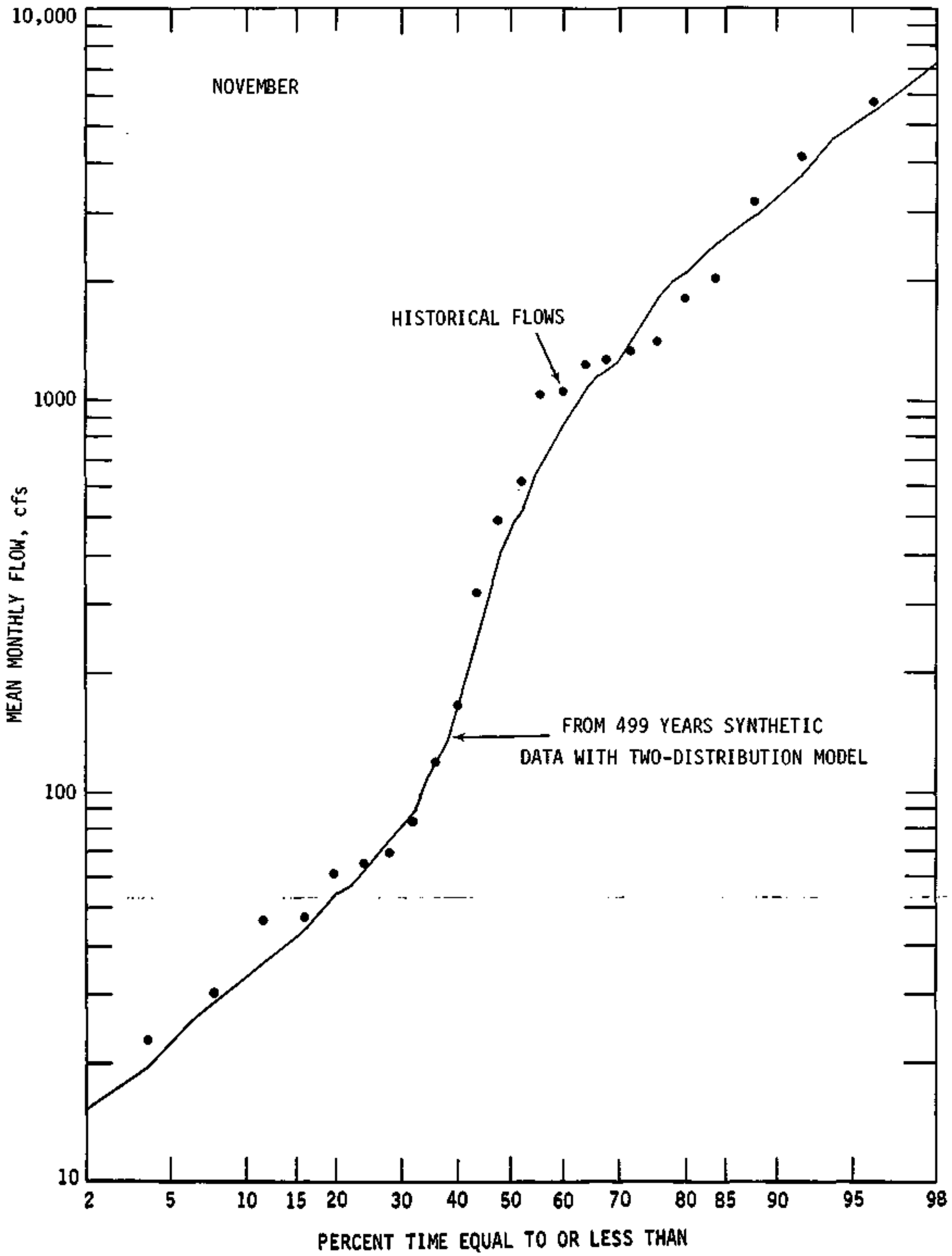


Figure 13. Distribution of November flows at Carlyle generated with the two-distribution model

Table 25. Two-Distribution Parameters for Monthly Flow Logarithms

Month	a	μ_1	σ_1	μ_2	σ_2	a	μ_1	σ_1	μ_2	σ_2
			Shelbyville					*Vandalia*		
Oct	0.7	1.606	1.101	1.253	0.249	0.5	2.470	0.695	1.744	0.384
Nov	0.5	2.729	0.399	1.245	0.448	0.5	3.076	0.345	1.802	0.430
Dec	0.6	2.689	0.371	1.148	0.300	0.6	3.050	0.320	1.684	0.334
Jan	0.7	2.880	0.636	1.300	0.320	0.7	3.160	0.630	1.922	0.292
Feb	0.5	3.283	0.405	2.136	0.503	0.5	3.490	0.390	2.569	0.563
Mar	0.5	2.962	0.243	3.110	0.300	0.6	3.400	0.196	3.132	0.404
Apr	0.5	3.344	0.244	2.818	0.263	0.5	3.450	0.310	3.156	0.455
May	0.5	3.118	0.386	2.758	0.177	0.6	3.282	0.576	3.083	0.100
Jun	0.5	3.234	0.310	2.698	0.326	0.5	3.300	0.390	3.018	0.471
Jul	0.5	2.742	0.535	2.380	0.594	0.5	3.100	0.430	2.713	0.583
Aug	0.6	2.060	0.621	1.633	0.684	0.5	2.580	0.560	2.291	0.447
Sep	0.7	1.350	0.990	1.264	0.283	0.5	2.220	0.590	1.929	0.372
			Carlyle					*New Athens*		
Oct	0.5	2.525	0.600	2.001	0.395	0.5	2.811	0.571	2.333	0.357
Nov	0.6	3.114	0.396	1.785	0.306	0.6	3.336	0.395	2.163	0.305
Dec	0.6	3.153	0.290	1.866	0.325	0.5	3.473	0.217	2.384	0.397
Jan	0.6	3.399	0.482	2.168	0.316	0.6	3.641	0.489	2.474	0.348
Feb	0.5	3.621	0.323	2.681	0.520	0.5	3.837	0.352	3.005	0.565
Mar	0.6	3.549	0.195	3.216	0.411	0.6	3.820	0.189	3.444	0.451
Apr	0.5	3.650	0.269	3.290	0.482	0.5	3.787	0.267	3.606	0.508
May	0.5	3.570	0.395	3.130	0.290	0.4	3.630	0.500	3.550	0.290
Jun	0.5	3.233	0.286	3.195	0.519	0.4	3.509	0.543	3.431	0.247
Jul	0.6	2.870	0.645	3.390	0.375	0.3	3.260	0.350	3.350	0.480
Aug	0.7	2.512	0.598	2.642	0.158	0.4	3.144	0.520	2.761	0.338
Sep	0.6	2.277	0.453	2.244	0.186	0.6	2.691	0.433	2.534	0.175

2-day PMP, and the mean and one standard deviation of the observed monthly precipitation at Urbana. For monthly PMP above Vandalia, the corresponding 2-day PMP and the mean and one standard deviation of monthly precipitation at Vandalia were added together.

The monthly PMP was converted to maximum probable mean monthly flow, Q_{mp}, by

$$Q_{mp} = C\,(PMP_2 + \text{mean} + \text{standard deviation}) \tag{15}$$

in which PMP_2 denotes the 2-day PMP estimate, and C is the factor for converting inch-square-mile to cfs — 910 cfs for Shelbyville and 1750 cfs for Vandalia. For a 499-year record, a storm frequency of 1 in 500 years will be of the same order as the standard project storm which is about 60 percent of the PMP. Historical monthly flows and precipitation were analyzed to derive runoff factors that could be expected with very high monthly precipitation. The monthly mean flows that may not be exceeded in the generated sequence, Q_L, are obtained from

$$Q_L = f\,g\,Q_{mp} \tag{16}$$

in which f is the fraction of PMP that may be expected to occur on the average once in 499 years, and g is the corresponding monthly runoff factor. The values of PMP_2; the mean and standard deviation of monthly precipitation at Urbana; f; g; and Q_L or the limit discharge are given in table 26 for the drainage basin above Shelbyville. Maximum monthly flows observed over the 24-year period are also included in the table. Similarly, values of Q_L were determined at the Vandalia gage.

Table 26. Limiting Flows at Shelbyville

Month	*PMP_2 (in)*	*Mean (in)*	*Std dev (in)*	*PMP (in)*	*Q_{mp} (cfs)*	*f*	*g*	*Q_L (cfs)*	*Q_{max} (cfs)*
Oct	22.45	2.80	1.92	27.17	24,740	0.6	0.5	7,400	1,349
Nov	16.34	2.62	1.29	20.25	18,430	0.6	0.7	7,700	2,396
Dec	13.88	2.37	1.55	17.80	16,200	0.7	0.7	7,900	1,532
Jan	11.55	2.12	1.61	15.28	13,910	0.7	1.0	9,700	7,097*
Feb	12.36	1.95	1.04	15.35	13,970	0.6	1.0	8,400	4,033
Mar	13.04	3.21	1.74	17.99	16,380	0.7	0.7	8,000	3,038
Apr	15.35	3.86	1.92	21.13	19,240	0.7	0.8	10,800	4,848
May	18.16	3.92	2.17	24.25	22,080	0.7	0.8	12,400	6,527
Jun	21.43	4.27	2.13	27.83	25,340	0.6	0.8	12,200	4,608
Jul	21.96	3.85	2.33	28.14	25,620	0.5	0.7	9,000	3,043
Aug	22.03	3.13	1.46	26.62	24,230	0.5	0.5	6,100	2,347
Sep	21.86	3.30	2.19	27.35	24,890	0.5	0.5	3,700	764

*Second highest for January is 3429 cfs

For drainage areas above Carlyle and New Athens, the extrapolation of depth-area-duration curves from 1000 to 2680 and 5120 square miles was not considered satisfactory. The monthly limit flows at Shelbyville and Vandalia were plotted against drainage area, and the straight line curves were extended to yield limit flows at Carlyle and New Athens. The limit flows for each of 12 months at Shelbyville, Vandalia, Carlyle, and New Athens are graphed in figure 14. Any generated monthly flows higher than these values were considered as outliers.

Filtering Outliers. The limit flow for each month was used in the computer program to curtail any higher flows generated to the limit value. The distribution of high flows modified to Q_L at the four gaging stations is given in table 27.

Out of a total of 499 x 12 or 5988 monthly flows at a station, 38 are modified at Shelbyville, 24 at Vandalia, 14 at Carlyle, and 22 at New Athens. Overall it means modification of one high flow out of 245 synthetic monthly flows. Such a low frequency of outliers in synthetic data underscores the merit of the two-distribution model.

Low Flow Adjustment. Comparison of the distribution of synthetic annual flows, obtained by adding monthly flows weighted for the number of days in a month, with the historical annual flows at the four gaging stations indicated that a minor modification of synthetic flows is needed at the lower end of the flow spectrum to preserve the drought characteristics. The modifications in synthetic annual flows at Shelbyville, Vandalia, and Carlyle are given in table 28. The synthetic monthly flows in the years with modified annual flow were made equal to the synthetic monthly flow multiplied by the ratio of the modified annual flow to the unmodified annual flow.

Statistics of Synthetic Flows. The 499-year synthetic data were partitioned into 10 sets of 49 years each by dropping the first 9 years. Means and standard deviations of the logarithms of flows in these sets, in the 499-year sequence and in the 24-year historical data at Carlyle, are given in table 29. The spread of these statistics about the historical values shows that the generated 49-year sequences have means and standard deviations which vary within an expected margin of the historical mean and standard deviation. The 499-year statistics are very close to the historical. Statistics for Shelbyville, Vandalia, and New Athens flows exhibited characteristics similar to those at Carlyle.

Table 27. Distribution of High Flows Modified to Flow Limits

	Number of months modified in 499 years of synthetic data at			
	Shelbyville	*Vandalia*	*Carlyle*	*New Athens*
October	6			
November				
December				
January	9	6		1
February	10	1		1
March	2	1	2	5
April	1	2	1	2
May		10		4
June	1	1	2	3
July	3	3	9	2
August				4
September	6			
Total	38	24	14	22

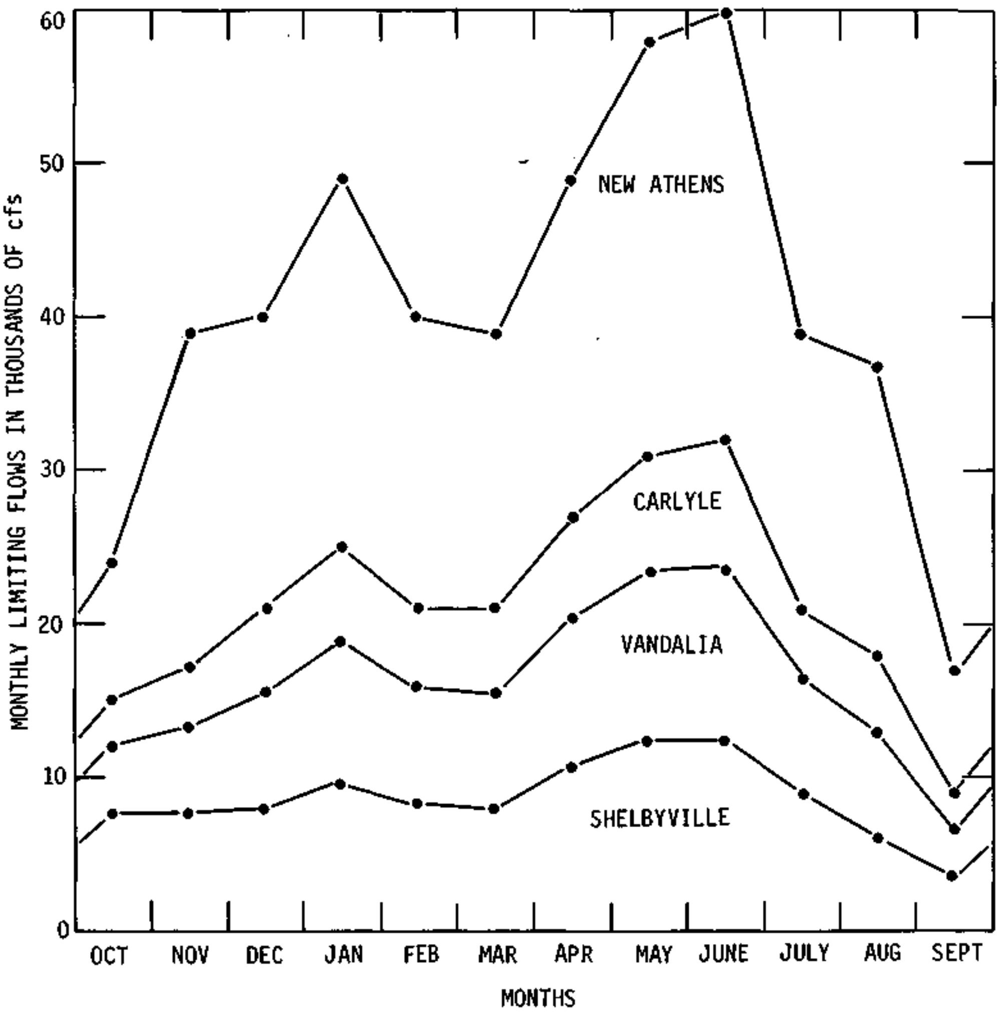


Figure 14. Limiting monthly flows for synthetic data

Table 28. Modification of Synthetic Annual Flows at the Lower End of the Flow Spectrum

	Shelbyville			*Vandalia*			*Carlyle*	
Year	*Q*	*Q**	*Year*	*Q*	*Q**	*Year*	*Q*	*Q**
19	238.0	104	86	564.6	50	186	577.7	67
24	264.6	214	98	562.6	180	294	358.9	64
67	196.5	25	107	573.6	515	366	372.9	62
98	235.8	210	195	568.7	260	398	685.6	340
143	228.9	205	300	530.7	90	430	457.6	118
221	281.2	68	370	575.5	462	450	204.4	60
246	183.2	154	394	551.5	160	472	374.4	230
251	188.8	161	414	574.6	463			
266	231.3	195	432	528.9	90			
294	228.9	185	450	551.2	170			
324	289.9	93						
330	234.2	114						
349	236.1	80						
366	145.8	123						
430	213.1	190						
473	238.4	66						

*Q is the adjusted value

Table 29. Synthetic and Historical Flow Logarithm Statistics

	Synthetic data set number											
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*499 years*	*Historical*
Means, $\overline{x}$												
Oct	2.198	2.255	2.244	2.439	2.364	2.058	2.263	2.377	1.900	2.559	2.273	2.263
Nov	2.513	2.553	2.633	2.658	2.764	2.442	2.579	2.418	2.163	2.705	2.551	2.582
Dec	2.549	2.593	2.718	2.757	2.750	2.638	2.573	2.443	2.259	2.753	2.608	2.638
Jan	2.766	2.889	3.070	3.035	2.884	2.860	2.911	2.711	2.691	3.073	2.895	2.906
Feb	3.097	3.134	3.224	3.117	3.215	3.031	3.086	2.949	3.006	3.266	3.114	3.151
Mar	3.338	3.267	3.518	3.498	3.359	3.353	3.344	3.272	3.322	3.361	3.364	3.416
Apr	3.364	3.388	3.456	3.425	3.439	3.387	3.304	3.316	3.374	3.467	3.388	3.410
May	3.316	3.353	3.283	3.350	3.378	3.302	3.503	3.330	3.255	3.139	3.322	3.342
Jun	3.237	3.027	3.232	3.237	3.133	3.078	3.310	3.146	3.093	3.109	3.165	3.214
Jul	3.088	2.744	2.888	3.044	2.929	2.890	3.130	2.970	2.801	2.961	2.953	3.033
Aug	2.581	2.575	2.432	2.636	2.585	2.469	2.486	2.450	2.286	2.718	2.525	2.551
Sep	2.293	2.194	2.248	2.237	2.270	2.180	2.151	2.291	2.044	2.382	2.237	2.264
Standard deviation, s												
Oct	0.624	0.549	0.571	0.592	0.603	0.543	0.621	0.620	0.685	0.694	0.636	0.571
Nov	0.805	0.768	0.721	0.786	0.725	0.790	0.750	0.824	0.800	0.831	0.792	0.745
Dec	0.751	0.751	0.679	0.693	0.695	0.735	0.818	0.758	0.741	0.668	0.739	0.700
Jan	0.688	0.790	0.665	0.693	0.699	0.713	0.861	0.777	0.757	0.678	0.739	0.737
Feb	0.661	0.763	0.628	0.647	0.576	0.677	0.739	0.757	0.718	0.632	0.683	0.639
Mar	0.364	0.518	0.266	0.350	0.397	0.450	0.528	0.438	0.426	0.375	0.421	0.342
Apr	0.402	0.528	0.477	0.489	0.499	0.511	0.539	0.490	0.494	0.475	0.492	0.409
May	0.415	0.424	0.439	0.465	0.528	0.465	0.541	0.453	0.443	0.439	0.467	0.440
Jun	0.416	0.654	0.490	0.451	0.392	0.401	0.510	0.429	0.452	0.619	0.503	0.420
Jul	0.630	0.855	0.706	0.693	0.578	0.690	0.609	0.704	0.702	0.785	0.702	0.556
Aug	0.614	0.633	0.483	0.662	0.552	0.569	0.803	0.628	0.459	0.540	0.605	0.512
Sep	0.454	0.430	0.422	0.431	0.351	0.447	0.486	0.470	0.496	0.412	0.450	0.370

Synthetic Weekly Flows

The synthetic monthly flows had to be converted to weekly flows because the system program for deriving the optimum operation rules for three levels of navigation flow requirement and six levels of water supply withdrawal used weekly flows at Shelbyville, Carlyle, and New Athens. The presently available model (Valencia and Schaake, 1973) could not be used for disaggregating the monthly flows to weekly flows because there is no integer number of weeks in a month. A new methodology was devised, based on the analysis of observed weekly and monthly flows, to disaggregate or convert monthly to weekly flows, retaining the monthly statistics and conforming to the Markov chain of up to third order between ratios of weekly to monthly mean flows.

Disaggregation Methodology. The observed weekly flows are first transformed to ratios by dividing them by the monthly flows, allowing for a week covering days that overlap 2 months and time lags of 2 days between Carlyle and Shelbyville and 3 days between New Athens and Shelbyville flows. The first week starts October 1 at Shelbyville, October 3 at Carlyle, and October 4 at New Athens. The weekly ratios are obtained from

$$\begin{aligned} R_{i,j} &= QW_{i,j}/QM_{1,j} \qquad\qquad i = 1, 2, 3, 4 \\ R_{5,j} &= QW_{5,j}/[(3QM_{1,j} + 4QM_{2,j})/7] \text{ at Shelbyville} \\ &= QW_{5,j}/[(QM_{1,j} + 6QM_{2,j})/7] \text{ at Carlyle and so on} \end{aligned} \tag{17}$$

In equation 17, subscripts i and j refer to the number of the week and the number of the year; QW and QM are the mean weekly and monthly flows; and the multiplier of 3 for $QM_{1,j}$ at Shelbyville changes to 1 at Carlyle because of 2 days lag. December 31 and February 29 in leap years were neglected; days in December and February were taken as 30 and 28, respectively. The weekly ratios were calculated for all 52 weeks of the 24 years of historical flow record at Shelbyville, Carlyle, and New Athens.

The weekly ratios, R, were used to determine a, b, c, and d in the following equation, which represents a Markov chain of the third order.

$$\begin{aligned} R_{1,j} &= a + b\,R_{52,j-1} + c\,R_{51,j-1} + d\,R_{50,j-1} + \epsilon \\ R_{2,j} &= a + b\,R_{1,j} + c\,R_{52,j-1} + d\,R_{51,j-1} + \epsilon \\ R_{3,j} &= a + b\,R_{2,j} + c\,R_{1,j} \; d\,R_{52,j-1} + \epsilon \\ R_{i,j} &= a + b\,R_{i-1,j} + c\,R_{i-2,j} + d\,R_{i-3,j} + \epsilon \\ & \qquad\qquad i = 4, 5, \ldots, 52 \end{aligned} \tag{18}$$

In equation 18 a is an intercept; b, c, and d are coefficients of regression; and e is distributed with zero mean and standard deviation equal to the standard error. For some weeks, inclusion of d, or d and c, or d, c, and b did not improve the multiple correlation coefficient (or the improvement was insignificant). When b, c, and d are zero, values of a and SE denote the mean and standard deviation of the observed ratios in that week over the 24 years. As an example, values of a, b, c, d, and SE for weekly ratios at Carlyle are given in table 30. Also included are the minimum and maximum allowable values of R for modifying any excessively low or high values of generated ratios. The minimum and maximum R are obtained from an analysis of the historical distribution of R, allowing for the higher and lower values that can be expected in 499 years of synthetic ratios compared with the 24 years of historical ratios.

Table 30. Statistics for Generating Weekly Ratios at Carlyle

Week	*a*	*b*	*c*	*d*	*SE*	*Min*	*Max*
1	0.9352				0.5369	0.2	3.0
2	0.9590				0.5249	0.2	3.0
3	0.9237				0.5427	0.3	3.0
4	3.5711	-0.7756	-0.8295	-0.9846	0.3110	0.3	3.0
5	0.6667	0.0375	0.2830	-0.2178	0.3825	0.2	2.5
6	0.1439	0.4729	-0.0061	0.4009	0.4404	0.2	2.5
7	1.1048				0.4952	0.3	2.5
8	2.7770	-0.5295	-0.5816	-0.6998	0.1826	0.3	2.5
9	2.2192	0.2544	-0.8314	-0.6758	0.5581	0.2	2.5
10	0.9431				0.4747	0.2	2.5
11	0.9925	0.3944	-0.1056	-0.3367	0.3128	0.2	2.5
12	1.5097	-0.0226	-0.3936	-0.2379	0.2684	0.3	2.5
13	1.8152	0.1959	-0.6641	-0.3891	0.4498	0.3	3.0
14	0.6724	0.7183	-0.3960	-0.0302	0.5562	0.2	3.0
15	0.9032				0.3773	0.3	2.5
16	1.1431	0.2371	-0.2924	-0.1406	0.3964	0.3	2.5
17	3.7800	-0.8063	-0.9233	-1.0372	0.1728	0.2	3.0
18	0.7538				0.5749	0.1	3.0
19	1.0380	0.3021	0.0251	-0.4546	0.4443	0.1	2.5
20	1.3657	-0.0918	-0.3584	0.0643	0.5080	0.2	2.5
21	3.1591	-0.6781	-0.8553	-0.6520	0.1556	0.2	3.0
22	0.9282				0.6412	0.2	3.0
23	0.5689	-0.0916	0.3500	0.0302	0.3251	0.2	2.5
24	1.0615	-0.4630	-0.2982	-0.1988	0.3274	0.3	2.5
25	2.1458	-0.2337	-0.4838	-0.4222	0.2344	0.4	2.5
26	1.7895	0.4609	-0.8048	-0.5050	0.6038	0.3	3.0
27	0.9274	0.4718	-0.4476	0.1075	0.4279	0.3	2.5
28	1.1358	0.1385	-0.2175	-0.0727	0.2996	0.3	2.5
29	1.8989	-0.2849	-0.5467	-0.1152	0.3533	0.3	3.0
30	3.1406	-0.6342	-0.3584	-1.2226	0.4224	0.2	3.0
31	-0.2762	0.5576	0.2268	0.7668	0.5492	0.2	3.0
32	0.9383				0.4156	0.3	3.0
33	2.1987	-0.2951	-0.5516	-0.2709	0.3406	0.3	2.5
34	3.1983	-0.6188	-0.7192	-0.8358	0.0992	0.2	2.5
35	0.7775	0.4427	-0.2648	-0.1119	0.4235	0.2	2.5
36	0.5841	0.3113	-0.3592	0.3135	0.4127	0.2	2.5
37	1.4120	-0.2021	-0.5675	0.3602	0.4337	0.2	2.5
38	2.1504	-0.2590	-0.7388	-0.3009	0.3261	0.3	2.5
39	3.8023	-0.8805	-0.6199	-1.2045	0.5936	0.4	3.0
40	1.2950				0.5663	0.3	3.0
41	0.9780				0.4487	0.3	2.5
42	1.8967	-0.1234	-0.6590	-0.0592	0.2817	0.2	2.5
43	3.2722	-0.6204	-0.7398	-1.1014	0.2796	0.2	2.5
44	1.4435	0.4203	-0.2215	-0.1835	0.5977	0.2	3.0
45	1.1838				0.5383	0.3	3.0
46	2.1390	-0.4557	-0.5319	0.1787	0.4349	0.3	3.0
47	2.2257	-0.3773	-0.5396	-0.4511	0.2422	0.2	3.0
48	1.1483	0.4915	-0.3136	-0.2281	0.6487	0.2	3.0
49	0.9220				0.4068	0.2	2.5
50	0.8618				0.3782	0.2	2.5
51	1.8872	-0.1506	-0.6657	-0.1685	0.5978	0.2	3.0
52	3.5236	-0.8890	-0.7403	-1.2024	0.4777	0.2	3.0

A matrix of 499 x 52 ratios is generated at each of the stations with the information exemplified in table 30. In equation 18, ϵ equals SE times $t_{i,j}$; t is the random normal deviate; i = 1, 2, . . . , 52, and j = 1, 2, . . . , 499.

The synthetic or generated ratios were transformed to weekly flows. The method is illustrated for flows at Carlyle. Monthly factors, or the sum of ratios occurring in a month divided by the norm for that month, are computed as

$$\begin{aligned} MFR_{1,j} &= [R_{52,j-1}\,(2/7) + \sum_{i=1}^{4} R_{i,j} + R_{5,j}\,(1/7)]\ 7/31 \\ MFR_{2,j} &= [R_{5,j}\,(6/7) + \sum_{i=6}^{8} R_{i,j} + R_{9,j}\,(3/7)]\ 7/30 \text{ and so on} \end{aligned} \tag{19}$$

In equation 19, MFR denotes the monthly factor for the year j; subscripts 1, 2, . . . , 12 for MFR represent the number of the month. The synthetic weekly ratios are adjusted to conform to the monthly norm by

$$\begin{aligned} R_{i,j} &= R_{i,j}/MFR_{1,j} \qquad i = 1, 2, 3, \text{ or } 4 \\ R_{5,j} &= R_{5,j}/[MFR_{1,j}\,(1/7) + MFR_{2,j}\,(6/7)] \text{ and so on} \end{aligned} \tag{20}$$

The synthetic weekly flows are obtained from adjusted ratios:

$$\begin{aligned} QW_{i,j} &= R_{i,j}\ QM_{1,j} \qquad i = 1, 2, 3, \text{ or } 4 \\ QW_{5,j} &= R_{5,j}\ [QM_{1,j}\,(1/7) + QM_{2,j}\,(6/7)] \text{ and so on} \end{aligned} \tag{21}$$

A computer program based on this methodology was developed. The computer cost was about $20 for generating the 499 x 52 ratio matrix, ratio adjustments, and computation, punching, and printing of the synthetic weekly flows from the 499 years of synthetic monthly flows.

Comparison of Historical and Synthetic Weekly Flow Statistics. The means, $\bar{x}$, and the standard deviations, s, of the logarithms of 24 historic flows in each of the 52 weeks were computed at both Shelbyville and Carlyle. The correlation coefficients, r, between the logarithms of flows at Carlyle and Shelbyville for each of the 52 weeks were also calculated. These are plotted as dots in figure 15.

The logarithms of the synthetic 499-year weekly flows at Shelbyville and Carlyle were analyzed to compute the means, standard deviations, and correlation coefficients. The resulting values are plotted as solid curves in figure 15.

The historical and synthetic means, $\bar{x}$, at Shelbyville and Carlyle are in excellent agreement. The standard deviations, s, are in good agreement. The scatter is rather small and is expected when the two different record lengths of 24 and 499 years are considered. The values of the correlation coefficient, r, fit very well for the months October to February, and are in good agreement for the rest of the year. Therefore, the synthetic weekly flows are considered a good representation of the historical weekly flows.

Synthetic Lake Precipitation and Evaporation

The observed or derived monthly precipitation and evaporation for Lake Shelbyville and Carlyle Lake are available for the 24-year historical record, 1942 through 1965. These data were used to adjust the weekly inflows to the lakes for precipitation on and evaporation from the lake surfaces. Similar data were needed for adjusting the 499 years of synthetic inflows.

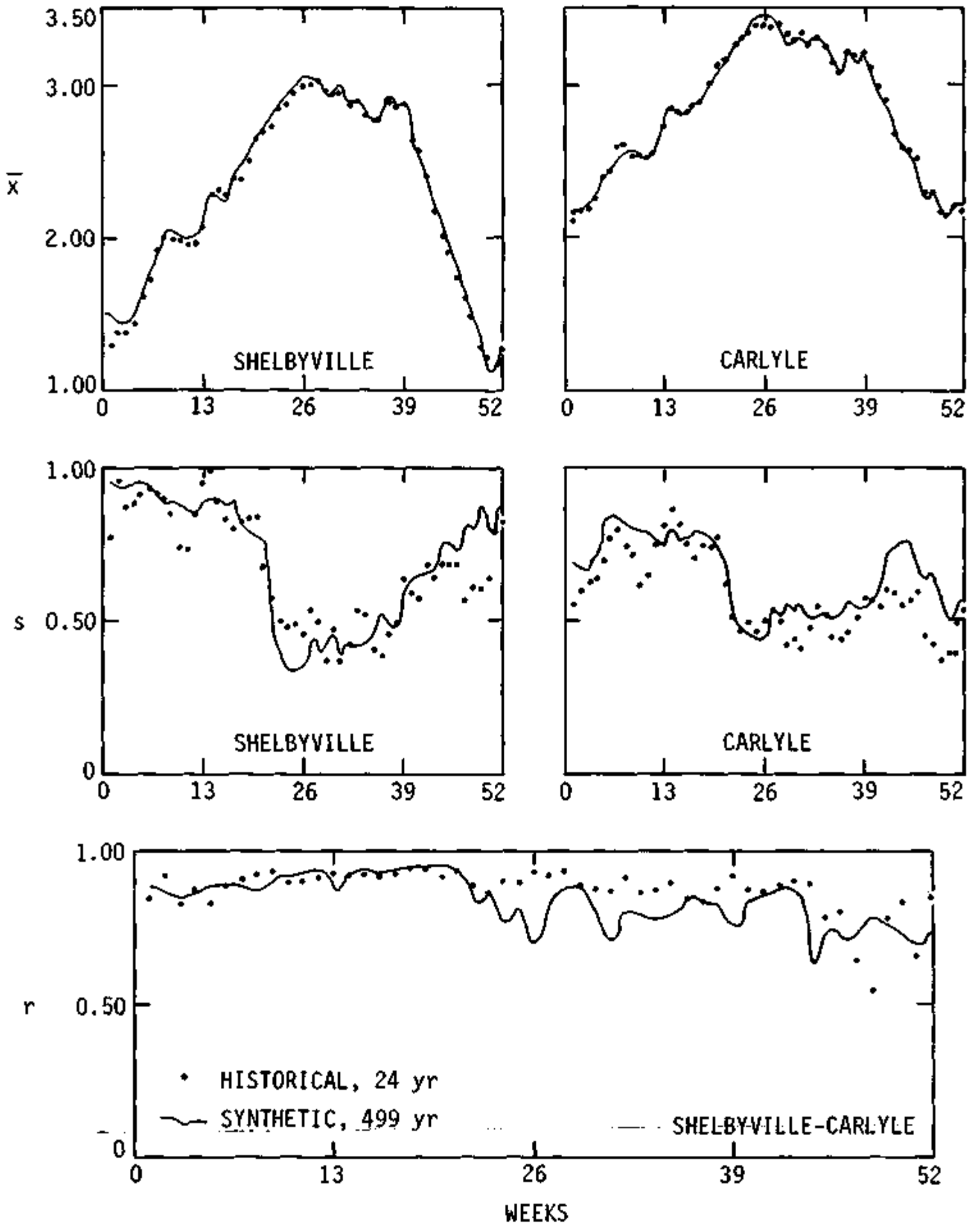


Figure 15. Comparison of historical and synthetic weekly flow statistics

Monthly Precipitation. Two types of models were tested for simulating the relationship between observed monthly precipitation and monthly flow, and their subsequent use for generating monthly precipitations given the synthetic monthly flows for 499 years at Shelbyville and Carlyle. The models are:

$$\log Q_i = a_i + b_i \log P_i + c_i \log P_{i-1} + d_i \log P_{i-2} + \epsilon \qquad (22)$$

and

$$\log P_i = a_i + b_i \log Q_i + c_i \log Q_{i+1} + \epsilon \qquad (23)$$

Table 31. Statistics for Generating Monthly Precipitation over Lakes

Month	*a*	*b*	*c*	*SE*	*Min*	*Max (inches)*
Lake Shelbyville						
Oct	0.1687	0.1491		0.2876	0.5	10
Nov	0.0183	0.1830		0.2052	0.6	9
Dec	-0.1478	-0.1679	0.3103	0.2322	0.4	7
Jan	-0.4811	0.2864		0.2513	0.3	9
Feb	-0.1518	0.1704		0.3003	0.2	7
Mar	-0.1425	0.2182		0.1857	0.8	10
Apr	-0.2994	0.2721		0.1669	1.4	10
May	-0.3253	0.3188		0.1441	1.4	10
Jun	-0.3143	0.1879	0.1516	0.1707	1.0	10
Jul	0.0731	0.0463	0.1797	0.1837	1.0	9
Aug	0.4257			0.2002	0.7	9
Sep	0.0912	-0.0873	0.2572	0.2100	0.4	9
Carlyle Lake						
Oct	-0.5352	0.3836		0.3596	0.3	11
Nov	-0.2487	0.2494		0.2719	0.4	9
Dec	-0.1147	-0.0394	0.1730	0.2580	0.4	7
Jan	-0.9251	0.3820		0.2995	0.2	10
Feb	-0.5460	0.2715		0.3576	0.2	8
Mar	-0.4355	0.2767		0.1934	0.7	11
Apr	-0.1989	0.2061		0.2002	1.0	11
May	-0.5965	0.3582		0.1663	1.4	12
Jun	-0.8398	0.1094	0.3540	0.2543	0.8	12
Jul	-0.4377	-0.0521	0.4002	0.3028	0.5	10
Aug	0.4028			0.2826	0.5	10
Sep	-0.7381	0.0718	0.4079	0.2468	0.3	10

in which Q and P denote the monthly flow and precipitation; a is the intercept and b, c, and d are regression coefficients; e is the random error term; and subscript i denotes the number of month, 1,2,..., 12. The model represented by equation 22 was not satisfactory because the division by b. in equation 24, i.e.,

$$\log P_i = (\log Q_i - a_i - c_i \log P_{i-1} - d_i \log P_{i-2} - \epsilon)/b_i \qquad (24)$$

greatly affects the results for small values of b. and the dependence of P. on the previous monthly values of P. Equation 23 was found to be quite satisfactory. It indicates that the effect of rainfall in a month affects the flow in that month and at the most the flow in the next month in the Kaskaskia River basin. The error term e equals the standard error times the random normal standard deviate. Values of a, b, c, and SE in equation 23 with respect to flows and precipitation at Shelbyville and Carlyle are given in table 31. One month's precipitation significantly affects the flows in the next month only for the months December, June, July, and September, at both stations. Values of minimum and maximum monthly precipitation expected to occur in the 499-year period are given in table 31 and were used for modifying any very high or very low precipitation generated. These were derived from an analysis of historical monthly precipitation, allowing for the probability of being higher or lower than the observed monthly highs and lows because of the increased record length to 499 years of synthetic record.

Synthetic monthly precipitation was calculated from synthetic monthly flows with equation 25 and parameters as in table 31:

$$\begin{aligned} \log P_{i,j} &= a_i + b_i \log Q_{i,j} + SE_i\, t_{i,j} \\ &\qquad i = 1, 2, 4, 5, 6, 7, 8, \text{ and } 11 \\ \log P_{i,j} &= a_i + b_i \log Q_{i,j} + c_i \log Q_{i-1,j} + SE_i\, t_{i,j} \\ &\qquad i = 3, 9, 10, \text{ and } 12 \end{aligned} \tag{25}$$

in which j is the year 1, 2, 3, ... , 499; and $t_{i,j}$ is the random normal standard deviate. The historical and synthetic annual means, obtained by finding the mean of annual precipitation, were in excellent agreement as shown below.

	Mean annual precipitation (inches)	
	24-year historical data	*499-year synthetic data*
Shelbyville	38.55	38.32
Carlyle	39.15	39.31

The generated monthly precipitation at Shelbyville and Carlyle was adjusted by multiplying with 38.55/38.32 and 39.15/39.31, respectively.

Monthly Evaporation. Three types of models were found satisfactory in correlating evaporation with flow or precipitation:

$$\begin{aligned} E_{i,j} &= a_i + b_i \log Q_{i,j} + SE_i\, t_{i,j} \\ &\qquad i \neq 3, 4, 5, \text{ and } 6 \end{aligned} \tag{26a}$$

$$\begin{aligned} E_{i,j} &= a_i + b_i\, P_{i,j} + SE_i\, t_{i,j} \\ &\qquad i = 5 \end{aligned} \tag{26b}$$

$$\begin{aligned} E_{i,j} &= a_i + SE_i\, t_{i,j} \\ &\qquad i = 3, 4, \text{ and } 6 \end{aligned} \tag{26c}$$

in which E is the monthly evaporation from lake surface in inches; subscript i and j denote the month and year; SE is the standard error of estimate obtained by regression of monthly evaporation on monthly flow or precipitation from the 24-year historical record; and t is the random normal standard deviate. Lake evaporation is nearly independent of rainfall and flow during the months December, January, and March at both the lakes. An analysis of historical data shows that evaporation in February decreases with increase in precipitation, P. For all other months, April through November, lake evaporation decreases as the monthly flow, Q, increases. The minimum and maximum E values were derived from an analysis of the historical data, keeping in mind the probability of lower and higher values in the 499-year synthetic record than in the 24-year historical record (table 32).

The mean annual evaporation with the 24-year and 499-year data are:

	Mean annual evaporation (inches)	
	24-year historical data	*499-year synthetic data*
Shelbyville	34.52	34.50
Carlyle	38.35	38.40

The generated monthly evaporation at Shelbyville and Carlyle was adjusted by multiplying with 34.52/34.50 and 38.35/38.40, respectively. Thus, the annual mean of the generated monthly values matches that from the historical record. It may be of interest to know that only 33 and 41

Table 32. Statistics for Generating Monthly Evaporation from Lakes

Month	*a*	*b*	*SE*	*Min (inches)*	*Max (inches)*	*a*	*b*	*SE*	*Min (inches)*	*Max (inches)*
	Lake Shelbyville					*Carlyle Lake*				
Oct	2.744	-0.283	0.532	0.6	4.1	3.658	-0.498	0.443	1.0	4.1
Nov	1.400	-0.156	0.316	0.1	2.1	2.063	-0.259	0.396	0.1	2.7
Dec	0.591		0.297	0.0	1.5	0.851		0.429	0.0	2.1
Jan	0.538		0.252	0.0	1.4	0.788		0.369	0.0	1.9
Feb	1.264	-0.153	0.288	0.0	1.8	1.543	-0.140	0.391	0.0	2.5
Mar	1.712		0.412	0.5	3.0	2.075		0.497	0.6	3.6
Apr	4.939	-0.573	0.644	1.2	5.2	5.278	-0.484	0.504	2.0	5.2
May	7.609	-1.073	0.779	1.9	7.1	6.500	-0.517	0.506	3.2	6.4
Jun	9.765	-1.504	0.728	2.6	8.1	7.813	-0.614	0.295	4.6	7.0
Jul	6.946	-0.476	0.335	4.4	7.0	6.725	-0.214	0.181	5.4	7.0
Aug	5.501	-0.158	0.518	3.3	6.7	5.467	-0.029	0.228	4.6	6.4
Sep	4.138	-0.404	0.561	1.8	5.4	4.577	-0.332	0.283	2.9	4.8

Table 33. Statistics of Historical and Synthetic Annual Net Precipitation

	Data (years)	*Shelbyville*	*Carlyle*
Mean, *inches*	24	3.80	0.96
	499	3.80	0.96
Standard deviation, *inches*	24	9.42	9.98
	499	8.74	11.73
Maximum, *inches*	24	20.42	19.28
	499	28.79	32.09
Minimum, *inches*	24	-12.63	-15.07
	499	-18.50	-31.52

generated monthly evaporation values were lower than the allowable minimum evaporation, and 18 and 12 were higher than the allowable maximum evaporation at Shelbyville and Carlyle, respectively. These were set equal to the minimum and maximum applicable values.

Net Precipitation over the Lakes. A computer program was developed to generate and adjust monthly precipitation and evaporation, as well as net precipitation or precipitation minus evaporation (positive if P exceeds E, zero if P equals E, and negative if P is less than E), for the Shelbyville and Carlyle lakes for each of the 12 months in the 499 years. The statistics of historical and synthetic annual net precipitation are shown in table 33.

The weekly values of synthetic precipitation and evaporation at Shelbyville and Carlyle were derived in the same manner as the weekly values of historical precipitation and evaporation from the respective monthly values.

Synthetic Navigation Flow Requirements

The navigation lock at mile 0.8 of the Kaskaskia River has a normal pool elevation of 368 ft above msl during low flow conditions. The weekly stages of the Mississippi River below the lock were determined for each of 52 weeks of the 24-year historical record as described under the heading "Data for the Years 1942-1965." The difference in level, 368 minus the Mississippi River stage, was designated as h. Flow is required to fill the lock if h is positive, i.e., when the normal navigation pool of 368 ft is above the water level in Mississippi River. For 10 synchronous and 6 other lockages, the flow requirement is 11.2 x h cfs, allowing a 20 percent increase to account for leakage and evaporation loss. The week at the lock starts 4 days later than at Shelbyville, 2 days later than at Carlyle, and 1 day later than at New Athens, to allow for proper travel times. In order to compute synthetic flow requirements, it is necessary to generate weekly values of h for the 499 years of synthetic record.

New Athens Weekly Flows and h. The synthetic weekly flows have already been generated for 499 years at Carlyle and New Athens. Weekly navigation flow requirements translated to weekly flow release (without additional flow for water supply), Q_{cn} , at Carlyle is:

$$Q_{cn} = 11.2\,h - (Q_{na} - Q_{ch}) \quad \text{but} \nless \text{minimum flow release} \tag{27}$$

in which Q_{na} denotes the weekly flow recorded at the New Athens gage, and Q_{ch} is the weekly flow at the Carlyle gage.

The historical weekly values of h were correlated with the corresponding weekly flows at New Athens for each of the 52 weeks. If Q_{na} equals 1000 cfs, the expected value of Q_{ch} is 1000 x 1960/3654, or 536 cfs; 1960 and 3654 cfs are average discharges at Carlyle and New Athens for the flow record up to 1965, taken from *Water Resources Data for Illinois, 1965,* published by the U.S. Geological Survey. Then, Q_{na} - Q_{ch} equals 464 cfs, which exceeds the maximum navigation flow requirement of 314 cfs when h equals 28 ft. Thus, data pairs in which Q_{na} exceeded 1000 cfs were removed from the weekly sets before attempting regressions.

Regression analyses were performed with various linear models. The final model, adopted for its simplicity and ability to provide meaningful results, is expressed by

$$h_{i,j} = a_i + b_i\,(\log Q_{na})_{i,j} + SE_i\,t_{i,j} \tag{28}$$

in which subscripts i and j refer to number of week and year; a is the intercept and b is the coefficient; and the error term equals the standard error of estimate multiplied by the random normal standard deviate because the error terms were found to be nearly normally distributed. Data pairs for some weeks were grouped together when the relation did not vary appreciably from week to week and the individual weekly sample size was rather small because of dropping data pairs with Q_{na} exceeding 1000 cfs. It was found that for the first three weeks the scatter diagrams with h and log Q showed two best fitting lines, one fit best to the higher 75 percent of the points, and the other fit best to the lower 25 percent of the points. Therefore, if t.. was equal to or less than -0.674, i.e., 25 percent probability or less, h was calculated from

$$h_{i,j} = 24.090 - 5.492\,(\log Q_{na})_{i,j} + 2.726\,t_{i,j} \qquad i = 1, 2, 3, \text{ and } t_{i,j} \leqslant -0.674 \tag{29}$$

The values of a, b, SE, mean h, and maximum h that could be expected to occur in 499 years as indicated by the distribution of historical h, are given in table 34. When b is zero, a and SE equal mean h and the standard deviation of h. The maximum value was used as a filter for modifying any higher h generated. Any negative values of h generated were set equal to zero and h equalled zero when Q_{na} exceeded 1000 cfs.

Comparison of Historical and Synthetic h. With the 23 years of Mississippi River gage data covering years 1943-1965, there were 659 weeks out of 1196 when h exceeded zero, or h exceeded zero on the average of 55.1 percent of the weeks in a year. The synthetic weekly data cover 499 years or 25,948 weeks. There were 29 weeks per year on the average, or 55.8 percent of the weeks in a year, when synthetic h exceeded zero.

Table 34. Relevant Data for Generating h

Week(s)	*a*	*b*	*SE*	*Mean b*	*Max b*
1-3	28.893	-4.393	2.349	19.074	26
4	32.064	-5.791	2.370	18.578	26
5	29.456	-4.630	2.188	18.668	26
6	25.579	-2.742	2.328	19.353	26
7	33.044	-6.572	4.036	17.978	27
8	19.362		1.535	19.362	27
9	20.619		1.566	20.619	27
10	20.189		3.014	20.189	27
11	20.985		2.276	20.985	27
12-13	21.889		1.907	21.889	28
14-16	21.934		2.708	21.934	28
17-19	21.711		2.844	21.711	28
20-24	20.054		3.613	20.054	28
25-30	24.047	-4.3984	2.087	12.532	23
31-35	-0.703	4.067	2.505	10.353	20
36-42	12.154		3.948	12.154	23
43	13.615		3.516	13.615	24
44	14.812		3.212	14.812	24
45	33.237	-7.255	3.346	14.259	25
46	15.535		4.203	15.535	25
47	16.545		2.725	16.545	24
48	16.973		2.582	16.973	24
49	17.164		2.489	17.164	24
50	32.673	-6.521	3.151	17.112	24
51	18.310		2.067	18.310	24
52	18.297		2.435	18.297	25

OPTIMAL OPERATING RULES WITH SYNTHETIC DATA

The optimal operation rules were derived with the historical data for 18 sets of varying navigation flow and water supply requirements. The weekly flows in the 24-year record were not high enough to raise lake levels beyond 618-620 ft in Shelbyville and 452.5-454.5 ft in Carlyle with the optimal rules derived from the 18 sets. Desirable values of SF and CF (or increase in release in cfs per foot of increase in lake level between the range ESF and 626.5 ft at Shelbyville and ECF and 462.5 ft at Carlyle) could not, therefore, be determined. An attempt is made here to find answers to the following questions by extending the simulation analysis to the ten 49-year synthetic sequences of weekly flows at Shelbyville, Carlyle, and New Athens; precipitation on and evaporation from the two lakes surfaces; and lockage requirements:

1) How often are flood pool levels exceeded and what are the resulting damage distributions, when optimal rules derived with the historical data are used for the synthetic sequences, assuming SF and CF equal to zero?
2) What possible changes in the operational policy during the "dump" period, that is, weeks 7 through 30, can increase the overall benefits, and what are the pros and cons of such changes?
3) What are the optimal values of SF and CF with the synthetic sequences, their variability within the sequences, and their effect on the distribution of damages?

More than 5000 simulation runs were made with the synthetic sequences. The results of these system simulations are presented here.

Table 35. Distribution of Damages with WS = 80 cfs and Full Navigation
(Optimal rules as derived with the historical data, and SF and CF equal to zero)

(Damages in thousands of dollars)

Percent of years	*Historical data*	*Synthetic data, 10 sets of 49 years each, for set number*									
		1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
4	1.1	1.5	0.0	4.1	2.0	1.4	0.1	1.1	0.0	0.0	0.0
8	3.7	3.2	1.0	4.9	4.3	4.0	1.0	3.6	1.4	1.5	1.2
12	4.9	3.5	2.6	5.7	4.7	4.7	1.5	4.8	4.4	2.6	3.5
16	5.5	3.6	5.0	5.9	4.9	4.8	2.9	5.2	4.7	4.4	4.3
20	5.9	4.1	5.7	6.1	5.2	5.0	3.9	5.4	4.7	4.8	5.1
24	6.3	5.0	6.1	6.3	5.6	5.3	4.4	6.4	5.0	5.3	5.8
28	7.0	5.1	6.2	6.5	5.8	5.6	4.8	7.0	5.5	5.9	5.9
32	7.0	5.4	6.4	6.8	6.6	5.9	5.0	10.4	5.7	6.1	6.3
36	7.0	5.5	6.9	7.0	6.7	6.0	5.3	13.7	5.9	6.3	6.6
40	7.0	5.6	7.0	7.0	7.0	6.2	5.4	19.8	6.3	6.5	6.9
44	7.2	5.8	7.1	9.4	7.8	6.4	5.7	32.4	6.5	6.5	7.0
48	7.6	6.7	10.4	12.7	10.6	6.6	6.3	42.8	7.0	6.8	7.0
52	8.3	7.0	13.5	24.4	13.0	6.9	6.5	105.6	8.9	7.0	15.3
56	22.7	7.2	24.1	25.1	19.3	7.0	7.2	140.0	18.7	7.6	18.9
60	23.9	12.5	34.2	30.5	47.5	7.0	8.6	196.4	54.0	20.8	34.8
64	32.2	38.0	40.7	120.8	97.6	15.7	12.7	225.1	84.3	23.7	118.7
68	47.2	44.8	123.1	180.1	123.8	23.0	13.2	285.4	123.3	27.3	181.1
72	55.0	195.0	163.9	254.2	148.4	44.4	32.5	469.1	135.7	29.6	241.3
76	74.8	224.0	186.1	340.5	165.2	52.0	36.2	595.1	154.2	58.6	306.0
80	144.8	250.7	458.7	572.2	304.3	115.8	66.7	731.4	198.1	63.9	406.8
84	157.8	261.8	709.8	632.3	759.9	197.5	138.9		267.1	116.3	1012.4
88	174.1	381.8	1040.2	864.8	827.0	463.3	210.0		632.1	121.0	
92	381.2		1482.2		1524.9	1042.9	462.3			217.9	
96	971.9				1827.1	1191.2	1790.4			323.8	
n	0	4	3	4	1	0	0	10	5	0	7

Note: n is the number of years when flood pool levels were exceeded

Comparison of Operating Rules with Historic and Synthetic Data

System simulations were carried out with the optimal operation rules derived from the historical data for full navigation and three levels of water supply withdrawals: 0, 80, and 160 cfs. Values of SF and CF were taken as zero and no flood regulation plan was evoked even when the lake levels exceeded the flood pools of 626.5 and 462.5 ft at Shelbyville and Carlyle, respectively. The distribution of resulting annual damages to recreation and agriculture, when the water supply withdrawal is 80 cfs, are given in table 35 for the historical record and 10 synthetic sequences. The number of years the lake levels exceeded the flood pool levels in one or both lakes are included in the table. Some interesting points worth noting are:

1) Synthetic sequence 9 yields a total damage of $2,248 million over 49 years compared with the $2,165 million with the historical record over 24 years. Maximum pool levels reached are 620.03 and 453.01 at Shelbyville and Carlyle compared with 618.31 and 452.35 with the historical data. This is the only sequence giving a smaller value of expected annual damage than the historical.

2) The high flows in synthetic sequences 1, 2, 3, 4, 7, 8, and 10 are much higher than in the historical sequence, and often these high flows occur in May, June, and July, causing heavy damages to recreation and agriculture. The total number of years when lake levels exceeded

Table 36. Distribution of Lake Levels with WS = 80 cfs and Full Navigation
(Optimal rules as derived with the historical data, and SF and CF equal to zero)

Percent of years	*Lake*	*Level*	*Historical data*	*Synthetic sequences of 49 years each*									
				1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
20	S	Min	588.6	589.0	588.8	588.8	589.3	589.0	588.7	589.0	588.9	588.6	589.3
		Max	596.0	596.0	596.0	596.0	596.0	596.0	596.0	596.0	596.0	596.0	596.0
	C	Min	439.4	439.3	439.3	439.4	439.4	439.3	439.3	439.4	439.3	439.3	439.3
		Max	444.0	444.0	444.0	444.0	444.0	444.0	444.0	444.0	444.0	444.0	444.0
40	S	Min	589.7	589.8	589.1	589.6	590.7	589.6	589.5	590.3	589.8	589.1	590.0
		Max	597.6	596.0	596.0	598.4	597.8	596.0	596.0	598.6	596.0	596.1	597.1
	C	Min	439.4	439.4	439.4	439.4	439.4	439.4	439.4	439.4	439.4	439.4	439.3
		Max	444.0	444.0	444.0	444.5	444.6	444.1	444.0	445.7	444.1	444.0	444.3
60	S	Min	591.9	592.1	590.7	590.5	592.4	590.6	590.4	592.5	591.2	589.7	592.5
		Max	601.4	598.6	597.9	601.3	603.5	597.7	598.4	606.5	601.3	597.6	602.1
	C	Min	439.4	439.4	439.4	439.4	439.4	439.4	439.4	439.4	439.4	439.4	439.4
		Max	447.4	446.0	446.9	446.0	448.6	445.9	445.3	449.6	446.8	444.5	446.4
80	S	Min	592.5	592.5	592.5	592.5	592.5	592.4	592.3	592.5	592.5	591.7	592.5
		Max	606.9	610.0	605.1	615.7	610.9	607.0	605.3	623.1	614.1	601.0	611.9
	C	Min	439.4	439.4	440.7	439.4	439.4	441.3	439.4	441.4	439.4	439.4	439.4
		Max	449.5	451.4	450.7	455.6	451.9	448.1	448.2	462.2	450.7	447.4	452.4

ECL = 442.0, ECLL = 439.4, ECH = 444.0, ESL = 592.5, and ESH = 596.0 ft.

flood pools in one or both lakes is 34, giving a frequency of once in 15 years. If sequences 7 and 10 (unusually high flow sequences) are neglected, this frequency becomes 1 in 24 years. Out of 34 years, the maximum level in Shelbyville exceeded the flood pool in 29 years and that in Carlyle in 15 years (8 out of the 15 years are in sequence 7).

3) For about 50 percent of the years, the distributions of annual damage with the historical and 10 synthetic sequences are quite similar, and the annual damage varies from zero to about $10,000 or less.

4) Rather high flows in a few years in the 7 out of 10 synthetic sequences can actually occur in a long record, though their exact frequency of occurrence cannot be spelled out because of various assumptions in generating synthetic data.

The minimum and maximum lake levels in Shelbyville and Carlyle occurring at 20, 40, 60, and 80 percent of the years with the historical and the 10 synthetic sequences are given in table 36, together with the optimum rule levels. A comparison of minimum and maximum levels with the historical and synthetic sequences shows that:

- At 20 percent probability, the historical minimum of 588.6 ft at Shelbyville lies in the range of 588.6 to 589.3 ft for the 10 synthetic data sets. The maximum is at ESH or 596.0 ft for both historical and synthetic sequences. Minima at Carlyle are practically at ECLL or 439.4 ft, and maxima are at ECH, or 444.0 ft, for both historical and synthetic flows.
- At 40 percent probability, synthetic minima at Shelbyville lie in the range 589.1 to 590.7 ft with the historical at 589.7 ft. Synthetic maxima vary from 596.0 to 598.6 ft with the historical at 597.6 ft. Carlyle minima are practically at ECLL, or 439.4 ft. The synthetic maxima vary from 444.0 to 445.7 ft with the historical at 444.0 ft.
- At 60 percent probability, synthetic minima at Shelbyville lie in the range 589.7 to 592.5 ft with the historical at 591.9 ft. Synthetic maxima range from 597.6 to 603.5 ft with the historical at 601.4 ft. Carlyle minima are all at 439.4 ft. Synthetic maxima range from 444.5 to 449.6 with the historical at 447.4 ft.

Table 37. Effect of Increase in Flow Releases, Weeks 7 through 30

					Percent of saving under the condition of					
					Above flood-pool			*Below flood-pool*		
Synthetic sequence	*DDRS*	*DDRC*	*Saving (million $)*	*(%)*	*Total*	*Rec*	*Agr*	*Total*	*Rec*	*Agr*
1	200	5000	2.362	16.0	97.5	66.2	31.3	2.5	3.0	-0.5
2	400	7000	3.622	25.2	64.1	41.8	22.3	35.9	24.4	11.5
3	300	3000	1.164	5.6				100	27.2	72.8
4	300	2500	2.277	18.1				100	58.4	41.6
5	300	500	0.101	1.3				100	84.6	15.4
6	250	1000	0.131	1.9				100	119.7	-19.7
7	250	3500	4.041	11.1	72.2	51.1	21.1	27.8	16.0	11.8
8	200	3000	0.479	3.8				100	77.9	22.1
9	300		0.131	5.8				100	99.2	0.8
10	300	3000	3.437	21.3	48.8	26.9	21.9	51.2	40.1	11.1
Average					28.3	18.6	9.7	71.7	55.0	16.7

- At 80 percent probability, synthetic minima at Shelbyville vary from 591.7 to 592.5 ft with the historical at 592.5 ft. Synthetic maxima vary from 601.0 to 623.1 with the historical at 606.9 ft. Carlyle minima are at 439.4 ft with the exception of synthetic sequences 2,5, and 7. Synthetic maxima range from 447.4 to 462.2 ft, with the historical at 449.5 ft.

The location of historical minimum and maximum levels in the range defined by the 10 synthetic sequences indicates that in the range of probabilities tested, the historical and synthetic results are very much in agreement.

Possible Changes in Operation for the 'Dump' Period

Optimal Operation in the 'Dump' Period, Weeks 7 through 30. Damages to wheat in the lake areas as well as to recreation in the weeks 7 through 30 (mid-November through April) can be significantly reduced if the lake levels are kept below the damage levels as far as possible. Damages in May and June can be reduced considerably in some years if the beginning levels at week 31 in Shelbyville and Carlyle are not high. Low levels in the weeks 7 through 30 can be achieved by allowing higher releases than the presently allowed maximum of 4500 and 10,000 cfs. System simulations made with the 10 sets of synthetic data for water supply withdrawals of 0, 80, and 160 cfs and full navigation flow requirement, indicated that the lake levels above which higher release may be allowed to bring about maximum reduction in damages are 600, 601, 601, 600, 601, 603, 602, 607, 602, and 602 ft at Shelbyville and 443, 444, 446, 446, 447, 447, 443, 445, 450, and 443 ft at Carlyle. Neglecting one much higher level, the average levels are 601 and 445 ft; the adopted levels for optimizing the desirable flow increase per foot of increase in level are 600 and 445 ft at Shelbyville and Carlyle, respectively.

DDRS and DDRC are the increases in flow release, in cfs, per foot of increase in level beyond 600 ft in Shelbyville and 445 ft in Carlyle, respectively. System simulations were made with the 10 synthetic sequences for three water supply withdrawals of 0, 80, and 160 cfs to find the optimal values of DDRS and DDRC, using the optimal operation rules derived with the historical data. The optimal values, percent reduction in damages over 49 years, and percent distribution of the reduction in damages shared between above-flood pool and below flood-pool conditions, as well as between recreation and agriculture under each of the two conditions, are given in table 37.

Provision of releases higher than 4500 and 10,000 cfs below Shelbyville and Carlyle during the 'dump' period, weeks 7 through 30, has the following ramifications:

1) In some years it can increase the releases up to 12,000 cfs at Shelbyville and 30,000 cfs at Carlyle.
2) Expected annual damage (or benefit foregone) will decrease by an average of 11 percent. The increased releases cause an increase in property (fences, farmsteads, roads, etc.) damages which are rather minor. The benefit to agriculture (allowing for increase in property damages) is expected to be about 26 percent of the overall benefit or reduction in damage. Thus, the expected decrease in agricultural damage with DDRS and DDRC is about 2.9 percent of the total annual expected damage without them.
3) The expected decrease in recreation damage with DDRS and DDRC is about 11 minus 2.9, or 8.1 percent of the total annual expected damage without them.
4) Out of a total of 34 years in the 10 synthetic sequences of 49 years each, the lake levels were brought down much lower than the flood pools in 9 years. In the remaining 25 years, the flood pools were exceeded mostly because of high flows in the corn and soybean growth period when releases were held to a maximum of 1800 and 4000 cfs at Shelbyville and Carlyle.
5) The increase in flow releases will flood the bottomlands some years. The sediment laid over the land may be beneficial to crops or it may be detrimental to them. This needs to be checked from field surveys.

System Results with Historical Record, DDRS, and DDRC. The system operation was optimized to evaluate DDRS and DDRC with the use of historical data and the optimal operation rules derived earlier. The optimal values of these two parameters and the resulting damages are given below for water supply withdrawals of 0, 80, and 160 cfs and full navigation.

			Damages in thousand dollars		
WS, cfs	*DDRS*	*DDRC*	*Rec*	*Agr*	*Total*
0	0	0	721.1	941.8	1668.9
0	250	2000	713.1	932.7	1645.8
80	0	0	1141.3	1023.6	2164.9
80	200	2500	1124.3	995.8	2120.1
160	0	0	1698.4	1329.7	3028.1
160	150	2000	1678.9	1279.2	2958.1

The damage decrease is 1.38, 2.07, and 2.31 percent for WS equal to 0, 80, and 160 cfs with higher flow releases allowed below Shelbyville and Carlyle when the lake levels exceed 600 and 445 ft during the 'dump' period, weeks 7 through 30. The results are similar to those with synthetic sequences 3, 5, 6, 8, and 9. The maximum flow release below Shelbyville is about 7000 and below Carlyle about 18,000 cfs.

Optimizations with the historical and synthetic sequences indicate a suitable value of 250 cfs/ft for DDRS and 2500 cfs/ft for DDRC.

Possible Changes in Operations for Growth-Harvest Period

The optimal operating rules derived with the historical data indicated that during the period of weeks 37 through 6 (the growth and harvesting season for corn and soybeans) the flow releases

Table 38. Optimal Values of SF and CF, and Resulting Reduction in Damages

(Damages in thousands of dollars)

Synthetic sequence	SF (cfs/ft)	CF (cfs/ft)	Reduction in damages				Percent reduction	
			Rec	Agr	Total	Percent	Rec	Agr
1	0	7,700	1592.0	-1253.6	338.4	2.7	470.4	-370.4
2	0	0						
3	0	14,000	3576.0	319.0	3895.0	20.4	91.8	8.2
4	0	0						
5	0	0						
6	0	5,000	672.8	-139.8	533.0	7.8	126.2	-26.2
7	0	10,000	4429.7	-509.3	3920.4	12.1	113.0	-13.0
8	0	0						
9	0	0						
10	0	0						

from Shelbyville and Carlyle are not to exceed the nondamaging flows of 1800 and 4000 cfs unless the lake levels exceed ESF and ECF, 615-618 and 452.5-453.5 ft, for the 18 combinations of water supply and navigation flow requirements. The optimal values of SF and CF could not be derived from the historical data because the optimum ESF and ECF levels were not exceeded, or in other words SF and CF were zero. There are quite a number of years with high flows in the synthetic data. System simulations were carried out to determine optimal values of SF and CF to minimize overall damages with the 10 synthetic sequences of 49 years each. Values of DDRS and DDRC were used as given in table 37. The optimization results are shown in table 38.

For 6 of the 10 synthetic sequences, the optimal values of SF and CF are zero. Even a CF value of 1000 to 3000 cfs/ft increased the overall damages by 10 to 25 percent with the exception of sequence 9. The optimal value of SF is zero for the remaining 4 sequences, but CF varies from 5000 to 14,000 cfs/ft. A significant reduction in damages occurred in sequences 3 and 7, but the flow releases went up to 81,000 and 64,000 cfs below Carlyle. Heavy releases below Carlyle in the corn and soybean crop growth and harvesting period may extend the damages beyond Crooked Creek. In 8 of the 10 sequences, agricultural damage reduction in the Carlyle Lake area is nullified by the much higher damage occurring in the damage reach below Carlyle. The overall benefit to recreation is rather small. An analysis of the system simulation results with synthetic sequences 1, 3,6, and 7 showed an increase in damages in some years and a decrease in others.

It is concluded from the above analyses that SF is zero and CF should be taken as zero, not only because significant benefits are indicated in only 3 of the 10 sequences and considerable damages occur in 5 other sequences with CF at nonzero, but also because very high flow releases occur increasing agricultural damages greatly downstream of Carlyle and even beyond the confluence with Crooked Creek. Out of the 25 years, in 10 x 49 or 490 years of synthetic data, with the maximum pool level exceeding the flood pool elevation in one or both lakes, 19 years still remained with the maximum Shelbyville level exceeding 626.5 ft and the maximum Carlyle level below 462.5 ft, with values of CF shown in table 38. Flood control regulation when lake levels exceed flood pools may be needed once in 20 years on the average with CF equal to zero.

Meeting Navigation Flow Requirements

The system optimization with the historical data and 18 combinations of water supply and navigation flow requirements indicated that 25, 20, and 10 percent of the navigation requirement may be met from storage in Lake Shelbyvile for only three combinations of full navigation and water supply of 40, 80, and 120 cfs, respectively. System simulations with the 10 synthetic sequences for 80 cfs water supply and full navigation, with 20 percent navigation requirement met from storage in Shelbyville, and with no navigation requirement met from storage in Shelbyville, indicate that all navigation flow can be met from Carlyle Lake.

The joint lake operation has a built-in provision to release water from Shelbyville (if lake level is above 590 ft) to Carlyle so that the lake level therein does not go too low. Therefore, no navigation withdrawals are assigned to Lake Shelbyville in the final optimal rules.

	Synthetic sequences									
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Δ	-28.6	-101.1	-18.4	-4.8	-43.1	-10.8	57.9	-10.0	-12.8	-17.8

Δ = damage in thousand dollars over 49 years with no navigation flow from Shelbyville storage minus that with 20 percent navigation flow from Shelbyville storage.

Average value of A is -$18,950; thus, it is beneficial to apportion no navigation flow to the storage in Lake Shelbyville.

Comparison of Lake Levels and Agricultural Damage

Minimum Lake Levels. The minimum lake levels reached in Shelbyville and Carlyle, considering full navigation and water supply withdrawals of 0, 80, and 160 cfs from the system, are given below for the historical and synthetic sequences. It is evident that minimum levels are higher than the dead storage pool elevations of 573.0 and 429.5 ft at Shelbyville and Carlyle, respectively. Therefore, water supply withdrawals of up to 200 cfs or even more can be met without the lake levels going below the dead storage pool elevations, with optimal rule levels and full navigation demand. The optimal rule levels are lower than the project normal pool levels. Navigation and up to 200 cfs firm water supply can be met from the lakes when the state and federal storages are not considered separate and distinct entities.

Minimum lake level reached, in ft above msl, with

	Historical data		*10 synthetic sequences*	
WS, cfs	*Shelbyville*	*Carlyle*	*Shelbyville*	*Carlyle*
0	588.88	437.80	588.76	436.18
80	583.73	436.50	586.08	435.96
160	582.07	433.32	581.00	434.14

Agricultural Damages with No Dams. The combined agricultural damage for the two river reaches below Shelbyville and Carlyle under natural flow conditions, or with no dams, averages $0.6596 million with the historical data. Under the same conditions, the average annual damage varies from $0.4698 to $0.8667 million with the 10 synthetic sequences; the overall average is $0.6316 million which agrees very well with $0.6596 million obtained with the historical record.

CONCLUSIONS

The availability of water from Lake Shelbyville and Carlyle Lake for municipal, industrial, and other purposes was analyzed under the conditions of the agreement between the state of Illinois and the federal government. Extensive analysis with the historical data of 24 years and synthetic data of 499 years indicates that 1) a total water supply withdrawal of 100 cfs can be sustained from the state storage in the two lakes with the expectation of a deficit of once in 50 years on the average, 2) a total minimum unused federal storage of about 244,000 ac-ft with the historical data and 221,000 ac-ft with the synthetic data are left in the lakes after meeting full navigation flow requirements, 3) a water supply of 200 cfs or even more can be met from the lakes without going below the dead storage pools when the two-lake system is operated optimally, i.e., without a distinct division between the water supply and navigation storages, and 4) the water supply can be used many times by an optimal system of water intake from and return to the Kaskaskia River.

More than 10,000 system simulations were made with the historical and synthetic data to analyze the system response from a broad spectrum of data samples and to arrive at the optimal rules for the operation of Shelbyville and Carlyle lakes for maximizing the benefits to recreation and agriculture under the various levels of navigation flow and water supply requirements. The optimizations with the synthetic data indicate that the optimal rules derived with the historical data are satisfactory, but that the following modifications might be beneficial and worth consideration:

1) All navigation flow requirements can be met from the storage in Carlyle Lake. The optimizations with historical data indicate that 25, 20, and 10 percent of the navigation flow requirement may be met from the storage in Lake Shelbyville when water supply withdrawals are 40, 80, and 120 cfs and navigation is fully developed. The system operation provides interaction of Shelbyville storage with Carlyle storage when Carlyle Lake is low, thus making it unnecessary to apportion navigation withdrawals to Lake Shelbyville.

2) In the 'dump' period, weeks 7 through 30, the flow releases below Shelbyville and Carlyle may be allowed to increase above the presently allowed 4500 and 10,000 cfs by 250 and 2500 cfs per foot of increase in level above 600 and 445 ft, respectively, up to the flood pool levels. This practically avoids flood regulation (when lake levels exceed flood pool elevations) in this period, and reduces beginning lake levels for the critical period of weeks 31 through 36 for corn and soybeans. This decreases the expected value of annual damage (or benefits foregone) by 11 percent, with 8.1 attributed to recreation and 2.9 to agriculture after allowing for an increase in property damage comprised mostly of flood damage to fences, farmsteads, roads, etc. The historical data show a reduction in damage of about 2 percent. Two out of the 10 synthetic sequences also show the reduction to be less than 2 percent (see table 37). The flow releases can increase up to 14,000 cfs below Shelbyville and 30,000 below Carlyle, say, once in every 100 years.

3) Holding the flow releases to a maximum of 1800 cfs below Shelbyville and 4000 cfs below Carlyle during the weeks 37 through 6 (the growth and harvesting period for corn and soybeans) unless the lake levels exceed flood pool elevations, minimizes the overall damage, or maximizes the overall benefit from the two-lake system. Any flow releases exceeding these nondamaging flows increase the damages very significantly. It is better to let the lake level rise

to flood pool, say once in 25 or 30 years, and then actuate the flood regulation plan than to cause severe agricultural damages at an average of once in 10 years if higher releases are allowed when the lake levels exceed ESF and/or ECF as defined by the optimal rules with the historical data, or once in 5 years if higher releases are allowed above the lake levels of 610 ft and 450 ft as envisaged under the Corps' present operations.

4) The synthetic data indicate the possibility of much higher flows than observed in the 24-year historical period at Shelbyville and Carlyle. The synthetic data thus provide a means of optimizing flood regulation if so desired.

The optimal operation of the lakes, the optimal rule levels, and the relevant parameters are described here, together with the system operation chart, to provide the final results in a concise and compact format. Figure 16 highlights the main features.

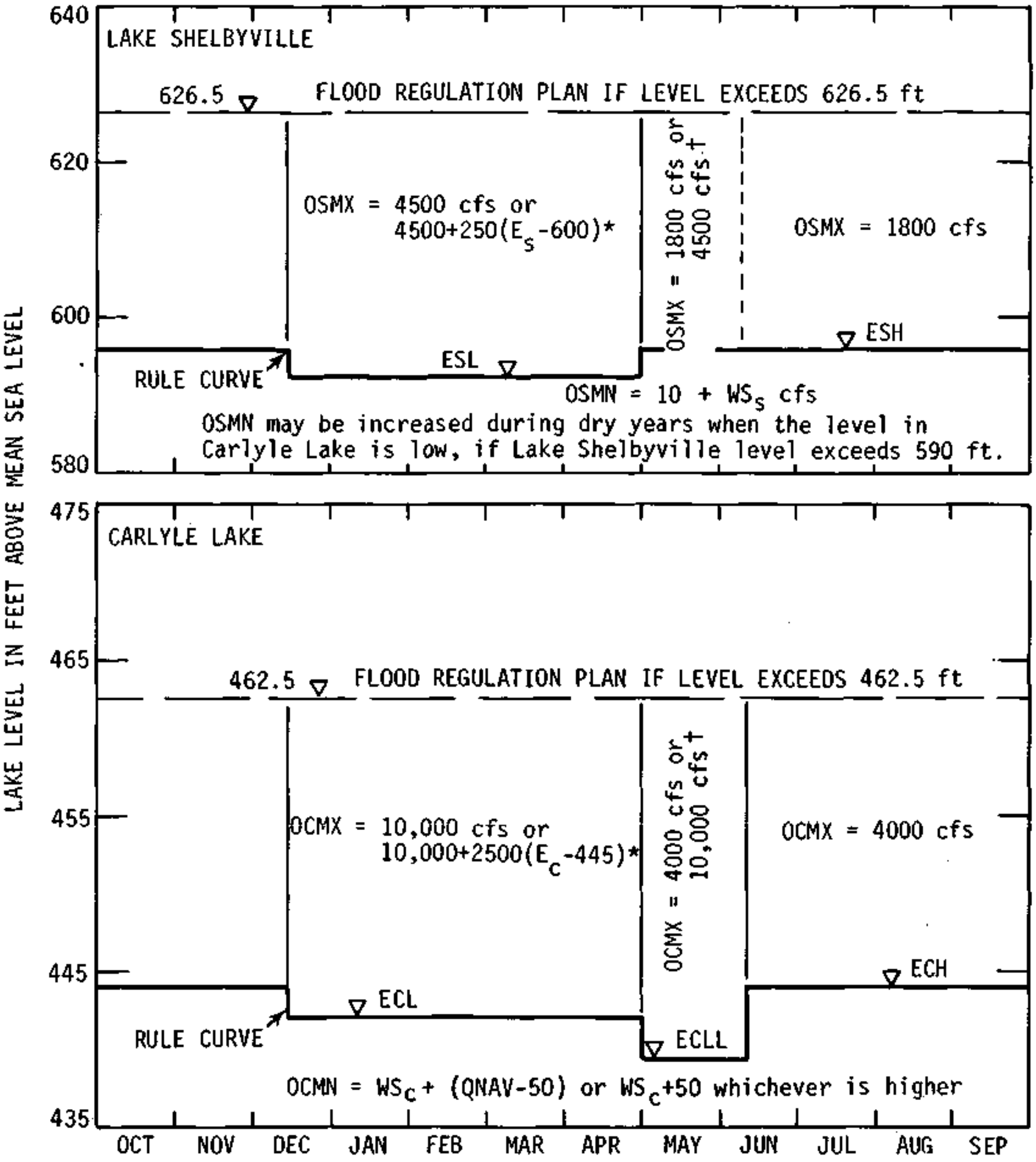
Optimal Operating Rules

The intent of the joint lake operation is to keep the lake levels as close to the rule levels as possible, without violating the seasonal flow release constraints. Various parameters used in explaining the optimal operating rules are given at the top of page 67. Their values are listed in table 39, along with any variations and their effects.

Table 39. Parameters for Optimal Operation of Shelbyville-Carlyle Lake System

WS (cfs)	*N (%)*	*ESL (ft)*	*ESH (ft)*	*ECL (ft)*	*ECLL (ft)*	*ECH (ft)*
0	0	590.0	593.5	440.5	438.7	443.0
	50	590.0	593.5	440.5	438.7	443.0
	100	590.5	594.0	441.0	438.9	443.0
40	0	590.0	593.5	441.5	439.1	443.0
	50	590.5	594.0	441.5	439.1	443.0
	100	591.0	594.5	441.5	439.1	443.0
80	0	592.0	595.0	441.5	439.1	443.0
	50	592.0	595.5	441.5	439.1	443.0
	100	592.5	596.0	442.0	439.4	444.0
120	0	592.5	596.0	442.0	439.4	443.0
	50	593.0	596.5	442.0	439.4	443.0
	100	593.5	597.0	442.5	440.2	444.0
160	0	593.0	596.5	442.0	440.3	443.0
	50	593.5	596.5	442.5	440.7	443.0
	100	594.0	597.0	443.0	441.2	444.0
200	0	594.0	597.0	443.0	441.0	443.0
	50	594.0	597.0	443.0	441.0	443.0
	100	594.0	597.0	443.0	442.2	444.0

Notes:
1) The above parameters for rule levels are shown in figure 16.
2) If higher flow releases are allowed during the 'dump' period; weeks 7 through 30, DDRS = 250 and DDRC = 2500 cfs/ft.
3) An increase of 0.5 ft in ESL will increase damages by about 2%, and a 1.0 ft increase will increase damages by 5%.
4) An increase of 0.5 ft in ESH will increase damages by about 2.5%, and a 1.0 ft increase will increase damages by 5.2%.
5) An increase in ECH to 444 increases damage by about 0.3% for WS = 0, 1.2% for WS = 40, 1.8% for WS = 80, 3.6% for WS = 120 cfs, 8.4% for WS = 160 cfs, and 12.2% for WS = 200 cfs.



Notes: Objective of regulation is to maintain rule curve elevation at all times.

*Increase in maximum flow release in "dump" period as indicated by synthetic data to result in about 11% decrease in average annual damage, but letting the flow releases to go as high as 12 and 30 thousand cfs.

†Conditional increase in maximum flow release for high inflow and high lake level conditions during the critical weeks 31 through 36.

Figure 16. Schematic sketch of optimal joint lake operation

Lake Shelbyville (week 1 starts October 1)
E_s = lake level at the beginning of week, ft
ESH = rule level for weeks 31 through 11, ft
ESL = rule level for weeks 12 through 30, ft
FRS = flow release, in cfs
FRSRC = flow release as per rule level, in cfs
I_s = net inflow, in cfs, during the week
OSMN = minimum allowable flow release, in cfs
OSMX = maximum allowable flow release, in cfs
Carlyle Lake (week 1 starts October 3)
E_c = lake level at the beginning of week, ft
ECH = rule level for weeks 37 through 11, ft
ECL = rule level for weeks 12 through 30, ft
ECLL = rule level for weeks 31 through 36, ft
FRC = flow release, in cfs
FRCRC = flow release as per rule level, in cfs
I_c = net inflow, in cfs, during the week
OCMN = minimum allowable flow release, in cfs
OCMX = maximum allowable flow release, in cfs
T = inflow contributed to Carlyle Lake by drainage area between Shelbyville and Carlyle, in cfs

Lake Shelbyville

Rule Levels.

ESH for weeks 31 through 11 (April 29 through December 17)
ESL for weeks 12 through 30 (December 18 through April 28)
Values of ESH and ESL for 3 levels of navigation and 6 levels of water requirements are given in table 39. (In the previous study, ESL was for weeks 12 through 36 with no water supply and navigation requirements.)

Release Rules. These rules vary over three periods.

1) Weeks 37 through 6 (June 10 through November 11)
OSMX = 1800 cfs if $E_s \leq 626.5$
The minimum flow, OSMN, is modified to maintain Carlyle Lake at 443 ft for hunting activity during dry years if the Lake Shelbyville level is higher than 590 ft.
OSMN = Max (OSMN, Min (1500, Min (Deficit storage in Carlyle, excess storage in Shelbyville above 590)))
Deficit storage in Carlyle
= storage in cfs-weeks (as per rule level + that for releasing minimum flow below Carlyle - that at beginning of the week - that contributed by flow, T, for the area between Shelbyville and Carlyle). [1 cfs-week = 13.88 ac-ft]

Excess storage in Shelbyville

= storage in cfs-weeks (at the beginning of week - that at 590 ft elevation + that from net inflow, I_s)

Provision of a maximum value of 1500 cfs for OSMN is more than ample for meeting the deficits. Thus, modified OSMN is the maximum of 1) old OSMN and 2) 1500 cfs or deficit or excess whichever is smaller.

FRS = Max (OSMN, Min (OSMX,FRSRC)) if $E_c < 447$; or $E_s \leq 597$ and $I_g <$ OSMX

The above statement means that the flow release from Lake Shelbyville is the maximum of OSMN and the smaller of OSMX and FRSRC. FRSRC equals the flow release during the week which will raise or lower the lake level to the rule level during the week, starting from a given lake level at the beginning of the week and the value of inflow during the week.

FRS = Max (OSMN, Min (OSMX,OCMX-T)), but ≤ FRSRC if level and flow conditions for the former FRS are not met.

This allows reduced flow releases from Lake Shelbyville to hold down the rise in level in Carlyle Lake.

2) Weeks 7 through 30 (November 12 through April 28)

OSMX = 4500 cfs if $E_s \leq 626.5$; or $4500+250(E_s - 600)$ if $600 \leq E_s \leq 626.5$ applies if higher flow releases are permitted in the 'dump' period.

The OSMN is modified to meet any deficits in Carlyle Lake during weeks 7 through 11, or up to the end of the hunting season, to maintain the Carlyle Lake at 443 ft, if the Lake Shelbyville level is above 590 ft elevation.

FRS = Max (OSMN, Min (OSMX.FRSRQ) if $E_c < 445$

= Max (OSMN, Min (OSMX,OCMX-T)) if $E_c \geq 445$

3) Weeks 31 through 36 (April 29 to June 9)

OSMX= 1800 cfs

= 4500 cfs if $E_s > 600$ and $I_s + 400(E_s - 600) \geq 4500$

FRS = Max (OSMN, Min (OSMX,FRSRC))

Release rules applicable throughout the year

OSMN = 10 + WS_s

WS_s is the water supply withdrawal below Shelbyville

Flood regulation plan effective if $E_s > 626.5$ ft

Lake Carlyle

Rule Levels.

ECH for weeks 37 through 11 (June 12 through December 19)
ECL for weeks 12 through 30 (December 20 through April 30)
ECLL for weeks 31 through 36 (May 1 through June 11)

Values of ECH, ECL, and ECLL for 3 levels of navigation and 6 levels of water supply requirements are given in table 39.

Release Rules. These rules are also different for the three periods.

1) Weeks 37 through 6 (June 12 through November 13)

OCMX = 4000 cfs if $E_c \leq 462.5$
FRC = Max (OCMN, Min (OCMX.FRCRC)) if $E_c < 447$; or $E_s \leq 597$ and $I_s <$ OSMX
Otherwise,
FRC = Min (OCMX.FRCRC)

2) Weeks 7 through 30 (November 14 through April 30)

OCMX= 10,000 cfs if $E_c \leq 462.5$;or $10{,}000 + 2500(E_c - 450)$ if $450 \leq E_c \leq 462.5$ applies if higher flow releases are permitted in the 'dump' period.
FRC = Max (OCMN, Min (OCMX,FRCRC)) if $E_c < 445$
FRC = Min (OCMX,FRCRC) if $E_c \geq 445$

3) Weeks 31 through 36 (May 1 through June 11)

OCMX = 4000 cfs if $E_c \leq 462.5$
It is modified for high level and high inflow conditions to allow lowering the lake levels for absorbing any later high inflows.
OCMX = 10,000 cfs if I_c + Max(Zero, $1000(E_c - 446)) > 10{,}000$
For weeks 31 through 36:
OCMX = Max (OCMX for the current week, FRC for last week)
This allows for higher maximum flow to insure low lake level when necessary. The DPI damages are not increased but some losses due to reduced yields because of late replanting will occur. Such losses will be considerably less than the loss of crops due to high lake level or high release in case high inflows persist beyond the 39th week.
FRC = Max (OCMN, Min (OCMX,FRCRC))
In order to minimize the chance of flooding in later weeks, the flow release in the 37th week is allowed to equal that in the 36th week, if the 36th week flow release is higher than 4000 cfs and the inflow, I , into the lake exceeds 10,000 cfs. No extra DPI losses are involved, though there may be some reduced income because of late planting or replanting.

Release rules applicable throughout the year

OCMN = WS_c + Max(50,QNAV)
WS is the water supply withdrawal below Carlyle
QNAV is the navigation flow requirement
Flood regulation plan effective if $E_c > 462.5$ ft

Shelbyville-Carlyle Operation System

The historical data file and synthetic sequence files were stored on disk. These files contain weekly values of flows in cfs at Shelbyville, Carlyle, and New Athens; net precipitation (precipitation on minus evaporation from the lake surface), in inches, at Shelbyville and Carlyle; and difference in feet between the navigation pool and Mississippi River stage, zero when negative. All these data are properly lagged for 2 days travel time from Shelbyville to Carlyle, 1 day from Carlyle to New Athens, and 1 day from New Athens to the lock and dam.

In addition, the stored information includes data on: lake surface area and storage versus elevation for the two lakes, area flooded versus flow release below Shelbyville and Carlyle, matrices of DPI and reduction in crop yields because of flooding and replanting, normal crop yields in the four damage reaches and unit monetary values, crop losses because of flooding during growth and harvesting, distribution of the seven recreation activities from week to week over the year at each of the two lakes, potential annual number of visitors for each activity, dollar value of each activity day, adverse high and low lake levels, and impairment of recreation per foot of lake rise or fall above or below the high and low damage levels.

A simplified system flow diagram is shown in figure 17. Explanation of various steps follows.

1) *Specify data file.* Historical sequence of 24 years or synthetic sequences 1 through 10 of 49 years each.
2) *Accept system parameter values.* Parameters are ESL, ESH, ECL, ECLL, ECH, ESF, ECF, NSH, N%, WS_s, WS_c, SF, CF, DDRS, and DDRC. NSH is fraction of navigation flow met from Shelbyville; and WS_s and WS_c are water supply withdrawals, in cfs below Shelbyville and Carlyle.
3) *Initialize lake levels.* Beginning level for Lake Shelbyville is set at ESH and that for Carlyle Lake at ECH.
4) *Set up a counter* for the total number of weeks in the record, i.e., 52 n where n is the number of years.
5) *Set up a counter* for the number of weeks in a year.
6) *Compute ARF1 and ARF2.* ARF1 and ARF2 are the areas flooded by high lake levels.

$$\text{ARF1} = A_s - 15{,}956 \text{ acres, zero if negative}$$
$$\text{ARF2} = A_c - 32{,}397 \text{ acres, zero if negative}$$

Lake Shelbyville surface area A_s, is 15,956 acres at elevation 610 ft, and Carlyle Lake surface area, A_c, is 32,397 acres at elevation 450 ft.

7) *Calculate I_s and T.* I_s is the net inflow into Lake Shelbyville

$$I_s = Q_s[1 - A_s/(640 \times 1030)] + [A_s(P - E)_s/166.6] \text{ cfs}$$

in which Q is the weekly flow at Shelbyville; P - E is the net precipitation, in inches, per week; 1030 is the drainage area in square miles abo've Shelbyville; and subscript s refers to Shelbyville.

$$T = (Q_c - Q_s)[1 - A_c/(640 \times 1650)] + [A_c(P - E)_c/166.6] \text{ cfs}$$

in which subscript c refers to Carlyle, and 1650 is the drainage area between Carlyle and Shelbyville gages.

8) *Find FRS.* FRS is the flow release, in cfs, from Shelbyville obtained as per given rules and navigation and water supply requirements.
9) *Compute ARF3.* ARF3 is the area flooded in the damage reach along the river below Shelbyville. It is zero if FRS $\leq$ 1800 cfs. For higher FRS, it is obtained by interpolation from the area flooded versus flow release information stored on disk.
10) *Calculate I_c.* I_c, or net inflow to Carlyle, equals FRS plus T.
11) *Find FRC.* FRC is the flow release, in cfs, from Carlyle obtained as per given rules and navigation and water supply requirements.
12) *Compute ARF4.* ARF4 is the area flooded in the damage reach along the river below Carlyle. It is zero if FRC $\leq$ 4000 cfs. For higher FRC, it is obtained by interpolation from the area flooded versus flow release information stored on disk.

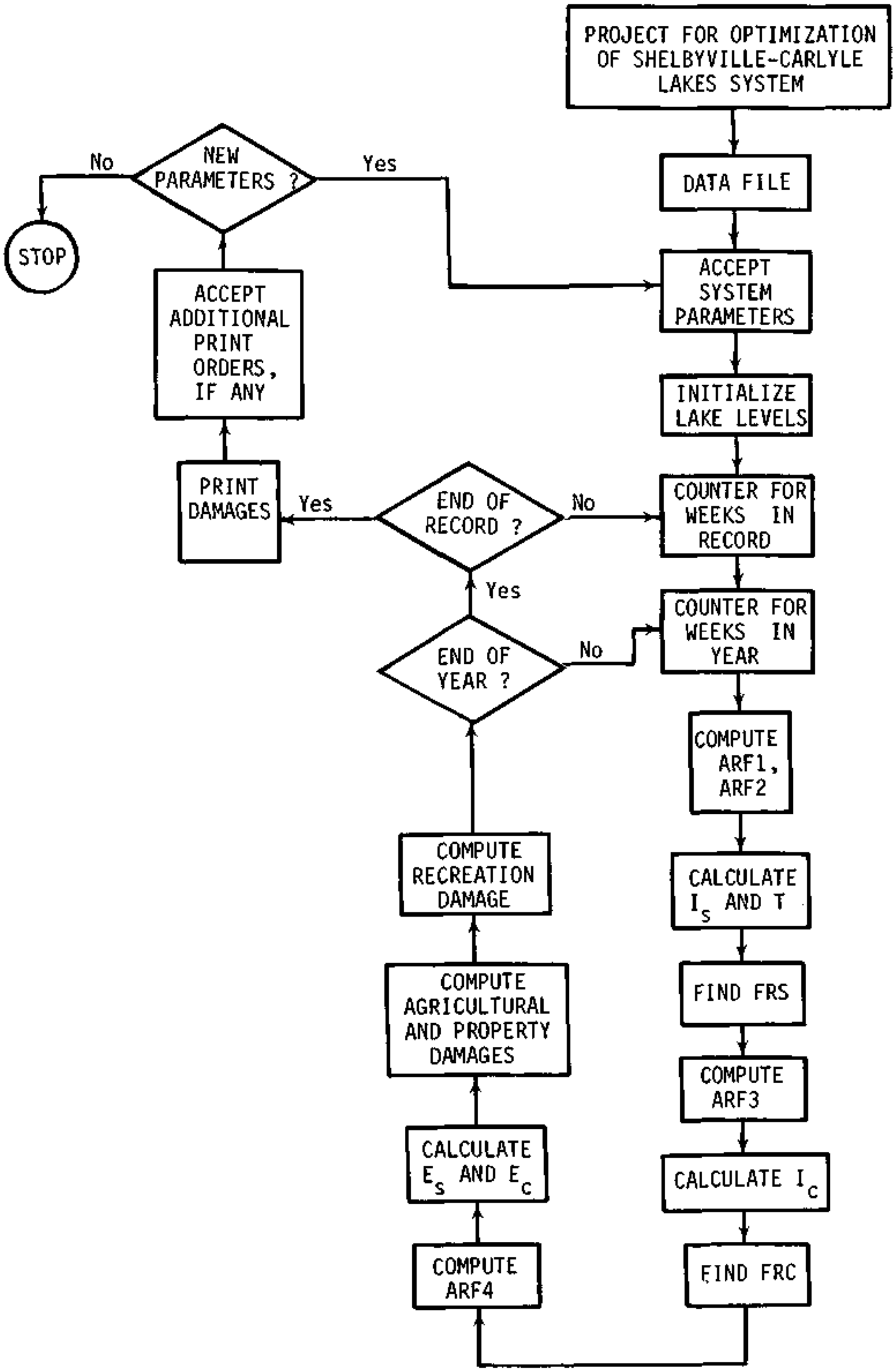


Figure 17. Simplified system flow diagram

13) *Calculate E_s and E_c* . E_s and E_c are lake levels at the end of week, or the lake levels at the beginning of next week, obtained from the storage equation and storage versus elevation information.
14) *Compute agricultural and property damages.* Agricultural damages, if any, are calculated for ARF1, ARF2, ARF3, and ARF4, via an agricultural damage assessment subroutine. Property damages are calculated for maximum flooding during the year.
15) *Compute recreation damages.* These are obtained for adverse lake level conditions via a recreation damage assessment subroutine.
16) *End of year?* If week 52 has been processed, the within-the-year week counter is set to 1.
17) *End of record?* The end is checked before starting printing.
18) *Print damages, NS and NC.* Total, recreation, and agricultural damages, or benefits foregone, for the length of record, and number of times, NS and NC, that Shelbyville and Carlyle go below dead storage, are printed.
19) *Accept additional print orders, if any.* The additional printouts can be of parameter values; annual values of total, recreation, and agricultural damage, minimum and maximum level at Shelbyville and Carlyle, and maximum flow release below the dams; and ranked values of annual damage and minimum and maximum levels at the two lakes, ranked low to high.
20) *New parameters?* A parameter value or values can be changed for a new system simulation.
21) *New data file?* System simulation can be run with another data file.

A sample output is shown in figure 18 for WS = 80 cfs and full navigation requirements. It delineates the optimal system response with the historical sequence.

The system program together with the subroutines was written for use on the DEC 10 time-sharing computer facilities of the University of Illinois. A simulation with the historical record costs about 35¢ and one with the 49-year synthetic sequence about 70¢. The weekly results of system operation can be printed on the main computer printer.

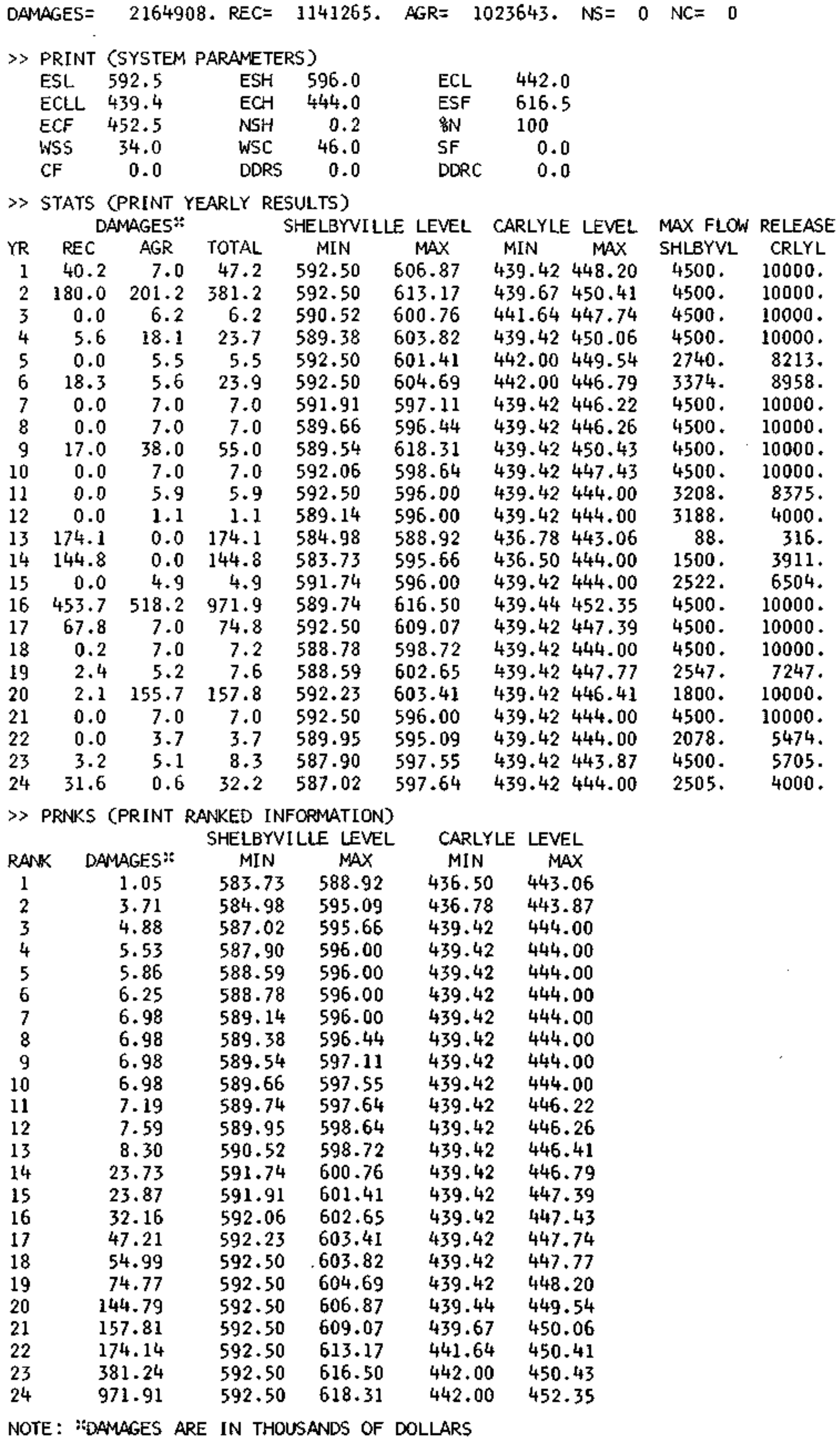
Major Policy Decisions

The optimization of joint lake operation in the overall best interests of agriculture, recreation, water supply, and navigation stresses the following departures in rule levels and release rules from the present operation policy of the Corps of Engineers. The suggested departures are items of major policy decisions.

Rule Levels: An optimal level ESH in Shelbyville for the period of weeks 31 through 11 (April 29 through December 17) is indicated instead of the Corps 599.7 ft for May 1 through November 30. ESH varies from 593.5 to 597.0 ft with increase in navigation and water supply requirements.

An optimal level of ESL in Shelbyville for the period of weeks 12 through 30 (December 18 through April 28) is indicated instead of the Corps 590.0 ft for December 1 through April 30. ESL varies from 590.0 to 594.0 ft with increase in navigation and water supply requirements.

An optimal level ECH in Carlyle for the period of weeks 37 through 11 (June 12 through December 19) is indicated instead of the Corps 445.0 ft for May 1 through November 30. ECH varies from 443.0 to 444.0 ft with increase in navigation and water supply requirements.



```
DAMAGES=    2164908. REC=  1141265.  AGR=  1023643.  NS=  0  NC=  0

>> PRINT (SYSTEM PARAMETERS)
   ESL   592.5        ESH   596.0        ECL    442.0
   ECLL  439.4        ECH   444.0        ESF    616.5
   ECF   452.5        NSH     0.2        %N     100
   WSS    34.0        WSC    46.0        SF       0.0
   CF      0.0        DDRS    0.0        DDRC     0.0
>> STATS (PRINT YEARLY RESULTS)
        DAMAGES*            SHELBYVILLE LEVEL   CARLYLE LEVEL   MAX FLOW RELEASE
YR   REC     AGR    TOTAL     MIN        MAX      MIN      MAX   SHLBYVL   CRLYL
 1   40.2     7.0    47.2   592.50    606.87   439.42 448.20   4500.    10000.
 2  180.0   201.2   381.2   592.50    613.17   439.67 450.41   4500.    10000.
 3    0.0     6.2     6.2   590.52    600.76   441.64 447.74   4500.    10000.
 4    5.6    18.1    23.7   589.38    603.82   439.42 450.06   4500.    10000.
 5    0.0     5.5     5.5   592.50    601.41   442.00 449.54   2740.     8213.
 6   18.3     5.6    23.9   592.50    604.69   442.00 446.79   3374.     8958.
 7    0.0     7.0     7.0   591.91    597.11   439.42 446.22   4500.    10000.
 8    0.0     7.0     7.0   589.66    596.44   439.42 446.26   4500.    10000.
 9   17.0    38.0    55.0   589.54    618.31   439.42 450.43   4500.    10000.
10    0.0     7.0     7.0   592.06    598.64   439.42 447.43   4500.    10000.
11    0.0     5.9     5.9   592.50    596.00   439.42 444.00   3208.     8375.
12    0.0     1.1     1.1   589.14    596.00   439.42 444.00   3188.     4000.
13  174.1     0.0   174.1   584.98    588.92   436.78 443.06     88.      316.
14  144.8     0.0   144.8   583.73    595.66   436.50 444.00   1500.     3911.
15    0.0     4.9     4.9   591.74    596.00   439.42 444.00   2522.     6504.
16  453.7   518.2   971.9   589.74    616.50   439.44 452.35   4500.    10000.
17   67.8     7.0    74.8   592.50    609.07   439.42 447.39   4500.    10000.
18    0.2     7.0     7.2   588.78    598.72   439.42 444.00   4500.    10000.
19    2.4     5.2     7.6   588.59    602.65   439.42 447.77   2547.     7247.
20    2.1   155.7   157.8   592.23    603.41   439.42 446.41   1800.    10000.
21    0.0     7.0     7.0   592.50    596.00   439.42 444.00   4500.    10000.
22    0.0     3.7     3.7   589.95    595.09   439.42 444.00   2078.     5474.
23    3.2     5.1     8.3   587.90    597.55   439.42 443.87   4500.     5705.
24   31.6     0.6    32.2   587.02    597.64   439.42 444.00   2505.     4000.
>> PRNKS (PRINT RANKED INFORMATION)
                      SHELBYVILLE LEVEL      CARLYLE LEVEL
RANK   DAMAGES*        MIN        MAX         MIN        MAX
 1         1.05      583.73     588.92      436.50     443.06
 2         3.71      584.98     595.09      436.78     443.87
 3         4.88      587.02     595.66      439.42     444.00
 4         5.53      587.90     596.00      439.42     444.00
 5         5.86      588.59     596.00      439.42     444.00
 6         6.25      588.78     596.00      439.42     444.00
 7         6.98      589.14     596.00      439.42     444.00
 8         6.98      589.38     596.44      439.42     444.00
 9         6.98      589.54     597.11      439.42     444.00
10         6.98      589.66     597.55      439.42     444.00
11         7.19      589.74     597.64      439.42     446.22
12         7.59      589.95     598.64      439.42     446.26
13         8.30      590.52     598.72      439.42     446.41
14        23.73      591.74     600.76      439.42     446.79
15        23.87      591.91     601.41      439.42     447.39
16        32.16      592.06     602.65      439.42     447.43
17        47.21      592.23     603.41      439.42     447.74
18        54.99      592.50     603.82      439.42     447.77
19        74.77      592.50     604.69      439.42     448.20
20       144.79      592.50     606.87      439.44     449.54
21       157.81      592.50     609.07      439.67     450.06
22       174.14      592.50     613.17      441.64     450.41
23       381.24      592.50     616.50      442.00     450.43
24       971.91      592.50     618.31      442.00     452.35

NOTE: *DAMAGES ARE IN THOUSANDS OF DOLLARS
```

Figure 18. A sample system output

An optimal level of ECL in Carlyle for the period of weeks 12 through 30 (December 20 through April 30) is indicated instead of the Corps 440.0 ft for December 1 through April 30. ECL varies from 440.5 to 443.0 ft with increase in navigation and water supply requirements.

An optimal level of ECLL is indicated in Carlyle for the period of weeks 31 through 36 (May 1 through June 11) instead of the Corps 445.0 ft. ECLL varies from 438.7 to 442.2 ft with increase in navigation and water supply requirements. System simulations with even ECLL = ECL indicate a tremendous increase in damages.

It is evident that optimal rule levels in the 'dump' period are somewhat higher than the present levels of the Corps, but lower in the growth and harvesting period for corn and soybeans. The greatest departure from the present operation occurs during the weeks 31 through 36 at Carlyle.

Release Rules. The optimal operation indicates restricting flow release from Shelbyville to 1800 cfs or less during the weeks 37 through 6 (June 10 through December 17) until the level exceeds the flood pool elevation of 626.5 ft when flood regulation will apply. It allows flow releases higher than 1800 cfs during the weeks 31 through 36 (April 29 through June 9) whenever high inflow and high lake level conditions (usually occurring once in 5 years) exist. This is to accept minor losses in the form of DPI loss and somewhat reduced yields rather than losing the entire crop due to flooding in the growth and harvesting period. The Corps present policy allows a flow release of up to 1800 cfs if the level is below 610.0 ft and increases the release linearly to 4500 cfs at 626.5 ft.

Similarly, in the case of Carlyle Lake, the optimal operation indicates restricting flow release to 4000 cfs or less during the weeks 37 through 6 (June 12 through December 19) until the lake level exceeds the flood pool elevation of 462.5 ft when flood regulation applies. It allows flow releases higher than 4000 cfs during the weeks 31 through 36 (May 1 through June 11) when high inflow and high lake level conditions exist. This is to accept minor losses in the form of DPI loss and somewhat reduced yields rather than losing the crop due to flooding in the growth and harvesting period. The Corps present policy allows a flow release of up to 4000 cfs if the level is below 450.0 ft and increases the release linearly to 10,000 cfs at 462.5 ft. These linear increases in flow releases at Shelbyville and Carlyle cause tremendous damages to agriculture as brought out in the previous report.

The added feature of manipulating releases from Shelbyville during low-inflow and low-lake-level, and high-inflow and high-lake-level conditions, to provide more flexibility in the operation of Carlyle Lake during the critical periods is missing in the Corps operation. This interaction of lake storages maximizes the overall benefits.

Higher Flow Releases during 'Dump' Period. The optimizations with synthetic sequences indicate that increases in allowable flow releases below Shelbyville and Carlyle above the presently allowable maximum of 4500 and 10,000 cfs during the weeks 7 through 30 yields an overall reduction of 11 percent in annual expected damage (8.1 for recreation and 2.9 for agriculture). The damage reduction is brought about mainly by 1) maintaining lower lake levels during weeks 7 through 30, and thereby decreasing recreation damage, damage to wheat in lake areas and practically obviating flood regulation plan, and 2) maintaining less high levels at the beginning of the crucial period, weeks 31 through 36, thereby reducing recreation damage in the lakes, damage to the wheat crop in lake areas, and DPI losses to corn and soybeans in the upstream and downstream damage reaches. The optimal values of DDRS and DDRC are 250 and 2500 cfs/ft; DDRS is the increase in cfs/ft of rise in level above 600 ft in Shelbyville, up to the flood pool of 626.5, and DDRC is the increase in cfs/ft of rise in level above 445 ft in Carlyle, up to the flood pool of 462.5 ft. The system simulation with the historical data and DDRS and DDRC gave a 2 percent reduction in damages.

Allowing high releases during the dump period will cause flooding of bottomlands with no crops in the 'dump' period, but the effect of sediment left over the land, once in 4 or 5 years on the average, on the land productivity needs to be investigated. The increased flow release may affect navigation for a few weeks every 4th or 5th year.

Suggestions for Further Research and Improvement

The flood regulation plans as laid out in the Design Memorandum No. 1 for Shelbyville and Carlyle Reservoirs (Corps of Engineers, 1958, 1962) when levels in these lakes exceed the flood pool elevations of 626.5 and 462.5 ft, respectively, need to be optimized after the major policy decisions have been made. These plans follow the conventional approach. Optimization studies may indicate if any modifications are needed to maximize the benefits. The synthetic data will have to be used for this purpose because the historical record does not contain flows high enough to top the flood pool elevations.

The following statement from the previous report on better data (Singh et al., 1975) still applies. "The need for better data may be exemplified by area flooded versus discharge information at various points along the river downstream of the two dams, particularly near the confluence with the tributaries. Such information coupled with flow information for the tributaries would help in modifying the flow releases below the dams to keep the overall damages to a minimum. The extent of a damage reach below a dam needs defining on the basis of specified criteria and considerations, because the flood control protection decreases with distance downstream from the dam. Information is needed on the advisability of changing the beginning week of the main crop season from week 31 (years starts October 1) to a later week for areas in the zone of maximum allowable flows, 4500 and 10,000 cfs from Shelbyville and Carlyle Lakes. Shifting the beginning week to week 35 may cut down the agricultural loss to one-half of that with week 31. The estimates of visitors in each recreation activity and the upper and lower lake levels at which the activity is adversely affected, need to be carefully analyzed with respect to the capacity of the recreation areas and the design of boating ramps and other recreation facilities."

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