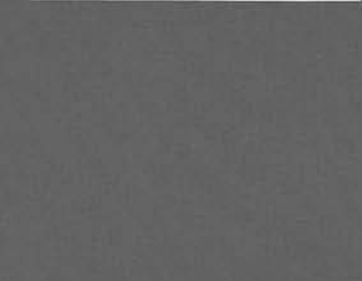
REPORT OF INVESTIGATION 69

STATE OF ILLINOIS

DEPARTMENT OF REGISTRATION AND EDUCATION

Use of Air-Water Relationships for Predicting Water Temperature

by V. KOTHANDARAMAN and R. L. EVANS



ILLINOIS STATE WATER SURVEY

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REPORT OF INVESTIGATION 69

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by V. KOTHANDARAMAN and R. L. EVANS

Tide: Use of Air-Water Relationships for Predicting Water Temperature.

Abstract: Application of harmonic analysis to average daily air and water temperature records for a given location indicates that the first harmonic accounts for a major portion of the total variance in the records. Water temperature residuals are well correlated with air temperature residuals. Parametric values of a mathematical model for predicting water temperatures from air temperature records are stable from year to year. The water-air temperature relationship appears to be a stationary linear process. Consequently, it is possible to predict water temperatures at a specific location from the air temperature records, provided both water and air temperature records are available for another similarly situated water body. Air and Illinois River water temperature data at Peoria and Havana, Illinois, were used for the study.

Reference: Kothandaraman, V., and R. L. Evans. Use of Air-Water Relationships for Predicting Water Temperature. Illinois State Water Survey, Urbana, Report of Investigation 69, 1972.

Indexing Terms: air temperature, Fourier analysis, harmonic analysis, Illinois River, nonseasonal temperature variations, seasonal temperature variations, water quality management, water resources, water temperature.

STATE OF ILLINOIS
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DEPARTMENT OF REGISTRATION AND EDUCATION
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SUMMARY

Fourier analysis applied to the observed average daily ambient air and Illinois River water temperatures at Peoria, Illinois, indicates that the harmonic with a periodicity of 1 year accounts for about 80 percent of the total variance in the average daily dry bulb air temperatures and about 95 percent of the total variance in the water temperatures. The nonseasonal variation in average daily water temperature on any given day could be predicted as a linear combination of the nonseasonal variations in the average daily dry bulb air temperatures for that day and the preceding 2 days. The prediction of nonseasonal variations in water temperatures improved when the residual temperature records for the months of December, January, and February were excluded. This is because water temperatures were close to the freezing point during these months and could not respond to the air temperature variations to the same degree as for other months. A combined model for seasonal and nonseasonal variations predicted the average daily water temperatures for the Illinois River at Peoria with a standard error of estimate of 1.9 and 1.4 F according to whether the residual records for the months of December, January, and February were included. The corresponding figures for the combined water temperature model for the Illinois River at Havana are 1 5 and 1.5 F.

The water-air temperature relationship was found to be essentially a stationary linear process. A mathematical model is presented to predict the annual cyclic trend in the water temperatures from the air temperature records. Values for the parameters of the mathematical model were evaluated by the use of the 1969 water and air temperature records for Peoria. These parametric values, in conjunction with the ambient air temperature records for Havana, were used to predict Illinois River water temperatures at Havana for the years 1968 and 1969 Excellent agreement was found between the predicted and observed water temperatures.

INTRODUCTION

Considerable interest has been shown in the past by limnologists, oceanographers, and meteorologists in predicting surface water temperatures. The interest stems from the need to provide insight regarding evaporation losses, air flow patterns, thermal stratification, fisheries' operations, etc.[1,2] The need for forecasting water temperatures is being increasingly felt also in the water quality management area. Several investigators have reported on the annual temperature variations in lakes, estuaries, and rivers.[3,4,5]

Among the water quality characteristics of surface waters, temperature is probably one of the most significant and widely measured parameters. Most of the physical properties of water are functions of temperature. For example, the ability of water to retain dissolved gases decreases with increase in temperature. The literature on temperature effects in the aquatic environment is voluminous, and several specific and detailed studies have been directed toward the response of fish life to temperature changes. The increase in raw water temperature has both beneficial and deleterious effects on water treatment technology.[3] The ability to predict temperature variations due to climatological changes can aid in the proper planning of water utility operations.

There is an immediate need for delineating the characteristics of water temperature variations due to natural causes in contrast to cases in which water bodies receive waste heat discharges from power plants. Power production in the United States has been reported to double every decade and consequently more and more surface waters are being used for cooling purposes. There has been increasing concern about the effects of thermal discharges on water quality of streams and lakes. The direct and immediate effect of these discharges is to increase the temperature of the receiving waters over and above that due solely to natural causes. Several studies have been and are being directed to delineate the mechanism affecting the decay of the rejected heat in the receiving waters. In these studies, a knowledge of the base temperature, or the naturally occurring temperature, is of fundamental importance if the effects due to added heat are to be isolated and investigated.

Literature Review

In the past, attempts have been made to estimate ocean and lake surface temperatures from such factors as radiation, wind velocity, cloudiness, and air temperature. Roden and Groves,[6] analyzing the past records of sea-surface temperatures, wind speeds, and solar radiation, found practically no relationship between radiation and sea-surface temperatures. However, they found that the mean monthly water temperature could be estimated by using the mean temperature values for the preceding 1 or 2 months. Later Roden[1] performed a time series analysis on sea-surface temperatures, cloudiness, and wind data and obtained similar results. Yu and Brutsaert,[2] analyzing Lake Ontario surface water temperature, air temperature, and sunshine percentage, found that there was practically no relationship between water temperature and sunshine anomalies and that the mean monthly water temperatures could be estimated from the present and the preceding 2 to 4 months of air temperature data. Harmeson and Schnepper[7] and the Texas Water Development Board[8] have indicated graphically that water temperatures in rivers follow closely the pattern of variations in mean dry bulb air temperatures.

Objective and Scope

The primary aim of this study was to investigate the nature of seasonal and nonseasonal variations in the daily mean water temperatures of the Illinois River at a given locale and to develop a method for predicting water temperatures based on observed meteorological data. Such a model for naturally occurring water temperature variations would be invaluable for predicting the responses of water bodies to any additions or alterations imposed by power plant operations.

The material presented in this report includes air and Illinois River water temperatures for Peoria and Havana. Daily records for 4 and 3 years, respectively, are given in appendixes A and B. Notations used throughout this report are given in the back *(see page 11).*

Acknowledgments

The results described in this report have been compiled by the authors as a part of the regular program of water quality research on Illinois water resources conducted by the Illinois State Water Survey.

This study was conducted under the general supervision of Ralph L. Evans, Head of the Water Quality Section, and Dr. William C. Ackermann, Chief, Illinois State Water Survey. Computer programming advice was provided by Robert Sinclair, System Analyst; Katherine Shemas, Clerk-Typist, typed the original manuscript; John W. Brother, Jr., Chief Draftsman, prepared the illustrations; and Mrs. Patricia A. Motherway, Assistant Technical Editor, edited the final report.

The authors extend special thanks to Mr. Marion Barringer, Central Illinois Light Company (Peoria) and Mr. H. W. McFadden of the Illinois Power Company (Havana) for making available Illinois River water temperature data. Professor S. A. Swami of the West Virginia Institute of Technology, Montgomery, provided valuable discussions on the subject by personal communications.

METHODOLOGY

Records of the average daily water temperatures for the Illinois River at Peoria for the years 1966-1969 and at Havana for 1967-1969 were analyzed for seasonal and nonseasonal variations. Similar analyses were performed on the dry bulb air temperatures at these locations, and the possible effects of air temperatures on water temperatures were investigated. Such meteorological parameters as cloud cover, wind speed, solar radiation, etc., were not considered in this investigation since they have not been found by previous investigators to be correlated with water temperatures.[1,2,6] Also, the National Weather Service does not make observations for solar radiation at Peoria and Havana. Therefore, only the average daily air temperatures were considered as the significant meteorological parameter influencing the river water temperatures.

Description and Source of Data

The river water temperature data for Peoria for 4 calendar years were obtained from the Central Illinois Light Company and are presented in appendix A. These records are for inlet cooling water at the company's Wallace Station generating plant located on the Illinois River at milepoint 162.6. These average daily water temperatures were extracted from continuous strip chart records as averages of 24 hourly data points.

The river water temperatures for Havana for 3 calendar years were obtained from the Illinois Power Company and are presented in appendix B. These records are for inlet cooling water at the company's Havana Station generating plant located on the Illinois River at milepoint 118.5. These average daily water temperatures were extracted from records of daily observations at approximately 8-hour intervals for 3 calendar years.

The intakes for both power plants are located on river banks and not on intake canals. Consequently, the inlet condenser water temperatures are representative of the river water temperatures at these locations.

The air temperatures for Peoria were recorded by the National Weather Service office located approximately 5 miles from the Wallace Station. These data were obtained from *Local Climatological Data* published by the Environmental Data Service, U. S. Department of Commerce. These average daily air temperatures are averages of 8 daily observations made at 3-hqur intervals. The air temperature records for Havana were obtained from the Havana Station power plant records as averages of 3 daily observations made at approximately 8-hour intervals.

Theoretical Considerations

A cursory glance at the average daily air and water temperatures plotted for Peoria (figures 1 and 2) and for Havana (figures 3 and 4) reveals the existence of annual cyclic variations. Kothandaraman and Evans,[3] Thomann,[4] and Ward[5] indicated that a harmonic with a periodicity of 1 year fits water temperature data in lakes, estuaries, and rivers, and accounts for about 95 percent of the total variance. In order to separate the cyclic or seasonal variations from the observed data, Fourier analysis procedures were used. Once the seasonal variations were removed from the observed data, it was possible to investigate the nature of nonseasonal or random variations. Some of the salient features of Fourier analysis are given below:

$$y(t) = A\cos(2\pi t/T) + B\sin(2\pi t/T) \quad (1)$$

which is termed an harmonic equation. This can also be written as

$$y(t) = (A^2 + B^2)^{1/2}\cos\left[(2\pi t/T) - \theta\right] \quad (2)$$

in which T is the period of the harmonic, the reciprocal of T is frequency, the quantity $(A^2 + B^2)^{1/2}$ or R is amplitude, and θ is the phase angle. The phase angle is given by

$$\theta = \arctan(B/A) \quad (3)$$

A series of the form

$$f(t) = (A_o/2) + \sum_{n=1}^{\infty}\left[A_n\cos(2n\pi t/T) + B_n\sin(2n\pi t/T)\right] \quad (4)$$

is called a Fourier series in which A_n and B_n are Fourier coefficients. The very first term on the right hand side of the Fourier series stands for the arithmetic average of the function $f(t)$. An important relationship between the coefficients of the Fourier series and the total variance σ^2, in the function $f(t)$ is given by

$$\sigma^2 = (1/2)\sum_{n=1}^{\infty}(A_n^2 + B_n^2) = (1/2)\sum_{n=1}^{\infty}R_n^2 \quad (5)$$

If there are N equally spaced discrete observations in a given period, T, then the Fourier coefficients are given by

$$A_n = (2/N)\sum_{t=1}^{N} f_t\cos(2n\pi t/T) \quad (6)$$

and

$$B_n = (2/N)\sum_{t=1}^{N} f_t\sin(2n\pi t/T) \quad (7)$$

If these observations could be approximated by the mth partial sum of the Fourier series, i.e., if only the first m harmonics are considered, then

$$f(t) \simeq f'(t) = (A_o/2) + \sum_{n=1}^{m}\left[A_n\cos(2n\pi t/N) + B_n\sin(2n\pi t/N)\right] \quad (8)$$

The variance in the observed record accounted for by the m harmonics is given by

$$\sigma_m^2 = (1/2)\sum_{n=1}^{m}(A_n^2 + B_n^2) = (1/2)\sum_{n=1}^{m}R_n^2 \quad (9)$$

For a given set of observations, the values of Fourier coefficients are independent of each other. Also if the linear expression (equation 8) were used to fit the observed data employing the least-square principle, the values of the coefficients in the equation would be identical to those given in equations 6 and 7.

DATA EVALUATION

Fourier Analysis

The characteristics of the water and air temperature records for Peoria and Havana are shown in table 1. The mean temperatures, variances, and standard deviations for both water and air show a high degree of consistency from year to year.

The results of Fourier analysis applied to the water temperature data for Peoria and Havana are shown in table 2. The amplitude of the first harmonic with a periodicity of 365 days is found to be the most significant. The characteristics of harmonics 2-10 show considerable variations. Their importance or lack of it could be judged better by the variances accounted for by these harmonics. It should be noted that the amplitude and phase angle of the first harmonic for each year of record under consideration in each case are within relatively close limits.

The results of Fourier analysis applied to the air temperature data for Peoria and Havana are shown in table 3. The same observations regarding the characteristics of the harmonics for water temperature data apply to air temperature data. The average daily water and air temperature ob-

Table 1. Characteristics of Water and Air Temperature Records for Peoria and Havana

(Temperatures in degrees Fahrenheit)

Year of record	Water Mean	Water Variance*	Water Standard deviation	Air Mean	Air Variance*	Air Standard deviation
Peoria						
1969	55.4	318 4	17.8	49.5	399.0	20.0
1968	55 8	294 6	17.2	50 6	398.5	20.0
1967	54.6	277.0	16 6	49 5	365.6	19.1
1966	55 5	293 8	17.1	50.0	405.0	20.1
Havana						
1969	57.3	316 1	17 8	51.3	408.6	20 2
1968	57 5	304.0	17.4	52.3	407.0	20 2
1967	55.7	288 5	17 0	51.3	350.2	18 7

** Variance in degrees Fahrenheit squared*

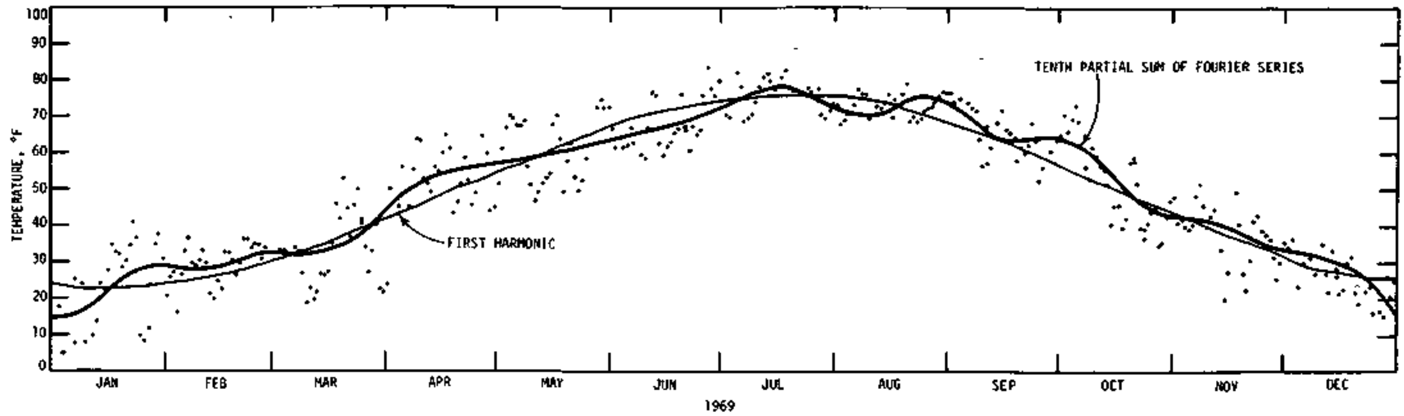

Figure 1. Average daily air temperatures for Peoria, 1969

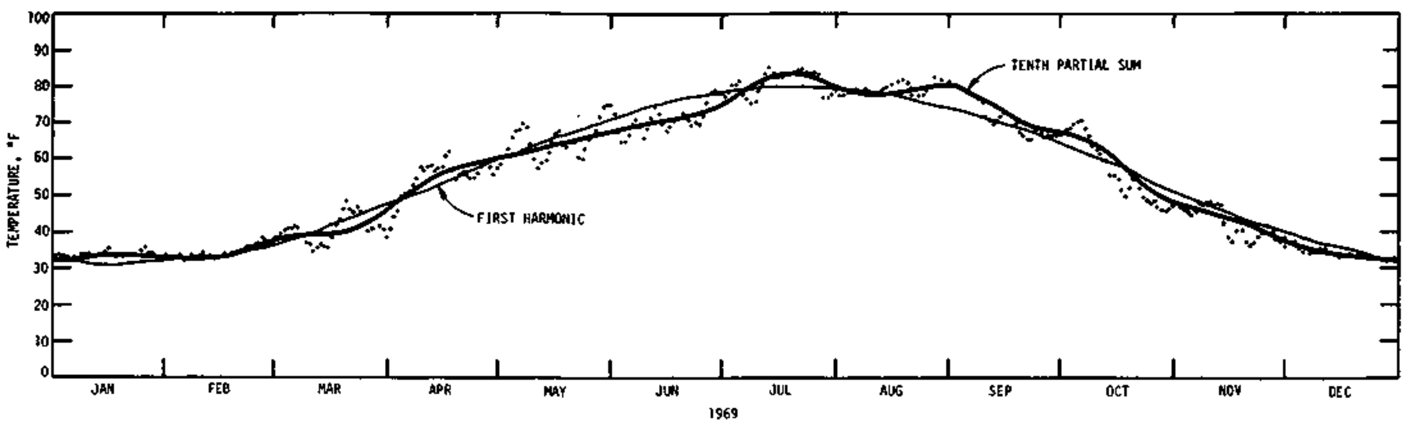

Figure 2. Average daily Illinois River water temperatures for Peoria, 1969

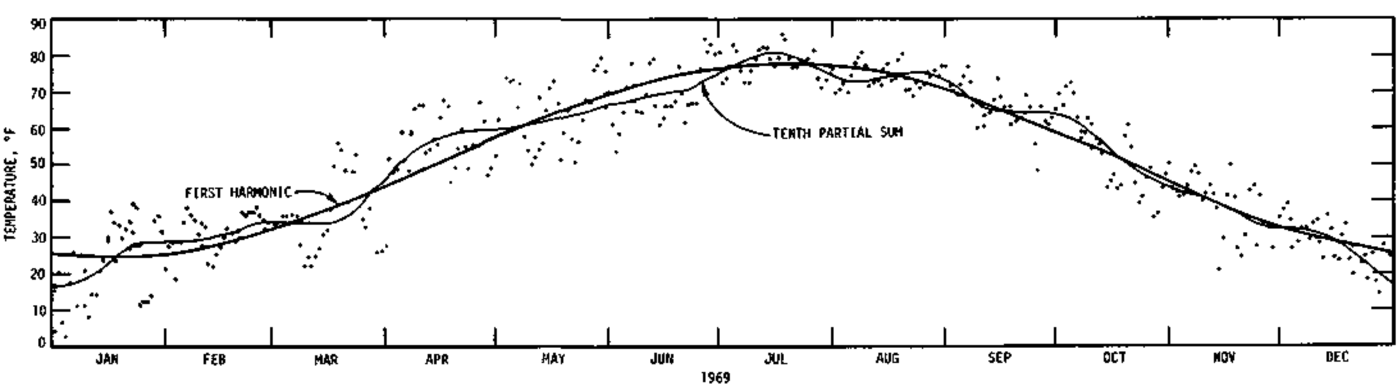

Figure 3. Average daily air temperatures for Havana, 1969

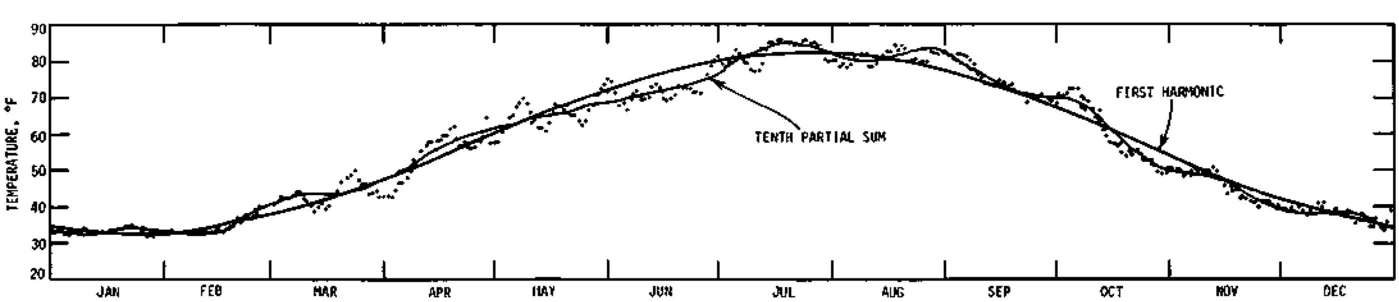

Figure 4. Average daily Illinois River water temperatures for Havana, 1969

servations approximated by the first and the tenth partial sums of the Fourier series are shown in figures 1-4. The dots on the figures show the observed daily mean temperatures, and the solid lines are their Fourier approximations.

Table 4 shows the total variance in the water temperature records for the Illinois River at Peoria and Havana and the magnitude of variance and percent variance accounted for by harmonics 1-10. Data in this table indicate that the first harmonic accounts for about 95 percent of the total variance; therefore, the variances accounted for by the higher harmonics are insignificant. In the case of air temperature records, the variance accounted for by the first harmonic as

Table 2. Results of Fourier Analysis for Water Temperatures at Peoria and Havana

Year of record	1st harmonic Amplitude (F)	Phase angle (deg)	2nd harmonic Amplitude (F)	Phase angle (deg)	3rd harmonic Amplitude (F)	Phase angle (deg)	4th harmonic Amplitude (F)	Phase angle (deg)	5th harmonic Amplitude (F)	Phase angle (deg)	6th harmonic Amplitude (F)	Phase angle (deg)	7th harmonic Amplitude (F)	Phase angle (deg)	8th harmonic Amplitude (F)	Phase angle (deg)	9th harmonic Amplitude (F)	Phase angle (deg)	10th harmonic Amplitude (F)	Phase angle (deg)
Peoria																				
1969	24 7	200	2 4	100	1 6	0	0 9	63	0 9	216	0 5	52	0 1	180	1.3	86	1 0	344	0 9	221
1968	23 6	200	1 4	128	0 8	315	1 4	56	1 0	96	1 7	230	0 6	141	0 7	254	1 1	20	1 2	143
1967	22 9	198	1 1	70	0 5	307	2 3	2	1 3	122	2.5	217	1.0	349	0 6	270	1 0	127	0 6	239
1966	23.6	200	2.5	53	1 3	241	1.4	339	1 0	169	1 2	31	0 8	157	0 7	153	1 5	323	1 8	22
Havana																				
1969	24 7	202	2 2	130	1 4	339	0 4	14	1 0	252	0.5	37	0.3	180	1 0	108	1 3	283	1 0	225
1968	24 0	202	1.4	160	0 7	315	1 4	23	1.3	90	1 4	236	1.1	146	1 0	241	1 0	32	1 4	188
1967	23.1	202	0 9	54	0 8	7	2.8	12	2 2	126	2 6	203	0 1	180	0.9	243	0 6	198	0 9	297

Table 3. Results of Fourier Analysis for Air Temperatures at Peoria and Havana

Year of record	1st harmonic Amplitude (F)	Phase angle (deg)	2nd harmonic Amplitude (F)	Phase angle (deg)	3rd harmonic Amplitude (F)	Phase angle (deg)	4th harmonic Amphtude (F)	Phase angle (deg)	5th harmonic Amplitude (F)	Phase angle (deg)	6th harmonic Amplitude (F)	Phase angle (deg)	7th harmonic Amplitude (F)	Phase angle (deg)	8th harmonic Amplitude (F)	Phase angle (deg)	9th harmonic Amplitude (F)	Phase angle (deg)	10th harmonic Amplitude (F)	Phase angle (deg)
Peoria																				
1969	26 1	197	2.5	156	0 3	270	1 1	112	3 0	195	1 2	211	0 8	201	1.0	90	1 5	245	1 6	197
1968	25 3	198	2 9	188	1 6	243	1.3	48	2 2	117	2 9	200	2 5	175	2 7	216	1 6	274	1 2	???
1967	24 0	196	1.5	217	1 0	276	2 9	348	2 4	107	4 0	180	0.9	263	1 3	189	1 2	142	1 5	212
1966	25 6	199	0 5	281	3 0	231	2 3	298	1 1	236	2.0	23	0 1	270	1 2	66	2 0	326	2 6	18
Havana																				
1969	26 6	195	2 4	166	0 3	270	1 1	95	2 6	198	1.2	222	0 6	211	1 1	90	1 4	236	1 4	192
1968	25 5	196	3.3	183	1 2	222	1.5	53	2 6	121	2 7	207	2 9	170	2.7	208	1.6	274	1 1	225
1967	23 3	195	1 5	212	0 9	270	2 4	358	2 1	104	3 9	184	1 2	275	1 0	186	1 3	132	1 6	230

Table 4. Variance in Water Temperature Records at Peoria and Havana

Year of record	Total variance (F^2)	1st harmonic Variance (F^2)	Percent of total	2nd harmonic Variance (F^2)	Percent of total	3rd harmonic Variance (F^2)	Percent of total	4th harmonic Variance (F^2)	Percent of total	5th harmonic Variance (F^2)	Percent of total	6-10th harmonics Variance (F^2)	Percent of total
Peoria													
1969	318.4	305 0	95 8	2.9	0.9	1.3	0 4	0.4	0.1	0 4	0.1	1 9	0 6
1968	294.6	278.5	94.5	1.0	0 3	0.3	0.1	1 0	0 3	0.5	0.2	3 6	1 2
1967	277.0	262 2	94 6	0.6	0 2	0.1	0.0	2 6	0 9	0.8	0 3	4.2	1 5
1966	293 8	278 5	94 8	3 1	1 1	0 8	0 3	1 0	0 3	0 5	0 2	2 9	1 0
Havana													
1969	316.1	305 0	96 5	2 4	0 8	1 0	0.3	0 1	0 0	1.0	0 3	2 0	0 6
1968	304 0	288.0	94.7	1 0	0 3	0 3	0 1	1 0	0.3	0.8	0 3	3.5	1 2
1967	288 5	266 8	92 4	0.4	0 1	0.3	0.1	3 9	1.4	2 4	0 8	4 4	1.5

Table 5. Variance in Air Temperature Records for Peoria and Havana

Year of record	Total variance (F^2)	1st harmonic Variance (F^2)	Percent of total	2nd harmonic Variance (F^2)	Percent of total	3rd harmonic Variance (F^2)	Percent of total	4th harmonic Variance (F^2)	Percent of total	5th harmonic Variance (F^2)	Percent of total	6-10th harmonics Variance (F^2)	Percent of total
Peoria													
1969	399 0	340.6	85.4	3 1	0 8	0 6	0 2	4 5	1 1	0 7	0 2	3.2	0 8
1968	398.5	320 0	80 3	4 2	1 1	1.3	0.3	0 8	0 2	2.4	0.6	13 9	3 4
1967	365 6	288 0	78 8	1.1	0.3	0 5	0 1	4.2	1 1	2 9	0 8	10 5	2 9
1966	405.4	327.7	80 8	0.1	0 0	4 5	1 1	2 6	0 6	0 6	0.1	8 8	2.2
Havana													
1969	408.6	353.8	86 6	2 9	0.7	0 5	0.1	0.6	0.1	3.4	0.8	3 5	0 9
1968	407.0	325.1	79.9	5.4	1 3	0 7	0.2	1.1	0.3	3.4	0 8	13.4	3.3
1967	350.2	271.4	77 5	1.1	0.3	0.4	0.1	2.9	0.8	2 2	0 6	9.5	2 7

shown in table 5 is only about 80 percent. Yet the higher harmonics (second through tenth) account for only about 4 percent of the variance. Therefore the cyclic variations in the water and air temperatures could be represented by the first harmonic alone with little loss in the degree of validity.

Nonseasonal Variations

Nonseasonal variations of water and air temperatures were determined by removing the Fourier approximations, f'(t), from their respective observed time series. The remaining residual records are random phenomena resulting from nonseasonal or random fluctuations in prevailing meteorological conditions.

The characteristics of the water and air temperature residual records for Peoria and Havana are shown in table 6. The variances in each class lie within a narrow range; therefore, it is reasonable to assume that the residual records are similar from year to year.

The following analysis pertains to the 1969 water and air temperature residual records for Peoria. The distribution of these residuals when plotted on normal probability paper is shown in figure 5.

The water and air temperature residuals were subjected to the Kolmogorov-Smirnov goodness-of-fit test under the hypothesis that they were normally distributed. The d statistic for the water and air temperature residuals was found to be 0.04 and 0.06, respectively, whereas the critical value for the d statistic at the 5 percent significance level was 0.07. Therefore, the hypothesis that these residuals were normally distributed is valid.

Figure 6 shows the normalized auto correlations, r_a and r_w, for water and air temperature residuals for lags ranging from 0 through 10 days. The auto correlations of water temperature residuals were significantly greater than zero for lags up to 2 days, whereas those for air temperature residuals were significant for lags up to 1 day. Figure 6 also shows the normalized cross correlation between water and air temperature residuals for lags ranging from 0 through 10 days with water temperature residuals lagging. Similar results were obtained for records of other years at Peoria and for all 3 years at Havana.

Table 6. Characteristics of Water and Air Temperature Residuals for Peoria and Havana

(Temperatures in degrees Fahrenheit)

Year of record	Water Mean	Water Variance*	Water Standard deviation	Air Mean	Air Variance*	Air Standard deviation
Peoria						
1969	0 0	6.1	2 5	0.0	45.8	6 8
1968	0 0	9.3	3 1	0.0	56.4	7 5
1967	0 0	6.2	2 5	0.0	58 0	7.6
1966	0 0	6 7	2.6	0.0	60.1	7 8
Havana						
1969	0.0	4 8	2 2	0.0	45 2	6.7
1968	0.0	9.2	3.0	0.0	56.2	7.5
1967	0.0	10 5	3.2	0 0	60 2	7.8

** Variance in degrees Fahrenheit squared*

Regression Analyses

Since air temperature has a significant influence on water temperature, the nonseasonal water temperature variations should respond to the nonseasonal air temperature variations. Multiple linear regression analysis procedures were used to estimate the nonseasonal variations in water tern-

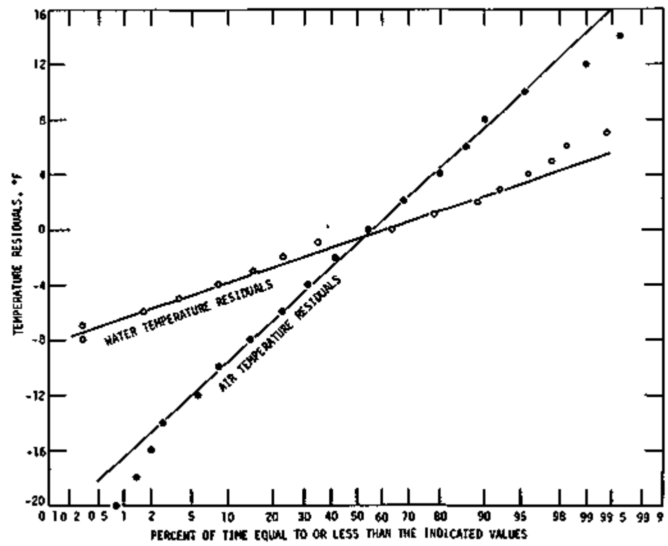

Figure 5. Normal probability plot of water and air temperature residuals for Peoria

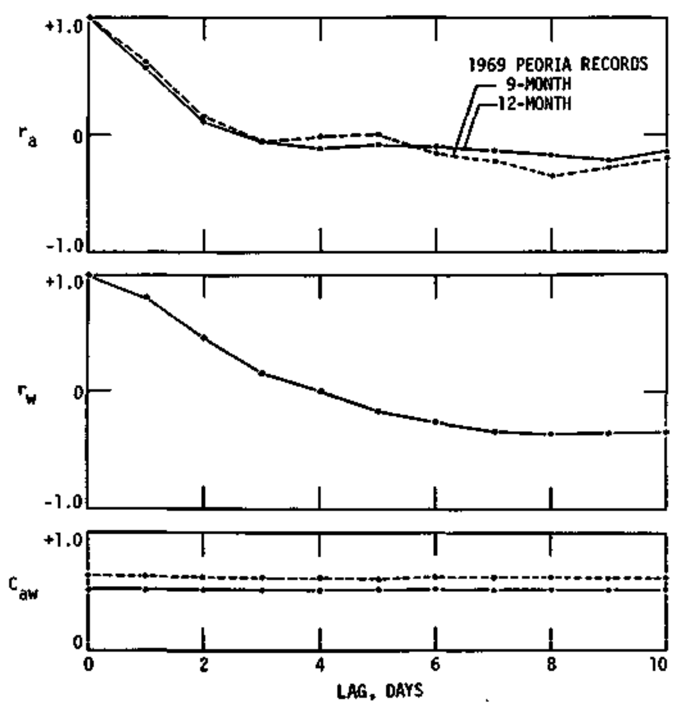

Figure 6. Auto correlations of air and water temperature residuals versus lag, and cross correlation between air and water temperature versus lag (water temperature residual lagging)

Table 7. Results of Regression Analysis for 12-Month Water and Air Temperature Residual Records for Peoria and Havana

Lag (days)	Dependent variable intercept	Regression coefficients β_1\	β_2	β_3	β_4	β_5	β_6	Multiple correlation coefficient	Standard error of estimate
Peoria									
0	-0 002	0 186						0 51	2 1
1	0 000	0 203	0 074					0 69	1 8
2	-0 002	0 128	0 115	0 109				0 75	1 6
3	-0 008	0 073	0 080	0 131	0 113			0 77	1 6
4	-0 018	0 048	0 048	0 087	0 134	0 119		0 78	1 6
5	-0 028	0 024	0 038	0 048	0 089	0 140	0 117	0 79	1 5
Havana									
1	0 005	0 182	-0 002					0 56	1 8
2	0 006	0.148	0 080	0 086				0 66	1 6
3	0 010	0 111	0 072	0 108	0.042			0 72	1 5

Table 8. Results of Regression Analysis for 9-Month Water and Air Temperature Residual Records for Peoria and Havana

Lag {days)	Dependent variable intercept	Regression coefficients β_4	β_2	β_3	β_4	β_5	β_6	Multiple correlation coefficient	Standard error of estimate
Peoria									
0	-0 001	0 263						0 62	2 2
1	0 012	0 288	0 088					0 82	1.6
2	0 013	0 172	0 142	0 154				0 87	1 4
3	0 009	0 115	0 074	0 192	0 148			0 89	1 3
4	0 004	0 060	0 064	0 099	0 190	0 146		0 90	1 2
5	-0 002	0 030	0 034	0 076	0 100	0 185	0 150	0 90	1 2
Havana									
1	-0 008	0 235						0 609	2 0
2	-0 003	0 197	0 078	0 024				0 732	1.7
3	-0 005	0 157	0 073	0 131	0 051			0 801	1 5

perature from air temperature variations. Equation 10 was used for lags 0 through 5 with the water temperature residuals lagging.

$$(R_w)_i = \alpha + \sum_{j=0}^{lag} \beta_{j+1} (R_a)_{i-j} \qquad (10)$$
$$lag = 0 \text{ through } 5$$

where

j = independent variable terms or summation index
$(R_w)_i$ = water temperature residual
$(R_a)_i$ = air temperature residual
α = dependent variable intercept
β = regression coefficient

The results of regression analysis for the Peoria data show that the dependent variable intercepts are practically zero (table 7). Multiple correlation coefficients increase from a value of 0.51 for zero lag to 0.79 for a lag of 5 days. The increase in multiple correlation coefficient values for lags beyond 2 days is not significant and does not warrant additional computational efforts. A regression equation based on a lag of 2 days accounts for about 60 percent of the variance in the water temperature residual records and appears to be adequate. The results of regression analysis for lags up to 3 days for the Havana data are also given in table 7.

In carrying out the regression analysis, residuals for all 365 days were considered. However, when the average daily water temperatures are close to 32 °F during the months of January, February, and December, the water temperatures cannot respond to air temperatures to the same degree as in other months *(see figures 2 and 4).* Therefore, a regression analysis with 9-month residual records, excluding the values for January, February, and December, was performed, and results for the Peoria and Havana data are shown in table 8. Once again, the dependent variable intercepts were practically zero, and the multiple correlation coefficient values increased considerably. A regression equation with a lag of 2 days appears to be adequate in explaining the nonseasonal variations in the average daily water temperatures. The dotted lines in figure 6 show the auto and cross correlations for water and air temperature residuals for Peoria with 9- and 12-month records for the calendar year 1969.

PREDICTIVE MODEL

A mathematical model was formulated permitting the prediction of water temperatures from observed air temperatures. The following equation represents the predictive model for the seasonal and nonseasonal variations in average daily water temperatures.

$$(T_w)_i = (A_o/2) + \sum_{n=1}^{m} [A_n \cos(2\pi ni/N) + B_n \sin(2\pi ni/N)] + \beta_1 (R_a)_i + \beta_2 (R_a)_{i-1} + \beta_3 (R_a)_{i-2} \qquad (11)$$

where $(T_w)i$ is the predicted average daily water temperature on the ith day from January 1 of any calendar year, and all other terms are defined in the notation list. In the analysis of water temperature data for the Illinois River at Peoria, the tenth partial sum of the Fourier approximation was used to define the seasonal variations. Since the contributions of harmonics higher than 1 were found to be insignificant, the value of *m* in the summation term of equation 11 can be taken as 1. A confidence band for the temperature predictive model can be constructed by adding appropriate multiples of the standard error of estimate. The proposed model should yield better results than those obtained in this study when it is used for surface waters which do not approach freezing temperatures.

The proposed model for water temperature predictions (equation 11) was verified by the water and air temperature records during 1968 for Peoria. The regression coefficients for the 9-month record during 1969 were used in predicting the water temperatures in the Illinois River at Peoria for the months of March through November 1968. Excellent agreement was found between the observed and predicted river water temperatures for these months. The mean and the standard deviation of the observed and predicted water temperatures for the 9-month period are 63.4 and 13.5F, and 63.2 and 13.3F, respectively.

The mathematical model (equation 11) proposed for forecasting the average daily water temperatures requires water and air temperature records at a similar location for a period of at least 1 year. However, situations can arise where there is a need for characterizing the variations in average daily water temperatures at a location where no temperature data are available. Such a situation can be visualized in the case of proposed power plants or other industries requiring large amounts of cooling water which are to be located near streams, lakes, or other water bodies.

Consequently there is a need to develop a method of predicting average daily water temperatures at a location based solely on the average daily air temperature records which are more likely to be readily available. Such a methodology becomes feasible only when both water and air temperature records are available for another similar water body. Parameters of the proposed mathematical model are determined by the use of the water and air temperature records. These parameters are employed in conjunction with the air temperature data for the location of interest to predict the mean water temperature.

The proposed methodology was used to predict Illinois River water temperatures at Havana for the years 1968 and 1969 with the use of only the air temperature records for those years. The results were then compared with the observed values. Parameters for the proposed mathematical model were determined by the use of the water and air temperature records at Peoria for the year 1969.

As pointed out earlier the first harmonic accounts for about 80 and 95 percent of the total variance respectively in the water and air temperature records. The higher harmonics, second through tenth, account for about 4 percent of the total variance. Consequently, it was concluded that the cyclic variations in water and air temperatures for Peoria could be represented by the first harmonic alone with little loss in the degree of validity.

Also, the nonseasonal variations in water temperatures were well correlated with the nonseasonal variations in observed air temperatures. If only the first harmonic with a periodicity of 1 year is considered to represent the cyclic variations in the water and air temperature records, it can be construed that a definite relationship exists between the water and air cyclic trends.

The ratios of the average annual water temperature to the average annual air temperature for both Peoria and Havana are given in table 9. The ratios of amplitudes of the first harmonic for water and air temperature records are also presented in table 9. These ratios vary within a very narrow range of values and can be considered constant. It should be noted that an increase in average annual air temperature generally results in an increase in the average annual water temperature. Also, when the amplitude of the first harmonic of the air temperature record increases, there is a similar trend in the amplitude of the first harmonic of the water temperature record. It is not fortuitous that a definite relationship exists between water and air temperature at the location investigated. Such a trend should be noticeable at other locations on this river also.

The nonseasonal variations in water and air temperatures, obtained after removing from the temperature records the cyclic variations represented by the tenth partial sum of Fourier series, were found to have a normal probability distribution with stable parameter values. Similar results were obtained for nonseasonal variations in water and air temperatures obtained after the removal of cyclic variations represented by the first harmonic with periodicity of 1 year. Since the cyclic variations and the nonseasonal variations in water and air temperature records are stable from year to year, it is reasonable to assume that the water-air temperature relationship is a stationary process.

If a constant parameter linear system is conceptually possible and stable, and if such a system is subjected to a sinusoidal input with certain frequency, the output from the system will also be sinusoidal with the same frequency.[9] Components of the input-output functions of such a system are shown in figure 7. The ratio, output amplitude/input amplitude under steady state conditions is called the magnitude ratio. The input-output functions may differ in that they may be out of phase, which means that they cross their respective mean values at different times. Since the dashed curve (output) in figure 7 crosses the mean value at a later time than the solid curve (input), the output is said to lag

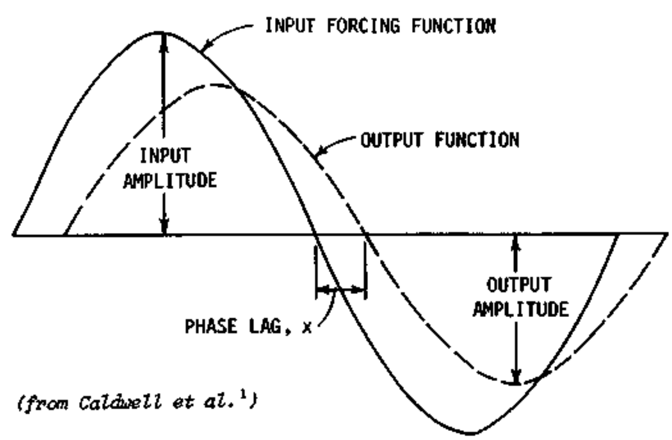

Figure 7. Steady slate response to a sine wave input in a stationary linear process

Table 9. Characteristics of Water-Air Temperature Relationships

Year of record	Ratio of means (a_1) Peoria	Havana	Amplitude ratio (a_2) Peoria	Havana
1969	1.119	1.117	0.946	0 929
1968	1.102	1.099	0.933	0.941
1967	1.103	1.086	0.954	0 991
1966	1.110		0.922	

the input. If x is the phase difference between the input and output waves, then

$$\text{phase lag} = x \times \text{frequency} \times 360°$$

For purposes of clarity, mean values for the input-output functions are shown as the same. Even if the mean values of input-output functions are different, the ratio output mean value/input mean value will remain constant and is called the ratio of means.

If only the annual cyclic variations in the water and air temperature records for Peoria are considered, the amplitude ratios and other parameters as presented in table 9 appear to be stable within practical limits. Consequently the water-air temperature relationship can be treated as a stationary linear process. Under this premise, these parametric values are also applicable to another similar water body in the neighboring area.

The annual cyclic variations in the air temperature record for Peoria can be expressed by

$$(T_a)^c_i = (A_a/2) + A_1 \cos(2\pi i/365) + B_1 \sin(2\pi i/365) \quad (12)$$

which is equivalent to

$$(T_a)^c_i = (A_a/2) + (A^2_1 + B^2_1)^{1/2} \cos[(2\pi i/365) - \theta] \quad (13)$$

where $(T_a)^c i$ is the trend component of the air temperature on the ith day commencing from January 1 of any calendar year, A_a is twice the annual mean air temperature, and A_1, B_1, and 8 are components of the harmonic describing the annual cyclic variations of air temperatures and their mean value. The annual cyclic variations in water temperature can then be represented by

$$(T_w)^c_i = (\alpha_1 A_a/2) + \alpha_2 (A^2_1 + B^2_1)^{1/2} \cos[(2\pi i/365) - (\theta + \theta_1)] \quad (14)$$

where $(T_w)^c{}_i$ is the trend component of the water temperature on the ith day from January 1 of the corresponding calendar year, a_1 is the ratio of means, a_2 is the amplitude ratio, and θ_1 is the phase difference between the water and air temperature harmonics. The mathematical model for predicting water temperature from air temperature records then takes the form

$$(T_w)_i = (\alpha_1 A_a/2) + \alpha_2(A^2_1 + B^2_1)^{1/2} \cos[(2\pi i/365) - (\theta + \theta_1)] + \beta_1(R_a)_i + \beta_2(R_a)_{i-1} + \beta_3(R_a)_{i-2} \quad (15)$$

where $(T_w)_i$ is the predicted water temperature on the ith day; β_1, β_2, and β_3 are regression coefficients; and $(R_a)_i$ is the residual in air temperature on the ith day after removing the annual cyclic trend component from the record.

Results

Table 10 gives a comparison of the characteristics of air temperature records for Peoria and Havana. The annual mean air temperature appears to be about 1.7F higher for Havana than for Peoria. If the assumption that the water-air temperature relationship is a stationary linear process is valid, then the annual mean water temperatures for Havana should be proportionately higher than those for Peoria. This was found to be true from the observed water temperature data, as will be presented later. All other parameters considered were similar for these two areas for both 1968 and 1969.

For predicting the water temperatures in the Illinois River at Havana with equation 15, the parametric values of α_1, α_2, θ_1, β_1, β_2, and β_3 were obtained by use of the 1969 air and water temperature records for Peoria. These values are 1.119, 0.946, 0.052, 0.137, 0.119, and 0.128, respectively. The regression coefficients β_1; β_2, and β_3 were evaluated with air and water temperature residuals obtained after removing the annual cyclic trends. It should be emphasized that these parametric values are assumed to be constant and are applicable to another similar water body in the general vicinity of the area for which these values directly apply. Such an assumption is justified if the water-air temperature relationship is essentially a stationary linear process.

Figure 8 shows the observed and predicted Illinois River water temperatures at Havana for the year 1969. For visual clarity, the origin for predicted water temperatures was shifted 10F higher than that for observed water temperatures. A statistical comparison of the observed and predicted water temperatures at Havana for the years 1968 and 1969 are given in table 11. The predicted water temperatures were in excellent agreement with the observed water temperatures.

Table 10. Characteristics of Air Temperature Records for Peoria and Havana

Parameters	1969 Peoria	1969 Havana	1968 Peoria	1968 Havana
Mean, F	49 5	51.3	50.6	52 3
Variance, F^2	399	409	399	407
Amplitude of 1st harmonic, F	26 1	26 6	25.3	25 5
Phase angle of 1st harmonic, deg	197	195	198	196
Variance accounted for by 1st harmonic, F^2	341	354	320	325
Percent of variance accounted for by 1st harmonic	85 4	86.6	80.3	79 9
Mean of residuals, F	0 0	0 0	0.0	0 0
Variance of residuals, F^2	45 8	45 2	56.4	56.2

Table 11. Comparison of Observed and Predicted Water Temperatures in Illinois River at Havana

Statistical parameters	1969 Observed	1969 Predieted	1968 Observed	1968 Predieted
Average, F	57.4	57.6	57.6	58.7
Variance, F^2	314.4	319.8	302 3	296.3
Standard deviation, F	17.7	17.9	17.4	17.2
Standard error of estimate, F		2.7		3.4

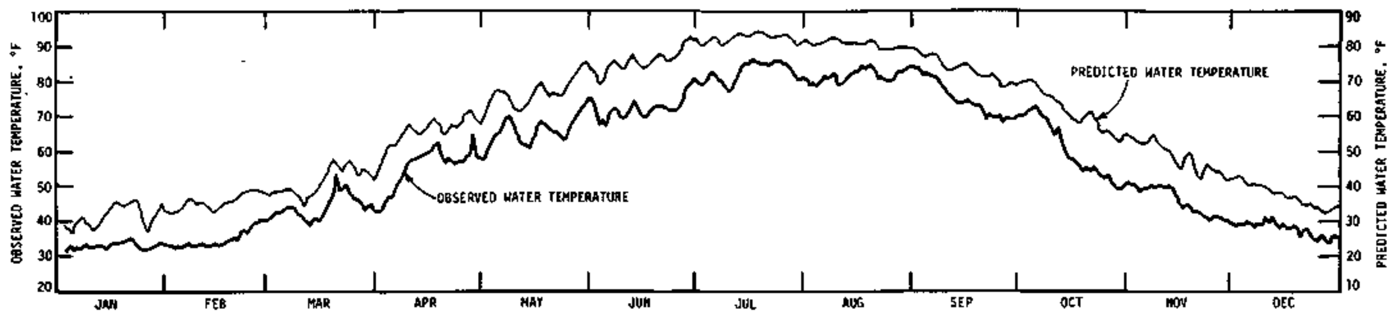

Figure 8. Observed and predicted water temperatures in Illinois River at Havana, 1969

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NOTATIONS

A, B = components of harmonic equation
A_a = twice the annual average of air temperature records
A_1, B_1 = components of the first harmonic of temperature records
A_n, B_n = Fourier coefficients
A_o = twice the average of time series records
C_{aw} = cross correlation between air and water temperature records
d = statistic in Kolmogorov-Smirnov goodness-of-fit test
$f(t), y(t)$ = functional representation of time series
$f'(t)$ = mth partial sum of Fourier approximation
f_t = individual values in time series at time *t*
i, n = integer indices
m = maximum number of harmonics considered in the Fourier series
N = total number of equally spaced observations in a given period
r_a = normalized auto correlation coefficient for air temperature residuals
r_w = normalized auto correlation coefficient for water temperature residuals
R = amplitude in degrees Fahrenheit
$(R_a)_i$ = residual in air temperature record on the *ith* day from January 1
$(R_w)_i$ = residual in water temperature record on the th day from January 1
R_n = amplitude of nth harmonic
t = time
T = period of cyclic changes
$(T_a)^c_i$ = trend component of the air temperature on the *i*th day
$(T_w)_i$ = predicted water temperature on the *i*th day .
$(T_w)^c_i$ = trend component of the predicted water temperature on the »'th day
x = phase difference between input and output functions
α = dependent variable intercept
α_1 = ratio of means
α_2 = amplitude ratio
$\beta_1, \beta_2, \ldots \beta_6$ = regression coefficients
θ = phase angle of the first harmonic of temperature records
θ_1 = phase difference between water and air temperature harmonics
σ^2 = total variance in observed record for a year
σ^2_m = variance accounted for by *m* harmonics

Appendix A. Average Daily Air and Illinois River Water Temperatures for Peoria, 1966-1969

(Temperatures in degrees Fahrenheit)

1966

Day	January air	January water	February air	February water	March air	March water	April air	April water	May air	May water	June air	June water	July air	July water	August air	August water	September air	September water	October air	October water	November air	November water	December air	December water
1	38 1	40.2	16 6	33 1	37 5	38.5	48 8	47 2	45.0	56 0	61 5	68.7	82.6	84 8	75 6	80 2	75.8	79.3	48.3	60.5	34 8	48 7	16 1	34 6
2	39 0	40 5	22 5	33 5	44 3	40 5	44 8	48 0	46 1	55 9	66 9	68 1	84 4	85 4	71.8	78 7	76.8	80 0	52 1	59 4	29 0	42 8	12 3	34 6
3	30 9	38.8	17 4	33 3	50 3	43.6	41 4	47.3	49 9	56 9	71 0	69 1	82 6	86 6	68.4	77 2	74 1	80 3	61 4	58 9	26 1	41 2	14.1	35.0
4	32 6	38 0	12 8	33 4	29 8	42 1	38 8	46 3	53 6	57 5	74 0	71 2	81 8	86 9	68 1	77 9	71 8	79 6	54.3	59.4	36 6	38.1	25 9	34 3
5	33 9	37 8	14 5	33 2	21.6	37 7	36 4	45 0	66 3	58 9	73 9	72 6	82 1	87 7	70 5	78 6	64 9	76.7	48.0	58.9	36 8	39 2	35 5	34.0
6	25 9	37.2	33 3	33 5	19 8	34 9	39 8	44 0	65 1	61 6	69 4	73 1	77.3	86 6	72 4	79 9	61 1	73.1	49 9	58 0	40 3	39 5	49 9	35 8
7	17 6	35 1	40 5	34.9	21 9	33 5	39 3	43.2	65 4	62 5	69 8	73 6	73 3	84 9	69 5	80 0	63 4	71.4	56 6	57 6	60 1	42 3	43 5	39.0
8	9 9	34 5	47 4	35 1	29 0	35 7	37.8	43 6	48 1	60 1	68 8	73 1	76 5	83.7	73 1	79 4	66 8	71 8	60 3	58 4	59 5	46 0	38 6	42 7
9	33 5	33 7	47 8	34 3	38 4	38 4	38 5	43 8	38 1	56 7	58 8	69 6	80 4	83 4	67 3	78 4	65 9	72.1	64 1	59 8	57 6	48 5	35 0	42 4
10	26 0	34 3	42.1	34 2	46 0	40 3	40.5	45 5	41 9	55 8	58 3	67 3	85.5	83 6	62 4	75 7	70 4	72.7	54 4	59.6	33.6	48 3	39 3	38.8
11	20 1	33.2	34 4	34 4	47.0	41 7	40 5	44.5	45 9	53 8	67 1	68 1	85 3	84 7	65 0	75 2	72.8	73 0	52 9	58 3	37 1	46 0	24 1	36.9
12	31 4	33.0	34 6	34 0	48 1	43.5	44.9	45 3	49 4	53 0	73.6	70 8	87.9	86 3	67.9	74 7	70.0	73 6	54 0	57 4	34 1	45.5	23 9	36 1
13	27 0	33.0	31 8	34 6	41 4	45 1	46.6	46 9	45 9	52 3	67 1	71.7	89 9	88 4	66 3	74 2	72.4	74 1	63.4	57.9	37 6	44 6	24 1	35 9
14	28 4	33 1	25 5	34 5	48 8	47 5	46 4	48 3	52 0	53 3	68 1	71 3	89 6	86 5	72 0	75 1	64 0	72 6	64.9	59 3	39 4	44 6	30 6	35 7
15	28 3	33.0	27 5	34 9	45 9	48 6	49 1	50.6	53 8	53 0	63 9	71 3	74 3	82.0	77 4	77 0	56 9	66 9	48 5	59 4	43 3	44 7	36 8	36.5
16	20 0	34 0	25 5	35 3	50 6	50 0	47 9	50 9	64 4	54 6	63 1	70 9	74 1	79.5	73 4	78 8	55.6	67 1	43 3	55 7	53 1	46 8	32 8	37.0
17	11 6	32 8	18 3	33 9	59 8	51.7	53 1	52 6	68 8	56 0	65 9	71 7	78 0	80 8	73 6	79 4	61 1	66.7	43 5	54 0	57 3	48 5	36 8	37 3
18	10 3	33 0	31 8	34 3	50.0	53 2	59 4	53 9	64 9	57 4	67 8	72 6	83 4	81 5	75 0	79 8	64 8	67 0	45 6	53 4	37 4	47 8	35 0	37 8
19	11 6	33 0	16 9	34 4	39.9	48 8	62.3	56.5	58 6	58 4	70 6	74 8	81 5	83 7	73 0	77 9	62 1	66 2	46.1	53.0	31 5	45 0	48 4	38 6
20	20 8	32 9	13 0	34 2	41 6	47 2	49.0	57 8	57 5	58 2	74 0	75 6	69.6	79 1	70 8	75 8	64 5	64 5	45 6	51 4	36 5	43 6	36 3	39 0
21	26 9	33 0	15 8	33 4	60 1	50 3	42.9	56 8	62 9	59 4	76 0	77 3	70 5	79 0	75 1	77 6	62 1	64 8	50 6	52 0	43 9	43 1	37 3	39 2
22	24 4	33 0	19 8	33 3	61 9	53.9	51 6	56 6	64 9	61 3	76 8	78 5	73 1	79 2	67 6	76 8	63 1	65 8	51 9	52 0	52 1	45 0	24 9	36.9
23	7 4	33 0	27 1	33 0	39 4	52 2	58 5	57.8	69 1	62 1	77 5	79 8	72 0	78 3	61 4	74 5	62.0	65 5	46 8	51 1	56 5	47 2	18 4	34 6
24	7 8	33 0	28 5	33 8	24 0	43 9	63 0	61.3	62 0	64 5	79 1	79 5	79.6	79 2	62 4	72 7	57 1	64 8	45 9	51 3	58 5	49 8	20 9	34 0
25	15 8	33 0	32.8	34 2	32 1	41 2	60 6	63 1	65 0	65 9	81.9	80 5	79.0	80 9	65 8	72 3	56 0	62 8	42.4	50 7	48 1	50 9	18 6	33 3
26	17 0	33 1	29 1	34 8	30 6	40 8	62 0	62.5	70 0	67 7	78 8	81 5	79.4	81 5	69 6	73 2	54 0	61 5	47 1	51.1	47 1	50 0	18 3	33 5
27	9 4	33 0	34 4	35 9	31 3	41 2	44.9	58.3	73 1	68 9	78 1	83 0	77 8	82 1	71 9	74 7	56.1	61 0	52 4	51.5	43 9	47 2	21 9	33 2
28	-2 8	32.8	34.1	36.9	34 4	43 6	49.4	57.1	63 6	70 0	74 0	81.5	74.5	82 0	73 3	75 9	55 8	61 8	56.8	53 1	32 0	44 3	28 0	33 0
29	-10 6	33 1			39 1	44 2	54.4	57.2	54 6	69 1	77 1	81.9	71 8	79 9	71 9	77 0	58 6	61 9	45.9	52 9	34 5	38 8	16 9	32 9
30	-0 1	32 8			33 9	45 1	51.1	57.4	56 9	68 5	80.0	83 4	69 4	79 1	72 9	77 0	50 6	61 3	42 1	51 6	34 6	37 3	20 5	33 3
31	12.6	33 0			44 9	45 0			57 3	68.0			73 4	79.8	74 0	78 2			46 3	51 3			30 5	33 7

1967

Day	January air	January water	February air	February water	March air	March water	April air	April water	May air	May water	June air	June water	July air	July water	August air	August water	September air	September water	October air	October water	November air	November water	December air	December water
1	27 1	33.2	31 3	33 7	29.5	33.6	59 3	56.8	62 4	58 8	59.5	60 7	75.1	80 0	77 0	80 8	58 9	66 4	63 9	57 3	48 9	50 5	33 3	36 1
2	28.9	33.4	20 8	34 8	45 0	33 5	63 6	56 8	43 1	56 3	64 9	62 0	68.0	79 1	78 9	82 5	62 6	66.5	68 3	60.6	44 9	49 4	34 6	36 5
3	20 5	33 1	18.0	34.9	35 5	33 3	47.4	56.7	43 7	55 6	67 6	64 9	62.0	75 8	73 0	81 6	63 5	66 5	69.0	64 4	34 9	46 1	32 5	36 5
4	22 9	33.1	31.4	33 9	31.5	32 8	48 1	56.7	45 6	55 4	67 8	67 7	61 1	73 7	70 0	81 3	63 9	66.8	73.1	66 5	28 4	42 3	31 5	36 7
5	21.5	33 0	28 0	34 0	30 3	32 7	64 3	55 4	42 9	54 0	69 5	70 2	62.9	71 6	69 6	80 5	68 0	67 1	64 6	67 6	28 3	40 1	40 5	37 7
6	36 4	33 5	11 9	34.7	24 8	33 2	63 9	55 8	45 1	52 2	72 6	72 0	67 5	72 7	70 0	79 8	67 5	68 2	58 0	65 6	32 5	38 9	44 6	39 2
7	24.4	33.6	8 6	34 6	24.5	33 0	47.1	57 9	51 6	53 4	75 3	73 8	72.6	74 5	74 8	79 4	67 5	70 6	60 0	64 6	30 9	39 0	41.8	40 0
8	18 0	33.1	17 6	34.7	19 3	33.4	45 9	57 3	52 0	54 2	76 9	75 0	75.8	77 3	72 1	78.9	68 8	71 5	52 1	63 6	38 3	40 1	39 8	41 2
9	24.9	32.8	29 4	33.6	33.6	34 8	62 8	55 6	53 4	54 4	75 4	75 8	72 3	77 3	73 0	78 7	65 8	72 3	46 6	60 0	49 3	42 6	39 3	41 5
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11	21 6	33.2	18 3	34 5	44 6	41.0	41 4	56 3	54 4	57 7	73 7	76.1	79 0	80 8	61 3	74 3	60 1	68 2	43 4	55 4	52 8	47.1	35 6	41 0
12	35 6	33.4	14 9	34 5	38 4	41.4	45.4	54 7	46.5	55.5	75.3	76 9	74.6	80 7	61 5	74 6	64 8	68.0	48 9	53 8	47 1	47 5	36 6	40 3
13	36 3	34.0	13 5	34 0	39 6	42 5	60 5	53 2	53 8	56 0	75 3	77 9	63 3	75.5	64 3	75 4	71 0	69.0	55 1	54 0	31 8	44 7	32 8	40.0
14	32 0	33 0	42 9	34 1	35 9	42 2	64.9	53.8	55 9	57 2	79 8	79.8	64 8	72.3	67 3	75 2	69 9	69 6	56 0	54 8	38 0	43 3	27 5	37 6
15	14.8	33 1	37 3	33.9	31 5	40 2	64.3	56 5	54 5	57 3	81 8	81.8	68 9	72.2	70 9	75 3	68 1	69 8	60 0	56 1	29 5	41 6	21 6	35 5
16	22 0	33 3	12.4	35 0	34 0	39.7	72 8	57 3	55 5	58 2	76 3	82 6	69 8	73 6	72.5	76 1	66 3	69 7	52 8	56 8	37 8	42.4	25 6	35.3
17	6.6	33 1	21 6	34 7	21.8	37 7	58 1	60 1	60 3	59 4	72 4	80 7	72 5	74 5	73 4	76 4	68 3	69 4	54 0	56 0	41 5	41.7	36 3	35.4
18	1 1	33 4	27.4	35 0	24 6	37.6	45 8	60 9	66.9	62 3	70 4	78 3	71 1	75.3	69 4	76 6	71 4	70 8	48 8	54 3	40 0	40 3	39 0	37 6
19	19 3	34 9	31.3	34 1	30 4	37 5	48 4	57 9	62 9	63 8	73.8	77 6	71 0	76 5	59 8	74 0	71 6	72 0	44 0	52 6	34 3	40 1	34 3	38 0
20	33 1	33.5	19 9	34 9	37 4	37.7	50 6	57 1	51 1	62.6	75.5	79 0	75 6	76 8	62 8	72 3	72 4	73 2	47 1	52 9	35 8	40 1	46 8	40 6
21	40 5	33 3	17 5	35.3	36 3	38 0	62.0	56 9	55 0	62 2	69 8	78 6	72 9	77 8	66 8	72 0	65 5	73 7	47 8	52 5	38 1	40 0	41 3	42 3
22	48 6	34 6	22 9	35 0	38 4	39 0	43 9	57 9	55.5	62 9	69 0	77 6	76 8	79 1	66 3	71 5	58 1	70 8	46 3	52 8	31 1	39 8	15 6	36 4
23	51 3	34 8	14 1	34.9	47 1	42 3	35 5	56 5	62 4	62 8	74.4	79 2	78 6	80.6	67 9	72 0	58 4	69 0	60 3	53 8	29 8	38 8	17 6	35 0
24	55 6	34.1	0 6	34 7	54 3	44 8	37 4	53 7	67 1	64 5	71 8	78 8	76.3	81 0	68 0	72 7	55 0	67 0	55 6	55 5	36 6	38 9	16 9	33.5
25	30 1	35.0	4 3	34.6	60 9	48 7	41 6	52 1	74 3	66 4	63 1	75 6	76 9	82 1	71 8	74 4	58 3	65 6	40 6	52 8	40 3	39 5	23 9	33 5
26	28.0	34 8	20 6	34 5	57 3	50 6	42 1	51 9	76.4	69.3	66 5	75 1	77 0	81 6	66 1	75 6	64 0	65 3	35 5	48 9	34 0	39 6	9 0	33.3
27	23.6	34.0	31 0	34 0	45 4	50 8	47 0	49 8	74 4	72 0	67.5	74 1	70 1	80.0	60 9	72 6	49 1	62 3	35.3	47 5	36 6	38 2	14 1	33 1
28	19 9	33.5	28 4	33 3	51 8	50 9	53.1	52.5	61 4	70 8	71 8	75 8	70 1	77 8	64 9	70 4	46 0	57 2	36 3	46 4	26 6	35 8	18 3	33.0
29	25 9	32 8			53 8	51 7	53.3	53 0	57.5	67 4	71 8	77 1	71.0	77 7	66 4	70 3	44 5	55.9	48 9	46 2	31 4	35 2	20.4	32.0
30	27 9	32.7			66.5	54.4	63.5	54 3	52 9	61.6	76 1	77.9	72.3	78.4	62 1	69 4	54.3	56 0	52.0	47.8	33 3	35 5	26 8	32.4
31	32 6	33 4			63.5	56.5			60 3	60 3			74.1	79.3	56 9	66 4			49 3	49.8			3 6	32.0

1968

Day	January air	January water	February air	February water	March air	March water	April air	April water	May air	May water	June air	June water	July air	July water	August air	August water	September air	September water	October air	October water	November air	November water	December air	December water
1	-1 3	32 4	47 5	39 6	31 3	33 5	43 5	54 5	65 0	61 6	67.0	65 4	73 4	74 2	64 6	75 5	62 6	72.2	67 8	68 1	62 9	50 7	34 1	39.0
2	170	33 0	32 0	40 1	30 5	33 3	46 1	52 2	66 1	62 9	68 8	66 4	63 9	74.2	67 0	75 6	65.1	72.1	66 8	68 1	48 9	52.4	39 5	39.4
3	8.0	33 0	28 9	37 2	26 1	34 1	54 1	52 1	67.3	63.5	71 1	68 7	63 5	73 6	74 3	76 6	72 5	72 0	49.4	65 4	44 9	50 5	38 5	40.4
4	1 3	33.0	34.6	37 5	38 8	34 0	44 4	52 3	54 3	65.3	73 3	71 5	67.1	73 3	76 1	77 5	69.3	72 5	43 6	61 0	47 0	50 5	36 6	40 4
5	14 1	33.0	35 4	37 4	38 5	34 2	37.0	49 0	48 0	65.2	73 6	73 5	72.0	74 1	80 8	79 5	63 5	73 0	47.3	58 3	47 5	50 9	29 5	38 7
6	7 8	33 0	31.1	37 6	37 3	35 7	40 6	48.5	53 8	62 5	77 0	74 4	71 6	75 1	81 8	81 8	61 1	71.5	50 4	57.1	45 1	50 5	25 9	35 3
7	-6 3	33.0	24.3	36 4	38 3	38 3	54.1	51.0	62 6	60 8	78 9	76 7	70 1	75 9	81 0	83 3	64 4	69 9	51 8	56 6	41.9	48 8	23 1	34.1
8	2 9	33 0	27 4	34 8	43 4	40 0	52 9	53 9	65 4	60 0	78 0	77 8	74 9	75 2	79 6	84 3	64 1	69 0	56 5	57 2	33 4	47 2	20 0	33.8
9	21 8	33.0	24 0	34 5	45 6	42 6	53 1	54 8	53 3	61 4	80 6	80 4	75.4	75 4	76 6	84 3	58 5	67 7	54 1	58.2	33 0	45 2	27 3	35 0
10	10 6	33 0	9 9	33 2	37 3	43.4	53.3	55.6	52.9	62 6	82 0	82 0	65 5	75 8	70 5	81 1	57 9	65 0	50 3	57 4	33 1	43 0	32 5	34 3
11	18 3	32.8	11 1	32 8	33 8	42 3	59 1	56 5	58 3	61 7	75 1	83 1	68.0	76 4	63 5	78 3	58 1	64 1	55 6	57 3	33 8	42.3	39 0	35 3
12	21.9	33 0	14 1	33.0	29 5	39 9	66 0	57 7	62 9	61 6	67.1	82 3	74 4	77 1	66 9	77 5	63.0	65 3	61 5	57.7	30 8	41 0	51 0	37.7
13	24 1	33 0	14 6	33 0	26 0	37 9	61.0	58 4	64 0	64 0	66 9	79 6	79.0	77 9	72 4	77 7	66 1	67 4	68 6	60 9	34 4	40 2	25 6	39 0
14	26 9	33 0	19 4	33 4	36 3	38 0	52.5	57.5	70 8	65 9	72 1	76 1	77.0	79 5	74 6	78 3	66 8	68 7	71.0	63 6	45 9	41.9	13 5	36.3
15	23.4	32 7	27 9	33.5	44 6	40 0	46 6	55.8	66 1	68 1	69 4	74 9	78 8	81 0	73 4	79 0	69 1	70 0	73 9	66 2	46 3	43.8	17 4	34 0
16	10 6	32 8	30 5	33 3	45 1	41 6	53.0	54.7	56 9	69 4	64 6	74 5	79.3	81.7	80 4	80 7	66 0	70 0	71 4	67.6	42 3	44 0	24 8	35 3
17	23 9	33 0	12 9	33 8	50 5	43 5	58 3	55 3	51 8	66 9	67 1	73 7	79.5	81 8	75 5	81 8	68 4	69 2	65 3	67 5	40 8	44 0	26 4	33 7
18	33 4	33.0	16 5	33 9	53 0	46 1	51 8	56 6	50 3	63.8	71 8	74 3	76.5	80 4	72 9	80 3	61.3	66 1	49.1	64 4	34 0	43 6	36 1	33 8
19	28 5	33 0	26 0	33 7	54 6	48 5	51 1	56 1	49 1	61 8	74 0	75 0	76 0	81 6	81 8	81 2	59 6	68 7	51 1	61 5	29 6	41 6	37 1	34 8
20	34 1	33 3	19 8	33 0	35 9	47 8	61 8	57 1	50 9	59.2	74 9	76 4	72 9	81 2	82 6	82 4	62 3	65 4	49 4	60 3	27 8	38 1	27 1	35.4
21	34 5	33 0	6 8	33 0	31 0	44 5	58 6	58 6	53 4	58 4	80 3	75 8	76 6	82 6	82 6	84 1	69 5	66 5	55 4	58 2	40 1	38 5	27 4	35 1
22	33 1	33.3	11 4	34 9	27 8	41 2	64 8	61.5	52 9	60 5	77 9	80 2	78 5	84 5	82 0	83 5	71 9	70 4	52 3	57 8	43.4	39 9	32.5	34 9
23	28 6	33 0	19 3	35 3	28 5	39 2	50 8	60.8	53.6	60 2	79.6	81 5	76 4	84 1	82 5	83 8	70 9	72 3	45.4	55 3	49 5	42.1	20 5	34 4
24	17 5	33 0	23 5	34 5	43 0	41 3	38 1	53 5	57.8	57 9	75.9	81 8	75 3	84 1	78 9	84 1	65 5	72 3	42 5	53 3	43 3	42 7	14 0	34 0
25	25 4	33 0	27 1	34 5	49 5	44 7	44.8	51 0	57 0	58 7	71 9	78 5	76 0	82 8	69 5	80 4	57 4	69 8	38 8	51 5	38 0	42 4	12 9	33 8
26	33 9	33 1	30 6	34 1	60 6	49 5	49 0	52 1	60 5	59 4	60 5	74 5	74 5	81 8	62 9	76 5	57 3	68 2	46 0	50 3	44 0	43 7	29 5	33.5
27	41 8	33.5	30.8	33 0	62.6	53.9	50 0	53.9	52 3	60 8	55 1	68 7	75 9	82 0	61 9	73 5	59 3	67 6	49 4	50 7	39 4	43 5	43 9	33.0
28	42 1	34 3	27 5	33 0	63 1	55 2	55.8	56.1	59 0	60 3	68 4	67.2	68 9	80 1	63 9	72 2	59 3	66 4	40 4	48 5	33.3	42 6	30 8	32 6
29	44 3	33 2	21 6	33 2	63.9	58 5	57 1	58.9	57 9	60.1	78 6	70 5	66 5	78 7	65 9	72.0	59 4	66 4	39 1	47 0	34 4	41 0	24.1	32.7
30	33 0	34.6			62 6	58 1	62 8	61 8	58 0	61.3	80 3	72 5	69 3	77 5	67 5	72 0	64 5	67 7	48 4	46 7	31 9	40 0	26 5	33 7
31	42 0	35 6			55 6	58.0			62 4	62 9			70 8	77.1	64 5	72 1			62 0	48 0			-2 3	33 6

Appendix A. Concluded

1969

Day	January air	January water	February air	February water	March air	March water	April air	April water	May air	May water	June air	June water
1	0.4	33.1	25.6	33 0	32.1	37 2	50.0	40.4	60.3	58.0	61.3	72 2
2	17.1	33.5	27.5	33.0	32.9	39 3	45 5	44 0	66 8	60 1	49 4	67.3
3	4.1	33.3	15.9	33 0	33 0	40 0	45 1	45 6	69 5	62 8	61.3	64.1
4	-2.4	33.3	27.1	33 4	32 3	40 8	56 8	49.3	68.6	65.4	61.9	64 7
5	14.8	32 8	33.0	32.0	31 3	40.8	48 8	50 0	66.6	67.1	66.8	66.6
6	24 8	33.0	36.3	33.0	33.8	41 4	44.8	50 1	67.1	67.5	62 5	69.3
7	7.4	32.7	34.4	33.2	32.6	40.9	54 5	52.3	67.8	69.4	64 9	70.5
8	23.9	33.3	33 8	33.0	26 9	39 0	63 1	54 5	56 0	68 6	59 4	68.4
9	7.5	32.7	30.1	33.0	18 6	37 0	63 0	57.3	50.4	63.9	58.4	65 8
10	4 4	33.0	32.5	33.0	22.4	36 2	52 4	56 8	46 5	59 5	66.7	68 0
11	9 5	32.9	29 3	33.0	19.4	34 3	51 0	57.6	48.8	57 0	76 1	71.1
12	13 3	33.0	21.3	33 5	21.8	35 4	48.9	57.9	51 1	58 5	75 8	74 2
13	23 5	33.0	19 8	32 0	26 6	36 1	55 4	56 2	52 9	59 0	62.5	72 3
14	22 0	34.3	24 8	33 3	26 1	35 8	54 5	57 0	59 4	61 2	59 4	69.3
15	27 4	34.9	22 4	33 3	27 0	35 3	59 8	58 3	67 5	64 4	61 3	67 8
16	34 5	33.3	32 6	34 0	34 9	38 8	64 1	60 5	69 8	66 1	62 6	69.0
17	32 9	32.7	32 3	33.5	45 3	41 5	61 1	61.5	63 5	67.1	65.4	70 4
18	31.9	33.3	30 3	33 8	51.6	43 7	43.1	57 0	49 0	64 0	66 6	71.0
19	28.3	33.0	28 1	34 0	52 4	46 0	46.0	54 0	57 6	63 3	75 9	71.6
20	30 1	33.1	29 8	34 8	44 1	48 4	51.0	55 2	62 5	65 1	66 1	72 7
21	34 5	33.8	36 0	35 0	36 3	46 0	58 3	56 0	53.0	64.6	60.9	70 5
22	41.0	33 0	35 9	35 9	45 1	45 0	52.0	55 5	49 3	60.4	66 9	69 8
23	36 6	33 5	33 8	35.9	49.9	46.4	45 6	54 2	52 1	60.1	64 9	70 0
24	9.1	34 5	34.8	36.3	41.3	45 3	48 6	54 8	58 1	62.4	65 6	69 7
25	7.9	35 1	34.9	36 8	34.0	42 7	56 3	55 7	62.3	65 8	75 5	71.7
26	11.3	34 0	30.9	38 3	27 3	40 0	64 0	57 6	62 6	66 3	83.0	75.5
27	23.8	34 0	33 9	37 3	32 3	40 8	59 3	58 4	72.6	68 0	77.5	78 2
28	34.8	33 0	32.9	36 4	39 9	44 1	43.3	57 4	74 4	71.5	75.3	78 8
29	37.1	33 0			22.1	41 2	44.9	55 5	72 8	74 1	79 8	78 8
30	30.4	33 0			21.3	40.2	51 0	57 0	72 8	74.9	72 8	78 3
31	20.4	33.2			23.3	38 1			66 6	74 8		

Day	July air	July water	August air	August water	September air	September water	October air	October water	November air	November water	December air	December water
1	70.3	77 1	67.5	77.8	73.6	80.9	70.1	67.1	40.8	47 2	34 4	37.1
2	70 0	79.2	68 6	77.2	72 4	80.7	65.6	68.0	41.8	46.8	36 5	37 2
3	74.5	80 4	70.6	78.1	74.4	80.1	69.6	67.7	39 9	46.3	32.9	37.1
4	81.8	80 8	70 3	78 9	70.3	79.9	72.9	69 8	39 0	45.9	23.6	35.1
5	78.0	81 7	72.9	78 7	73.3	78 9	68.9	70.0	40 6	44.9	30.3	34 5
6	68.5	77.5	77.3	79 1	72.1	78.1	61 1	70.5	48.8	45.8	32.9	35.0
7	69 4	76 7	76 0	78 8	71.0	77 3	56 0	68.5	51 9	47.5	31 6	34.4
8	70 6	75.1	76 0	79 1	63.4	76 8	53.0	66 2	45 4	47.5	27 0	34.3
9	73 9	75 6	69.3	77.5	56 6	75.5	61 4	63 6	40 0	47 8	32.5	34.7
10	77 9	78 3	69.9	77 4	57.4	71 3	59 4	62 6	43 0	48 0	35.6	35 5
11	80 1	80 8	72 3	78 1	61.4	70 3	56 5	61 2	44 8	47 7	27 0	35 6
12	81.1	83 4	72 4	78.8	64.1	69.7	55.8	59.1	40.5	47 5	22.0	34 9
13	79 3	84 9	73 0	79 4	68.9	71 2	51.6	59.3	33 6	46 1	33 6	34 0
14	77.3	83 9	76 1	79 8	71.8	72 1	40.4	55 2	20 0	42 2	28 4	34 0
15	75 1	82 5	69 9	80 3	66.5	72.6	45.5	55.3	27.5	38 7	21 6	33 1
16	80 5	82 5	74 6	80 1	65 8	72 2	45.5	54.8	38 9	37.9	22 3	33 2
17	82.3	83 7	73 3	81 2	63.9	71.3	39 6	51 9	49 0	40 2	30 0	33 5
18	78 4	83 5	75.9	81 8	57 9	69.3	42 0	49 9	40 5	42.6	31 8	34.0
19	76 8	83 5	78.8	81.8	61 5	67.6	57 6	52 0	27 0	40 2	22 0	33.1
20	76 5	83.5	68 3	80 7	60.0	66 5	57.4	54.4	22.5	37 0	18 6	33.7
21	76 1	84.6	69 4	77.5	62.1	65 4	51 4	54 1	30.3	36 0	26 3	33 5
22	76.1	84 3	68.5	76.3	68 0	65.8	39.4	52 0	41.4	36 9	22 1	33 2
23	77 3	83.6	69 1	77 3	63 3	67 9	36.5	49.2	42.1	38 7	23.3	33 0
24	76.3	83 8	71.3	77 6	52 9	68 5	41 0	48.5	38 8	39 5	16 5	33.0
25	74 5	83 8	71 8	78 8	55.9	66 7	43.3	48 0	38 7	40.7	24 0	33.0
26	77 1	83.3	72.9	79 8	62.3	65 8	45.3	48.3	36 3	41.0	16 6	33.0
27	70.5	81 9	74 0	82 6	60.9	66.3	34 5	46 8	31.9	40 0	15 8	33 0
28	69 4	77 0	75.4	80 8	56.9	66 4	35.3	46 0	25 9	37 9	26 6	33 0
29	71 3	76 9	76 3	81.3	63 4	66.3	42 8	46 0	30.1	37 2	21 3	33 0
30	73.0	77.7	76 5	80 9	64.5	66 5	47.0	46 5	35 5	36 5	24.9	32 9
31	73 0	78 6	76.4	80 9			48.0	47 5			26.9	32.7

Appendix B. Average Daily Air and Illinois River Water Temperatures for Havana, 1967-1969

(Temperatures in degrees Fahrenheit)

Day	January air	January water	February air	February water	March air	March water	April air	April water	May air	May water	June air	June water	July air	July water	August air	August water	September air	September water	October air	October water	November air	November water	December air	December water
1967																								
1	28.3	33.7	32 5	38.0	30 0	36 0	60.0	56.0	65.0	56.0	57.7	62.3	86.0	79.0	79 7	78.0	60.0	70.0	62 0	61.0	50.3	50.3	34 0	38.0
2	31.0	33.3	25.7	38.7	53.0	37.0	66.0	55.7	50.3	56.7	65.7	62 0	72.3	79.3	78 3	80.0	60.3	70.0	69.0	63.0	45.0	51.0	36 0	37.3
3	25.7	33 3	21 0	34 3	39 0	41.0	50.0	56.7	47.7	57 3	69.7	63 7	63.0	79.0	76 0	82.0	64.3	70.3	70.3	63.7	36.3	50 7	35 3	37.7
4	25 3	33 0	34 2	35.3	32.3	38.0	53 7	56 7	49 3	56 3	69 0	66 7	60.0	77 3	71.5	82 3	68.0	70 3	71 0	64.3	30 0	49.0	33 0	38.0
5	23 0	33 7	31 0	36 3	31.0	34.0	68.0	57 0	42 0	54 0		69 0	63.7	75 7	71.0	82 0	66.7	70 3	65.7	66.0	31.3	46.3	42 0	37.0
6	36 7	34 3	15.0	35.0	27 7	34.0	68.0	57 3	45 0	54 0	73.0	70.7	66.7	73 7	69 7	81.7	68.3	70.3	57 3	67.3	33 0	44.3	46.3	39.3
7	25 7	36.0	12 3	35.7	28.3	35 0	48.7	57.3	52 3	53 0	73.7	73.0	71.7	75.0	76 3	80 7	68.7	71.7	61 0	68.0	36 5	43 0	42.3	40 7
8	19 3	33.3	19 0	36.7	20 7	35 3	48.7	57.3	56 0	54 7	79 0	74 7	76.3	76 0	71 3	80 7	67.3	72.7	51.7	67.0	40 0	43.3	39 3	40 7
9	25 3	37 0	33 0	36.7	36 3	37.0	62.3	56 0	55 0	54 0	76 7	77 0	71.7	77 3	74.0	80.0	66.3	73.7	48 7	64.0	52 3	42.7	40 5	42 0
10	25 0	36.7	38.3	37.7	56 0	37 3	50 0	56 3	58.7	54 7	70.0	77.5	75.7	78 3	64.5	79.3	59.3	72 7	41.0	61.3	49 0	44.3	38 0	41 7
11	21.7	37 0	24 0	35 7	48 3	39 0	42.3	56.0	58.0	55 7	74 3	76 7	78.7	80.0	63.7	78 0	63.3	71.7	44 7	60.0	53 3	46.0	36.3	42 0
12	37.0	36.0	19 0	34.3	38.0	41.3	44 7	54.3	46.8	56 3	74 0	77.7	77.0	81.7	62 3	76 7	62.7	70.7	50.3	59 7	49.7	46.0	38 0	42 0
13	37 0	39 3	37 3	36.7	41.7	43 0	64 0	54 0	55 0	56 5	74 7	77.7	65 7	80 3	65 0	76.7	71.0	71.7	59.3	59 7	33 3	43 0	35 0	42.3
14	32 3	36 3	51 3	35.7	37.3	43 3	66 3	55 7	56 3	56 3	80 3	80 0	65.7	78.0	67.0	72 7	70.7	72.3	59.3	59 0	40.0	46.3	30 3	42.0
15	19 7	34 3	39 7	40 3	31.3	41 7	67.3	57.0	58 3	57 3	83.0	81 0	69.3	77.3	70 7	76 3	67.0	72.3	63.7	59 3	32 7	45.3	23 7	40 5
16	27 3	38.0	12 3	37 7	28 0	42 0	75 0	58 7	59.7	58 7	78 0	82.3	71.0	76 0	75.5	76.7	65.7	71 3	54 0	58.0	40 0	44.3	28 0	40 0
17	10 0	33 7	23 3	35.0	24 0	40 0	63 0	61 0	62 0	59 7	72.7	82.0	73.7	76.3	78 0	77 7	67 0	70 7	58 0	57 8	42 7	44 7	38.3	38 0
18	3 3	37 0	29 3	35 0	23 0	39.5	47 7	60 0	72 3	59 0	70 3	81 0	73 7	76 7	69 7	78.0	70 7	71 3	50 0	57.0	44 3	44 0	35.0	37 3
19	19 0	39 0	34 3	35.0	32 0	38 7	53 8	59 3	67 3	64 0	74 3	79.7	69 3	76 7	60 7	76.3	69 7	72 3	48 0	56 7	33 5	45 0	35.3	40 7
20	38 7	38 3	22 0	34.0	40.7	38 7	52 0	59 0	56.3	65 0	78.3	79 7	74.7	77 0	64 3	75 0	69.3	74 0	50.7	55.3	37 3	44 3	46 0	40 7
21	41 5	34 0	19 3	34 0	39.7	38.7	68 7	58.3	58 0	63.0	72 3	80 0	75.3	75.3	67 0	76 0	68.0	74.3	52 0	54.3	41.7	43.3	41 7	38.7
22	49 7	35 7	26 3	34 0	43 0	40 7	47 3	57 7	58.7	63 7	72.3	79 0	75.3	78.3	67.7	74 7	60.3	73 0	60 0	54.0	31 0	42.0	18.3	39.7
23	55 0	40 3	19 0	34 0	51.0	40 3	35.7	57.0	61.7	64.3	74 0	79 3	82.0	80 0	69 3	73.0	59.3	72.3	63 0	54 3	32.3	41 7	21 0	37.3
24	56 0	40 7	4 7	33 0	59 0	42 7	40.3	54.3	74.3	65 0	75.3	79 0	74.3	77.7	68.0	74.3	57.3	70 7	58 0	55 0	38.0	42.0	29 0	35 0
25	31.0	42 7	9 7	34 0	63 0	44 7	42 7	53 7	80 0	67 0	65 3	78 7	76 3	78.3	73 0	75 0	58.7	69.3	44 0	54.0	45.0	40 7	26.0	33 3
26	28 2	40 0	23 0	33 0	58 7	47 0	44.0	52.3	78.0	68 7	67.7	78 7	78.3	80.0	68 0	75.7	64 3	69 0	37.7	52.7	36.7	41 0	14 0	32.3
27	26 3	37 0	33 0	33 0	46.3	48.3	49.5	51 5	80.3	70.3	69 3	77.3	81.0	81.0	62 3	73 7	56.7	66 3	35.7	52.0	26 0	39.3	16 3	33 7
28	23.2	36.0	33 7	34.0	52.0	49.7	54 0	52 0	66 0	70 7	67 0	76 7	79.7	79 7	68 0	74 0	45.3	64.7	37 7	49 7	30 0	39 3	19.3	33 7
29	30 5	34.0			57.7	51 3	56.3	53.7	57.7	71.7	72.8	77.3	77.3	77.3	69 0	74 7	48.0	62.7	49.2	48.0	33.0	39.7	24 3	33 7
30	27 3	37 0			63.3	52.7	63 0	54.3	52.7	68.3	75.3	77.3	78.5	78 0	64.7	73.3	56 0	61 3	51.3	49 0	33.3	38.0	28.0	32 7
31	32.7	36.7			65 0	55.3			57 3	64 3			73.3	78 0	59 3	69 7			50 3	48.3			6 3	32.7
1968																								
1	-0 3	32.2	49.7	37 3	34 7	35 0	47 0	58 0	69.7	63.0			75 0	73 0	63 3	78.0	65 3	74 0	70 7	69.3	66 0	53.3	33 3	41 7
2	19 3	33 3	34 3	39 0	33.0	35 7	44.5	56 5	68 3	64 7	68.0	60 7	69 3	75 0	67.0	79 0	68 7	73.7	65.5	69 7	49 3	53 7	39.7	40 7
3	6 7	33 7	32 0	40 3	27 0	36 0	55.3	55.7	70.5	67.3	76 0	68.7	65 7	74.7	74.0	77.7	75.3	73.7	53.0	69.0	44 7	52 3	38 3	41 0
4	12 3	34 0	36.7	39 7	36.0	35.3	45 0	54.3	58 3	66 7	77 3	71 3	68 0	75.0	75.0	79 2	73 3	74 3	49.7	65.7	50.7	52 7	38 3	42 0
5	12 7	33 0	36 7	38 0	43.3	37.3	37 3	53 3	51.0	65 7	76 7	73 7	72 3	77.7	81.0	79.3	67 3	74 7	48 0	65.3	46.3	52.3	32.3	41.3
6	10.7	33 7	33 7	38.3	36 0	37.7	41 0	51 3	56.0	65 7	78.3	75.7	73.0	75.0	84.3	82 0	64 0	74 0	52 3	63.0	45 3	52.7	27 3	40 7
7	-1.7	33 3	26 0	37.3	42 7	40.3	56 7	50 3	60 0	64 3	77.0	78 0	70.3	75 3	83.7	84 7	66 7	73 3	53.7	60.3	43 0	52.7	24.0	39.7
8	4 3	34 0	29 7	37.3	43.0	41 3	57 3	52.3	68 7	64 3	79.0	79 3	74 0	76 3	82.3	85.7	65.3	72 0	55.3	60 0	34 3	51 3	21 3	37.3
9	20 7	34 0	25 7	35.7	50 7	42.0	58 7	54 0	58 0	65 3	83 3	81.7	76 7	76 7	79 0	86 3	59.0	70 7	54 0	60 7	31 7	49 0	27.7	37 3
10	13 7	37 0	13 7	34 7	39 3	43 0	56 3	55 0	53 7	63 3	85 0	82 7	71 0	71 0	71.0	87.0	60.3	68 7	52 0	60 0	32 3	48.7		
11	15 7	33 7	14 0	32.7	38 0	48 0	61 3	55 3	58 0	64.3	77 3	83.7	71 3	71 3	68 3	82.0	59.0	68.3	61 0	61.3	33.0	54.3	36.7	36 7
12	24 3	33 7	18.3	32.0	34 3	41.0	69 3	57.3	64 3	65.3	71 8	82 7	73 3	77 0	66 3	80 3	62 7	69.3	62.3	61.3	33 7	46 3	51 7	40 7
13	28.0	33 0	16 7	33 0	30.0	42 0	62 0	58 0	63 0	66.7	70 3	81.3	80 3	78 3	73 7	79 7	68 7	68.7	67 0	62 0	33.7	45.0	25 3	38 3
14	24 0	33 7	19 0	33.0	37.7	41 0	54.0	58 3	72.3	67 0	71.7	80.0	78.0	80 7	72 0	80.3	67.3	68 7	70 7	62 7	47 0	45.3	13 0	36.3
15	27 7	33 7	31 7	33.7	45.5	41 0	50 7	57 7	68.0	69 0	71 0	78 3	81.3	80.7	75.7	80 0	67.3	70.3	76.0	66.0	48.0	45 3	14.3	36 0
16	13.7	34 0	33 0	33 3	48 3	43.0	54 7	57 3	59 0	70 0	66 7	77 0	78 0	82 0	77 7	80.0	65 0	70.7	70.3	67 3	42 7	45.7	27 0	39 7
17	23 7	34 7	15 0	32.7	54 0	44 0	59 7	64.3	56.7	66 0	69 0	75 3	79 7	82.3	78.5	82 3	65.7	70.0	66 3	69.0	41.7	45 7	25 5	36.0
18	35 3	34 7	18 7	33 0	52.0	45 7	56 3	58 3	53.0	65.0	72 7	76.0	79 7	82 3	75 3	83.7	58.0	70 0	53.3	67.0	36 0	45 3	37 0	35.7
19	32 7	35.3	28 3	33 3	56 3	45 3	53 3	58 0	52.0	62 0	78.0	76.3	78 3	82.7	84 0	82 3	61 3	68 7	53 3	65.7	31 0	44.5	38 3	36 0
20	38 3	35 3	23.7	33.0	36.3	47.3	64.0	60 0	54.7	60 7	77.3	78.0	74.3	82.3	85 0	84 0	66 3	70 0	55.0	62.7	27 7	43.3	27 3	35 0
21	37 7	35 3	8	33 5	31 3	48.0	62 3	57 7	57.7	60 0	82.3	78.7	79.0	83 0	84 3	83 7	71 0	68 3	60.7	64.5	41 3	43.0	27 0	34 7
22	31 0	35 0	15 3	32.7	31 0	46 7	66 3	59 0	54.3	62 7	80 3	80.3	78.5	83.5	83.3	86 0	74 5	68.0	56 8	61 0	43.3	42 3	33 0	34 3
23	36 7	35 0	21.0	33 0	30 0	44 3	52 7	59 7	53 0	60 3	81 7	82 3	77 3	85 0	84 7	87 7	70 0	70.7	57 7	58.7	51 7	42 3	21 0	34 3
24	20 0	35 0	26.0	33.7	43 7	42 0	43 7	59 3	54 5	63 0	81 7	83.0	77.3	85.3	81 7	86 3	69 0	71 7	44.3	58 0	42 5	43 0	16 0	32 3
25	28.3	34 7	30 0	34.3	49 3	43 3	49 7	59 0	56.7	60 0			74.0	84 3	73 3	83 7	63 0	71 3	41 0	55 7	39 3	43 3	14 0	32 3
26	34 3	34 0	33 0	34 7	60 3	45 0	53 5	55 0	60 7	61 3	74 5	83.3	72.3	84.7	68.7	83.7	61 0	71 3	46 0	54 7	46 3	44 3	27.3	33 7
27	47 3	34 7	33 3	35 0	66 3	49.0	52 3	54 0	55.3	60 3	56 3	72.7	77.7	84.0	66 3	78.7	62 0	70.7	53 3	54.7	38 0	45 0	45.3	32 7
28	44 0	35 3	31 7	35 0	64 3	53 3	59 3	56 0	60 7	61 7	71 3	70 7	72 0	82 7	67 0	80 3	60 7	69 0	41.0	53 7	39 0	44 7	31 7	32 0
29	48 7	35 7	24 0	35 0	69 0	56.3	61 3	58 7	60 0	62 0	80.0	69 3	68.3	80 7	66 7	75.0	62 0	68 3	41.3	52.3	35 0	44 0	25.7	32 7
30	33 7	36 0			64 7	58.3	68 7	60 7	59 3	62.0	82.7	71.0	68.0	80 3	69 0	74.7	67 0	68 7	46 3	64.0	29.7	42.7	0	33 3
31	44.7	35.3			56 7	59 3			64 7	63.0			72.3	81.0	65 3	75 0			62 7	52.0			32 0	33 3
1969																								
1	3 7	32 3	27 3	33 0	33 3	40 0	51.7	42 7	62 0	57.7	60 3	74 3	72 7	79.3	69 3	80 7	72.0	83.7	69 3	70.0	43 7	50 7	37.7	39 3
2	20 3	33 0	28 7	33 0	34.0	41 3	47 7	42 7	70 0	60.3	53.0	70 7	73.1	78.3	70 3	78 7	70 0	81 0	66 0	70 3	42.7	50 0	39 0	38 7
3	6 6	32 0	18.3	32.7	35.3	42.7	48 3	44 7	73 7	61 3	58.7	67.3	77.0	79 7	72 5	79 0	69 5	82.0	71 7	71.0	41.0	50 0	34 7	39 7
4	2 5	33.0	29 0	33 0	35 7	42.3	58.7	46.7	73.0	64 3	63.7	68.3	82.7	81.0	69.7	78.7	72 0	82.0	72 3	72 0	41 3	48 7	26.0	39 0
5	17 0	32 0	34.0	32 5	33 3	42 7	51 3	46 7	73.0	64 7	71.0	66.7	80.7	83.0	74 3	79 7	74.3	81 3	69 3	72.3	43 7	49 0	27.3	40 0
6	25 5	32 7	37.7	33.0	35.7	44.0	48.0	48 0	65.3	67.7	78 0	70 0	75 3	82.0	77.3	81 3	76.3	81.0	62 3	72.3	47.3	49 3	32.3	39.0
7	10 3	32 7	35.5	33.3	35.3	44.0	58 7	50 0	71 7	68.8	71.7	71.0	72 7	80 0	79 0	80.3	72.3	80.0	58.8	70.7	49 7	50 7	32.0	38.0
8	25.0	33.0	34 3	33.0	27 7	44 0	65 7	52 3	57 0	69.7	64.3	71 7	72.3	79.7	80.7	82.0	65 7	78 7	58 7	69.3	47.0	49 3	29.3	39.3
9	11.0	33 3	30 3	33.0	22.0	42 0	66.0	55.3	53.5	68 0	63 0	69.7	75 7	77.3	76 3	82 0	60.0	76 0	62.7	69.0	39.3	49 7	34.0	39.0
10	7 7	32 3	33 7	33.3	24.3	41 7	66.0	57.0	49 7	65 7	69.3	69 3	79 3	77 0	71 7	78.7	60 0	75 3	61.0	67.0	41 3	50 0	34.3	41 0
11	14.0	32 3	32 7	33.3	22 0	40 3	53.0	57.7	51.3	63 3	77.3	70.0	81.0	77.3	72 7	78.7	63.3	74 0	57.0	64 7	44.7	49.7	28.3	39.7
12	14 0	33 0	22 0	32.7	24.7	38 7	53.7	57.7	54.0	61.3	78.7	73.0	81.7	79.3	73 0	79 7	64.3	73.3	55 0	66.7	41.7	50 7	24.0	41 3
13	25.3	33 0	21 7	33 0	26 7	40 0	56 7	58 0	55 3	61 0	66 0	74 0	81.7	83 0	71.7	80 7	68 0	73.3	53.0	60.7	37.7	50 0	36.0	38 7
14	23 3	33.0	25 0	33.3	30.3	40 7	55.3	58.3	64 7	60 3	60 3	72 5	78.7	85.0	75 3	81.3	73 7	74 0	43.3	59 0	21 0	47.7	31.7	37.7
15	29.0	32.3	27 0	33.7	31.7	39 7	62 3	59.0	70.3	63.0	63.0	70 0	77.0	85.0	77.0	82.7	68 0	74 3	45.7	57.3	29.3	45 3	24.0	39 0
16	36 7	33.7	29 5	33 0	37.7	40 7	67 3	59 7	72 7	65.3	65.7	69.7	78 7	85.5	73 0	82.7	65 7	73.7	47.7	57.3	41.7	43.3	23.7	38 7
17	33 7	33 7	32 0	33 7	49 3	43 0	65 0	61 3	67.3	68 0	66.0	69 7	85.5	86.0	74 3	84 7	65 7	73.0	43.3	56.3	50.0	44.3	30.0	37.3
18	33 0	33 7	30.7	34.3	55 3	44 0	45 3	62.0	51 0	67 7	67 5	70 7	84.0	85.7	78 3	83.0	60 7	73 3	44.3	55.0	41.0	44.7	33.7	38 7
19	29 7	34 0	29 0	34 3	54 0	47.0	48 7	58.3	58 0	66 3	77 0	72 7	79 0	85.0	80 7	84 7	62 0	71.7	57.7	54 0	26.7	42.0	24.0	38 5
20	32 0	34 3	29.7	35 3	48 0	53 0	54 0	56 3	64.3	66 0	69.3	72.7	77.7	85.0	70 0	83.7	62.3	68 3	61.0	55 0	24 3	42 7	19.3	35 3
21	34 0	34 3	36 7	34 7	39.3	48.0	59.5	57 5	56.8	65.0	61 1	72 3	77 7	84 3	70 3	80.0	64.3	70 3	54.3	54 7	30.3	42.3	27.3	36.7
22	39 0	35.0	36.0	37.0	47.7	48 7	54.7	56.3	50.0	65 0	66.7	72.3	78.3	84.3	68.7	80 7	69 3	69.7	45.0	55 7	42.7	41.7	23 0	37.3
23	38.0	34.0	36 3	37.3	52.3	50.0	48 3	56 0	55 3	63 7	67 0	71 3	77.3	85 3	72.3	80.7	65.5	69 7	39 3	54 7	44 0	41.0	25 0	36.7
24	11.0	33.0	36 5	36 5	41 3	47 7	55 0	56.3	61.0	62.3	67 0	71 7	78 3	86 0	71.0	79.7	55 0	70 3	41 3	53.0	27.5	40 0	18 3	34.3
25	11.7	32.0	38 0	38.3	35 0	46 0	59 0	56.7	67.3	63.7	76.0	71 5	77.3	85.0	71 3	80.0	48 0	68 0	46 7	52 3	41 3	41.7	25 3	34.7
26	12.0	32 0	35 5	39 7	32 3	46 3	66 7	58 0	67 0	66 3	84 3	74 7	81 3	85.0	75.0	80 0	66 0	69.3	48.3	52 7	33.7	41.7	18 0	36.7
27	13.7	32.0	34 7	39 7	37 7	45 7	61 7	59 0	76 0	68.3	81 0	76 7	73.7	84.3	74.0	83 0	62 0	69 0	35.3	51 0	33 0	41.3	14 3	34 0
28	34.3	32.7	33 0	40.0	42.3	43.0	46.3	64.7	77.3	70 7	82.3	79 0	70.3	82.3	75.7	82.7	60.7	69.7	36 3	49 7	26 7	40 7	27.3	33 7
29	35.7	33.0			26 0	44.0	49 0	57 3	79.3	72 0	80 0	80 0	73 0	80 7	74 0	83.0	62 7	68.7	43.7	49 0	32.7	40 3	27.3	36 0
30	31.0	33.7			25 0	44.3	52 0	57.7	75.7	73 3	80 8	81 0	75 0	80 0	77 0	84 7	65 0	68.7	47 3	49.7	35.3	39.7	25 0	39.3
31	21 0	33 3			27.0	42.7			70.0	75 0			76 0	80 5	76 7	83 7			50 3	50 3			24 7	35 3