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ILLINOIS STATE WATER SURVEY
METEOROLOGIC LABORATORY
at the
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Urbana, Illinois

# RAINFALL DROP SIZE-D3STRIBUTION AND RADAR REFLECTIVITY

by

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SUMMARY

This report contains the results of a tvo-year study on the size and number of raindrops as observed in central Illinois. The raindrop samples have been classified as to the rainfall type which produced the raindrops. Within the limits of accuracy of the measuring device, the Z-R relationships have been found for all storms which had at least seven minutes of data collected. A general Z-R relationship using all 1211 one-minute observations was found to be

$$z = 396\ R^{1.35}$$

where R is the rainfall rate in mm $hr^{-1}$ and $Z = \sum Nd^6$ in mm $m^{-3}$ . The 1211 one-minute observations were further classified into rainfall types, and a general Z-R relationship for each of three rainfall types was determined. For thundershowers it was found to be

$$Z = 486\ R^{1.37}.$$

For rainshowers it was found to be

$$Z = 380\ R^{1.24}.$$

For continuous rain it was found to be

$$Z = 313\ R^{1.25}.$$

Because it was desired to determine the rainfall over an area from radar power measurements, it was decided that the preferable relationship to use was

$$R = R\ (Z).$$

When the observed rainfall rate-reflectivity points from the 1211 one-minute observations were plotted on log-log paper, it was noted that the higher rainfall rates tended to cause the relationship between log Z and log R to be non-linear. Accordingly, the following second-order logarithmic equation relating log Z and log R was derived from the data

$$\log Z = 2.569 + 1.317\ (\log R) + 0.072\ (\log R)^2 .$$

The drop size distribution as exemplified by the radar reflectivity, Z, was examined for its changes during the passage of time. It was found that, in general, the mean drop size tended to increase just prior to an increase in rainfall rate and it decreased as the rainfall rate decreased. It was also found that there was considerable variability in drop size-distribution which could not be explained by rainfall rate changes.

Expressions were found relating liquid water content (W) and median volume diameter ( .50) with rainfall rate. These expressions are

$$W = .052\ R^{.97}$$

and

$$d_{50} = 0.80\ R^{0.34}.$$

The observed rainfall rates, raindrop shape, and number of raindrops were used to determine a more exact relationship between rainfall rate and Stevenson's radar back-scattering function, $\sigma$ . For a continuous rain the $\sigma - R$ relationship was found to be

$$\sigma = 0.072\ R^{1.32}.$$

For a rainshower the $\sigma - R$ relationship was found to be

$$\sigma = 0.168\ R^{1.43}.$$

For a thunderstorm the $\sigma$ - R relationship was found to be

$$\sigma = 0.198\, R^{1.56}.$$

In general, the scatter of points relating $\sigma$ - R is greater than is the scatter of points relating Z-R.

It is shown that the inherent variability about the mean of the drop size distribution within an individual rain, in addition to the variability of the mean drop size-distribution between rains, limits the possible determination of rainfall rate by the Rayleigh approximate scattering function to the, range 0.5 - 2.1 mm per hour at $Z = 4 \times 10^2 \text{mm}^6 \text{m}^{-3}$ and 17-140 mm per hour at $Z = 2 \times 10^5 \text{mm}^6 \text{m}^{-3}$.

ACKNOWLEDGMENT

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The manuscript vas reviewed by D. M. Swingle and W. J. Richards of the Signal Corps Engineering Laboratories and H. R. Byers, Head, Department of Meteorology, the University of Chicago.

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## INTRODUCTION

Interest in raindrop sizes has developed sporadically through the years since about 1895. The earliest work appears to have sprung purely out of scientific curiosity while later interest was generated by particular problems. Thus, Laws and Parsons[9] studied the effect of rainfall on soil erosion, and more recently a large number of investigators, Marshall and Palmer[10], Boucher[4], Bowen[5], Cooper[6], Best[3], Anderson[1], Hood[7], and Wexler[15,16], have correlated rainfall rate and raindrop size and number in an attempt to relate the microwave power return from precipitation to the rainfall rate. For the most part these investigators have concerned themselves with the less intense rates of rainfall. The investigation in this report was also undertaken to further explore the relation of raindrop size and number to the rainfall rate. In addition, the two-dimensional shapes of the raindrops have been observed.

The sampling procedure was designed to explore the short time variations in raindrop size and number as related to rainfall rate, the variations in drop size-distribution from storm to storm, and to determine a general relationship between drop size-distribution and rainfall rate. In order to facilitate the application of the results to the measurement of rainfall rate with radar, the drop size-distributions have been reported as Z-R relationships; i.e., the relationship of rainfall rate to $\sum N d_i^6$, where N is the number of drops in the diameter class interval, d., and all class intervals are summed. Emphasis has been placed upon the measurement of raindrop size-distribution in the heavier rates of rainfall, particularly in thunderstorms, since few observations of high rainfall rates have been reported by previous investigators.

## APPARATUS

When methods of obtaining data were reviewed at the beginning of the study, it became apparent that the methods used by previous investigators were not easily adaptable to the measurement of raindrop sizes in heavy rates of rainfall. A literature search revealed that Laws[8] had photographed raindrops as they fell. He used a very short exposure to reduce blurring of the drops due to movement and a special type of optical system to eliminate the apparent change in size of an object with increasing distance from the camera. The raindrop images used in this study have been recorded with an improved design of his instrument. The instrument has been described in Research Report Wo. 3[7] under an earlier contract. Major features of the instrument include: a 35-mm movie camera triggered by an intervalometer to take pictures 1/3 second apart for 12 seconds of each minute of rainfall, a lens and mirror system forming a telecentric optical system to eliminate normal perspective and form an image which could be projected to a size large enough to be measured with vernier calipers, and a flash tube light source of 10 microseconds flash duration synchronized with the opening of the camera shutter. The date and time were recorded on the film with the raindrops. A schematic drawing of the optical system is shown in Figure 1 while Figure 2 is a photograph of the instrument. An example of one of the photographs obtained with the instrument is shown in Figure 3. Shown on the photograph are two clocks, a day counter, and the silhouettes of raindrops.

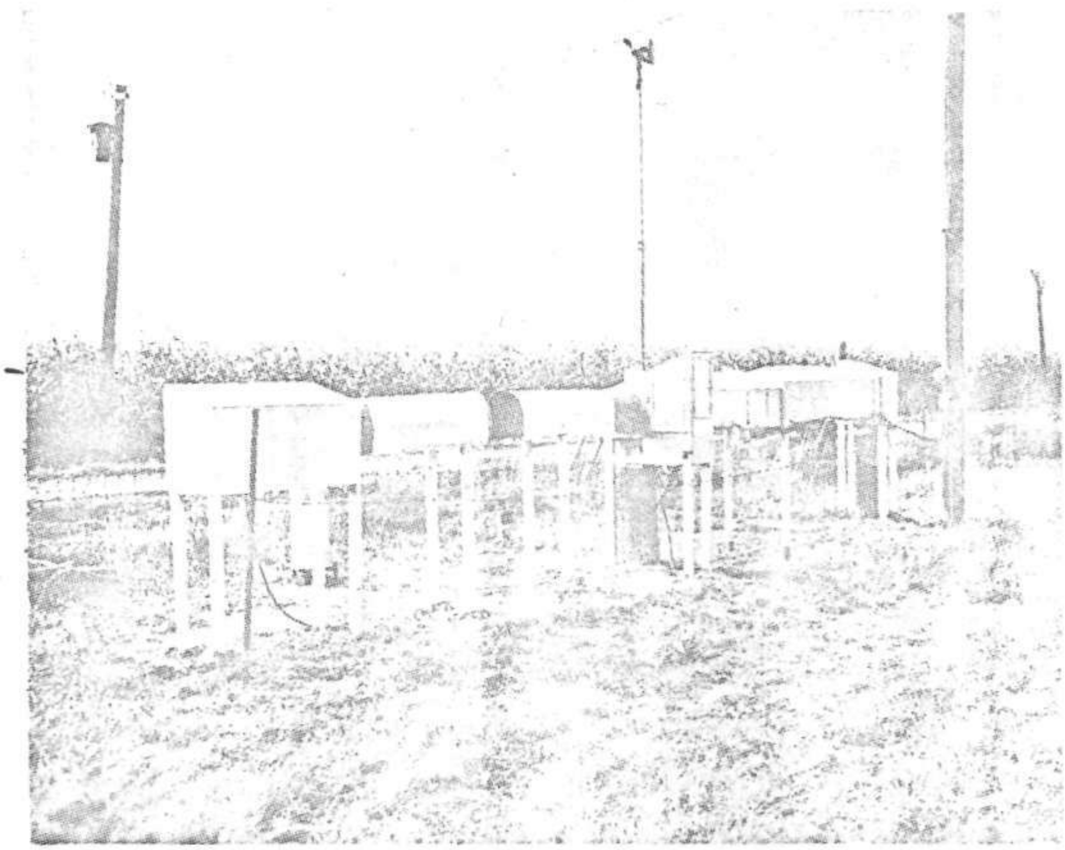

FIG.2 THE RAINDROP CAMERA SHOWING FROM LEFT TO RIGHT: FLASH HOUSING, SAMPLING VOLUME AND CAMERA AND OPTICS HOUSING

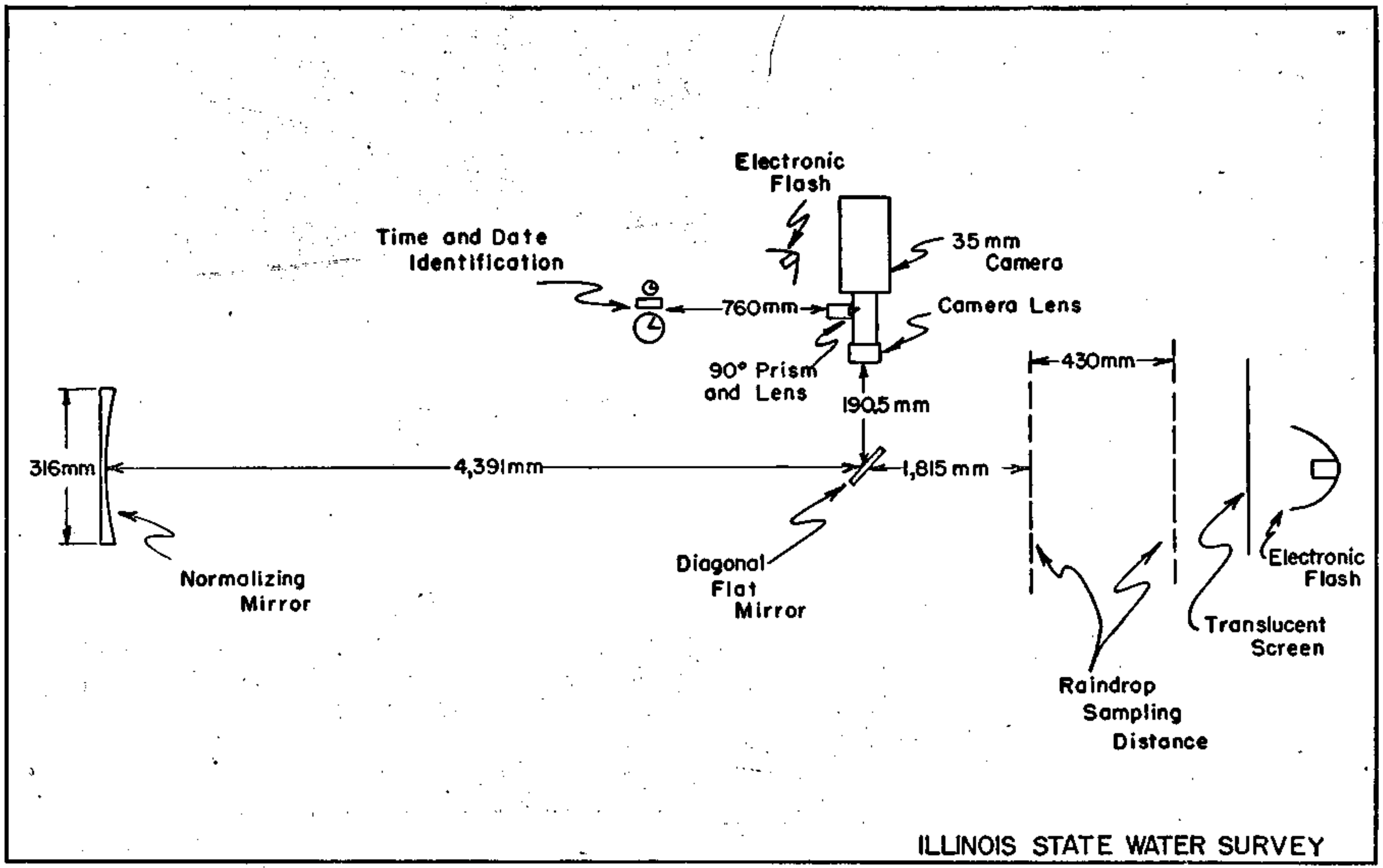


FIG. 1 RAINDROP CAMERA OPTICAL SYSTEM
(Not To Scale)

# DATA REDUCTION

The sizes of the raindrops were obtained from the photographic records by optically enlarging the photographic images to twice the normal size of the raindrops and carefully measuring them with a pair of vernier calipers coupled to a Streeter-Amet recorder by a flexible shaft. The optical system was designed for an accuracy of measurement of ± 0.15 mm through a depth of 17 in. for drops 0.5 mm and larger with a lens aperture of f/18.0. In August 1954, a series of photographs was made of known-size zeolite beads ranging from 0.4-mm diameter to 1.3-mm diameter with the lens opening set at f/4.5, the standard opening used for all data collection during 1953-54. For beads of 0.5-mm or greater diameter a measurement accuracy of ± 0.05 mm was found to be attainable.

Since it was found in August 1954 that the expected accuracy of ± 0.15 mm was exceeded and that an accuracy of 0.05 mm could be achieved, a change was made in the class intervals used in the data. Those storms preceding 26 August 1954 were classified by 0.3-mm intervals while the storms for 26 August 1954 through 3 January 1955 were classified by 0.1-mm intervals. Comparison of the same data classified by both intervals indicates that the shift from the 0.3-mm interval to the 0.1-mm interval causes approximately an 8 percent decrease in the coefficient of R and an approximate 2 percent increase in the exponent of R. Within the range of rainfall rates covered by this investigation, the change in class intervals causes no more than a 10 percent change in Z.

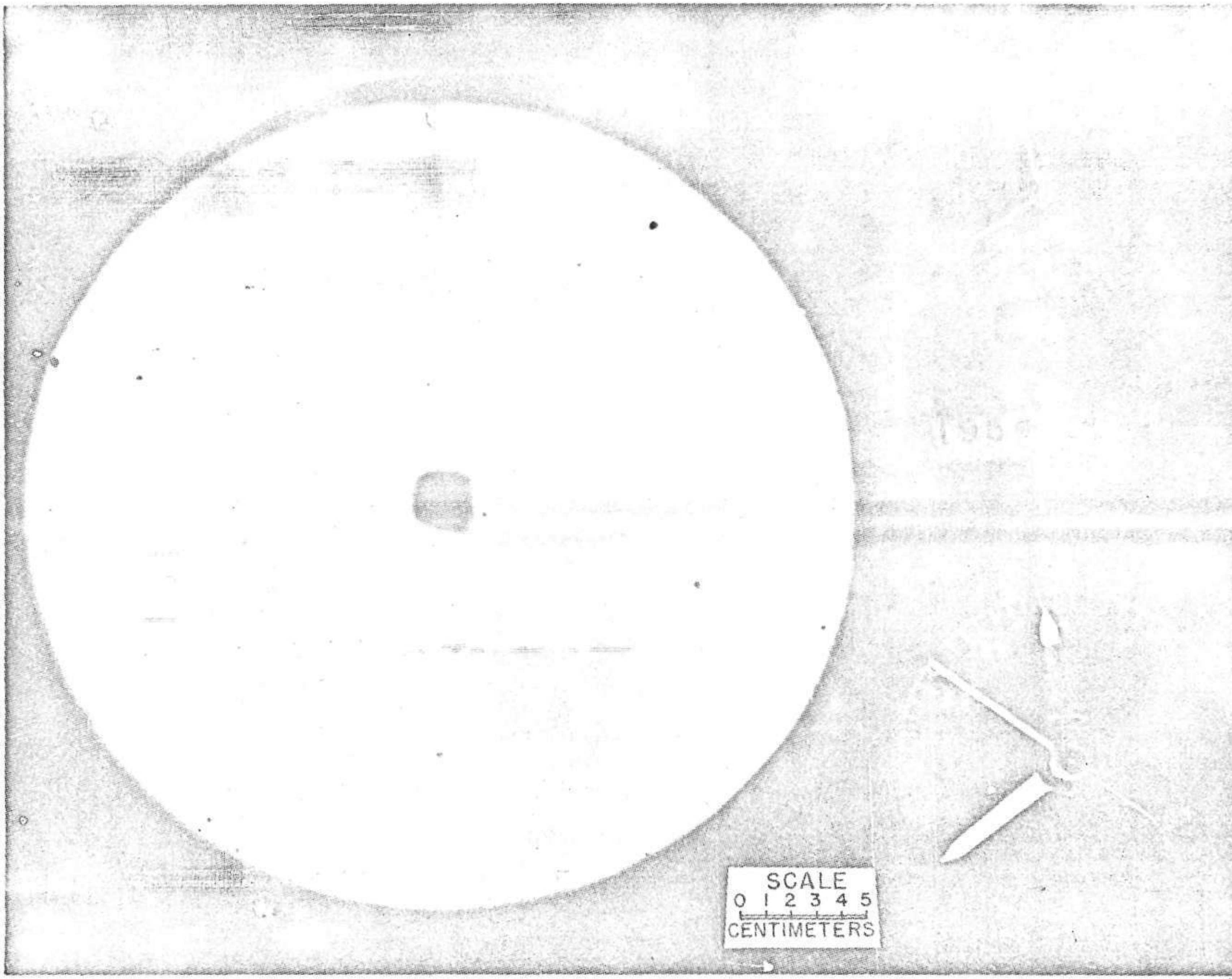


FIG. 3 RAINDROP PHOTOGRAPH

## OBSERVATIONS

Rainfall rates as high as 157 mm per hour and as low as 0.004 mm per hour have been calculated from the photographs. Observations have been made in thunderstorms with hail, high level thunderstorms, continuous rains from cyclonic storms, rainshowers, and post-frontal instability showers. Wind speeds have varied from calm to as much as 50 miles per hour. Samples of raindrops were obtained from storms ranging from light rains to violent thunderstorms. The raindrop shapes varied from small spherical drops of 0.5-mm diameter to large non-spherical and spherical drops of 9-mm diameter.

One observation was defined as a series of 36 exposures in 12 seconds. This was normally repeated each minute. A total of 1211 observations of drop size-distributions from 45 storms during the period from July 1953 to January 1955 has been included in the equations which relate the rainfall intensity to $Z = \sum Nd^6$ for all storms. Whenever possible the samples were made from the beginning until the end of the rain at the sampling site.

## Z-R RELATIONSHIPS

### Choice of Regression of Log R on Log Z

The Z-R relationship used in this report assumes that the rainfall rate is a function of $Z = \sum Nd^6$ where Z is the back-scattering function used in Rayleigh's approximation. Since it is desired to measure the rainfall rate from the radar echo power return, it is assumed that R = R (Z). In general, the choice of Z = Z (R) or of R = R (Z) is an arbitrary one since both variables have been obtained from the same drop size-distribution. Analysis shows that the linear regression of log Z on log R will always have a larger exponent and smaller coefficient of R than the linear regression of log R on log Z for a given set of data. For example the storm record of 18 August 1954 yields

$$z = 429 \ R^{1.43} \tag{1}$$

where R = R (Z) and

$$Z = 405 \ R^{1.47} \tag{2}$$

where Z = Z (R).

The amounts of the change in coefficient and exponent depend on the linear regression correlation coefficient, r, of the logarithms of the two variables. Most previous Z-R relationships reported by other investigators have shown the correlation of Z = Z (R). Because the other investigators plotted Z = Z (R), the Z-R graphs in this report show Z as the ordinate and R as the abscissa in order to make the graphs easily comparable.

Definition of 'Storm'

A 'storm' was assumed to be a period of rainfall which had no complete stoppage of water accumulation at the drop camera site. Table I summarizes 19 thunderstorms, 19 rainshowers, and 7 continuous rains which constitute the 45 storms analyzed. It will be noted from Table I that on several occasions more than one storm occurred on a single day, and that, in general, the Z-R relationships for the two or more storms are not the same. This may be illustrated by the data collected 10 October 1954. The thundershower which started at 2033 CST and continued until'2056 CST yielded the relationship

$$Z = 624 \ R^{1.38} \tag{3}$$

## TABLE I

## SUMMARY OF ANALYZED DATA

| Date | Time CST | Rainfall Type | Air Mass | n | R Max mm $hr^{-1}$ | R Min mm $hr^{-1}$ | r* | Z-R Relationshi] |
|---|---|---|---|---|---|---|---|---|
| 7-16-53 | 1931-2025 | TRW+ | mT | 45 | 31.6 | 0.5 | .98 | $Z = 313R^{1.46}$ |
| 7-21-53 | 1658-1720 | R | mT | 23 | 5.0 | 0.1 | .99 | $Z = 225R^{1.37}$ |
| 8-7-53 | 1745-1801 | TRW+A | mT | 13 | 103-4 | 0.1 | .98 | $z = 686R^{1.49}$ |
| 10-26-53 | 1315-1435 | R- | mP | 40 | 1.0 | 0.1 | .64 | $Z = 179R^{1.01}$ |
| 10-27-53 | 1147-1331 | R | mP | 100 | 9.9 | 0.2 | .92 | $Z = 202R^{1.50}$ |
| 11-20-53 | 0424-0929 | R- | nmT | 40 | 4.3 | 0.1 | .95 | $Z = 391R^{1.40}$ |
| 11-20-53 | 1242-1252 | RW | nmT | 7 | 6.8 | 0.5 | .87 | $Z = 306R^{1.08}$ |
| 3-2-54 | 1713-1732 | RW | ncP | 20 | 2.7 | 0.8 | .73 | $Z = 453R^{1.20}$ |
| 3-19-54 | 1153-1214 | TRW+ | mT+cP | 22 | 14.3 | 0.1 | .95 | $Z = 333R^{1.35}$ |
| 4-30-54 | 2049-2102 | RW | mT | 14 | 4.2 | 0.1 | .99 | $Z = 310R^{1.44}$ |
| 5-2-54 | 0448-0504 | RW+ | cA+cP | 17 | 14.5 | 0.7 | .96 | $Z = 344R^{1.33}$ |
| 5-16-54 | 0944-1013 | RW+ | cP | 17 | 24.6 | 0.7 | .95 | $Z = 374R^{1.31}$ |
| 5-27-54 | 0824-0937 | RW | mT/$_{cp}$ | 15 | 12.0 | 0.4 | .98 | $Z = 313R^{1.34}$ |
| 5-31-54 | 1428-1433 | TRW | mT | 6 | 5.6 | 0.3 | - | ** |
| 5-31-54 | 1559-1624 | TRW+A | mT | 23 | 75.9 | 0.2 | .97 | $Z = 614R^{1.49}$ |

*r - correlation coefficient of log R and log Z
** - insufficient data

TABLE I (cont'd)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 5-31-54 | 2342-0016 | TRW+A | mT | 34 | 157.0 | 1.4 | .95 | $z = 374R^{1.35}$, |
| 6-3-54 | 0316-0339 | RW | cP | 19 | 8.2 | 0.1 | .97 | $Z = 324R^{1.21}$ |
| 6-3-54 | 1517-1526 | RW | cP | 6 | 8.1 | 0.4 | - | ** |
| 7-2-54 | 1158-1249 | RW+ | mT | 30 | 23.1 | 0.1 | .99 | $Z = 395R^{1.33}$ |
| 7-3-54 | 1942-1945 | TRW+ | mT | 12 | 89.7 | 9.0 | .94 | $Z = 1119R^{1.52}$ |
| 7-20-54 | 2214-2242 | TRW+ | mT | 29 | 19.7 | 0.1 | .97 | $Z = 634R^{1.42}$ |
| 7-20-54 | 2356-0047 | TRW+ | mT/cP | 47 | 15.6 | 0.1 | .99 | $Z = 439R^{1.27}$ |
| 7-21-54 | 0758-0914 | TRW | mT/cP | 44 | 7.7 | 0.1 | .95 | $z = 550R^{1.28}$ |
| 8-2-54 | 0808-0850 | RW- | cP | 6 | 0.7 | 0.2 | - | ** |
| 8-2-54 | 1801-1815 | RW- | mT/cP | 14 | 6.3 | 0.1 | .97 | $z = 871R^{1.29}$ |
| 8-3-54 | .2222-2240 | TRW+ | mT/cP | 19 | 52.2 | 0.3 | .91 | $Z = 695R^{1.27}$ |
| 8-3-54 | 2315-2322 | TRW+ | mT/cP | 8 | 13.7 | 0.2 | .98 | $Z = 543R^{1.65}$ |
| 8-3-54 | 2346-0009 | TRW+ | mT/cP | 23 | 48.7 | 0.2 | .94 | $Z = 611R^{1.26}$ |
| 8-4-54 | 0407-0413 | TRW | mT/cP | 7 | 8.5 | 0.1 | .99 | $z = 555R^{1.45}$ |
| 8-4-54 | 0419-0428 | TRW+ | mT/cP | 10 | 19.4 | 0.2 | .99 | $z = 542R^{1.48}$ |
| 8-8-54 | 0553-0611 | R | _ cP | 19 | 2.8 | 0.6 | .95 | $Z = 556R^{1.51}$ |
| 8-8-54 | 1155-1245 | R+ | cP | 33 | 23.4 | 0.2 | .99 | $Z = 218R^{1.21}$ |
| 8-9-54 | 2253-2348 | TRW+ | mT | 56 | 23.7 | 0.2 | .96 | $Z = 418R^{1.47}$ |

TABLE I (cont'd)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 8-11-54 | 0955-1008 | RW | cP | 12 | 6.4 | 0.2 | - | ** |
| 8-13-54 | 0530-0553 | RW | cP | 10 | 6.2 | 0.1 | .95 | $z = 570R^{1.57}$ |
| 8-14-54 | 2335-2358 | TRW+ | mT | 23 | 36.2 | 0.1 | .98 | $Z = 673R^{1.29}$ |
| 8-18-54 | 2146-2223 | TRW+ | mT | 38 | 47.2 | 0.2 | .99 | $Z = 429R^{1.43}$ |
| 8-19-54 | 2224-0001 | RW+ | mT/cP | 59 | 19.5 | 0.1 | .99 | $Z = 413R^{1.17}$ |
| 8-26-54 | 0827-0848 | RW+ | cP | 22 | 44.2 | 0.1 | .99 | $Z = 358R^{1.36}$ |
| 9-19-54 | 2107-2314 | R | mP | 127 | 7.5 | 0.1 | .97 | $Z = 457R^{1.35}$ |
| 9-30-54 | 0701-0730 | RW+ | mT | 25 | 12.9 | 0.1 | .99 | $Z = 249R^{1.31}$ |
| 10-4-54 | 0206-0227 | RW+ | mT/cP | 22 | 25.0 | 0.6 | .97 | $Z = 295R^{1.20}$ |
| 10-10-54 | 2032-2125 | TRW+ | mT | 53 | 106.8 | 0.2 | .93 | $Z = 475R^{1.38}$ |
| 10-10-54 | 2032-2056 | TRW+ | mT | 24 | 106.8 | 0.2 | .94 | $Z = 624R^{1.38}$ |
| 10-10-54 | 2057-2125 | TRW+ | mT | 29 | 45.1 | 0.4 | .94 | $Z = 318R^{1.46}$ |
| 10-10-54 | 2145-2207 | TRW+ | mT | 22 | 34.3 | 0.9 | .98 | $Z = 149R^{1.62}$ |
| 10-11-54 | 0053-0118 | TRW+ | mT | 26 | 32.0 | 0.3 | .98 | $Z = 279R^{1.31}$ |
| 1-3-55 | 1121-1157 | RW+ | mT/cP | 17 | 45.3 | 0.8 | .98 | $Z = 161R^{1.50}$ |

while the second burst of the storm beginning at 2057 CST yielded the relationship

$$Z = 318\ R^{1.46}. \tag{4}$$

Qualitatively, it was noticed in the analysis procedure that the number of large non-spherical drops decreased sharply after 2056 CST. Equations (3) and (4) may be compared with equation (5) which combines the data from equations (3) and (4).

$$Z = 475\ R^{1.38}. \tag{5}$$

A graph of Z vs. R for equations (3), (4) and (5) is shown in Figure 4. It will be noted that there is a tendency for the log R - log Z relationship to become non-linear above $Z = 10^5 mm^6 m^{-3}$. Figure 5 shows the raingage trace of the two bursts of rainfall.

Comparison of Z-R Relationships

In order to compare the Z-R relationships obtained from analysis of individual storms, it is necessary to make the comparison at a given rainfall rate. The greatest differences in the Z-R relationships usually occur at the higher rainfall rates; but some storms did not exceed a 1-mm per hour rate. For this reason, all comparisons of individual storms have been made at a rate of 1-mm per hour unless otherwise stated.

Figure 7 indicates that the maximum variation in Z for all storms is 14.3 db at 1.0 mm per hour and 13.3 db at 100 mm per hour. The variability of the, rainfall rate at $Z = 4 \times 10^2 mm^6 m^{-3}$ is from 0.5 mm per hour to 2.1 mm per hour, and at $Z = 2 \times 10^5 mm^6 m^{-3}$ is from 17 mm per hour to 140 mm per hour. These are extreme values; but the scatter of observed points within these extremes does not include any points which do not appear reasonable.

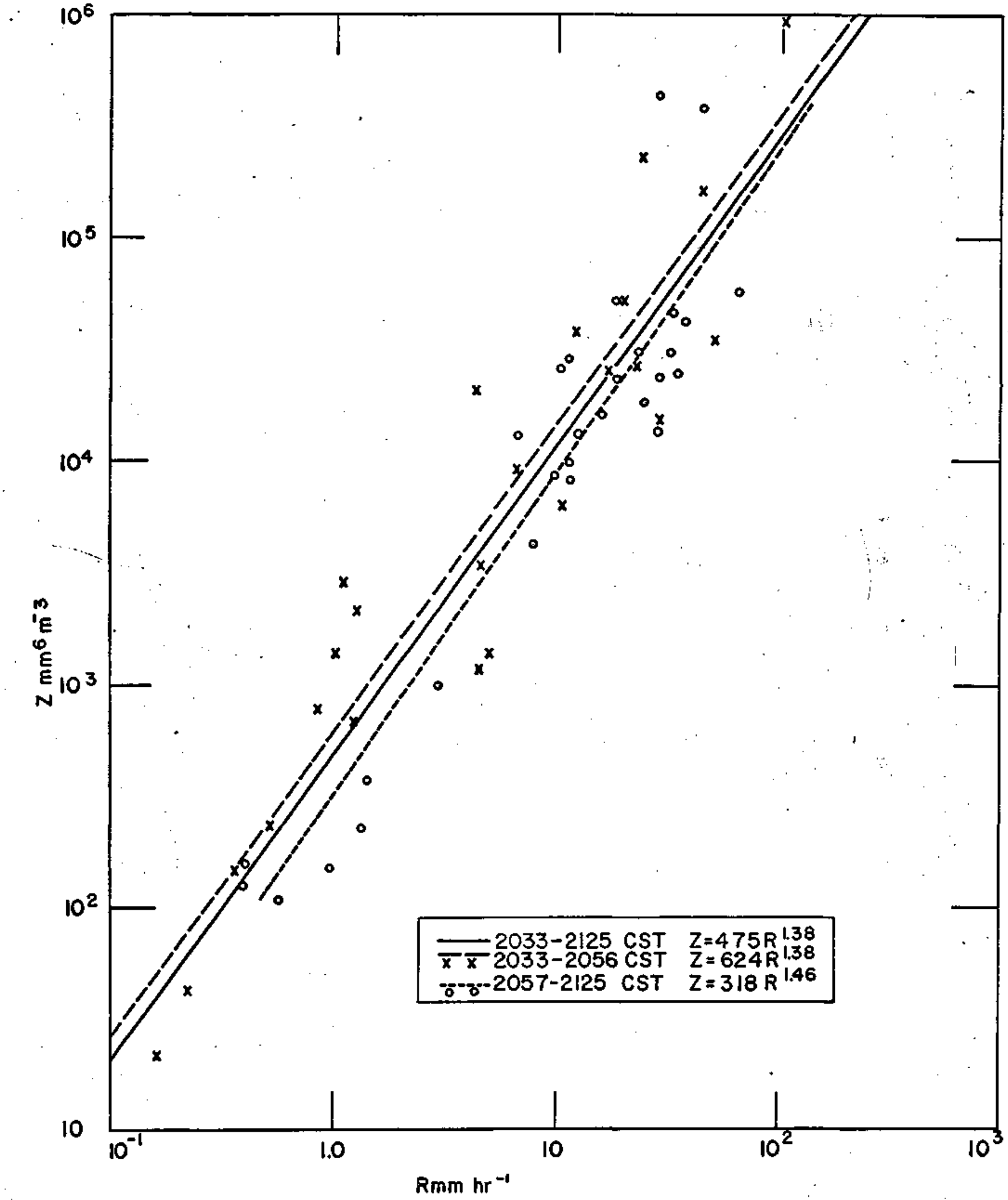


FIG.4 Z-R RELATIONSHIPS FOR STORM OF 10 OCTOBER 1954

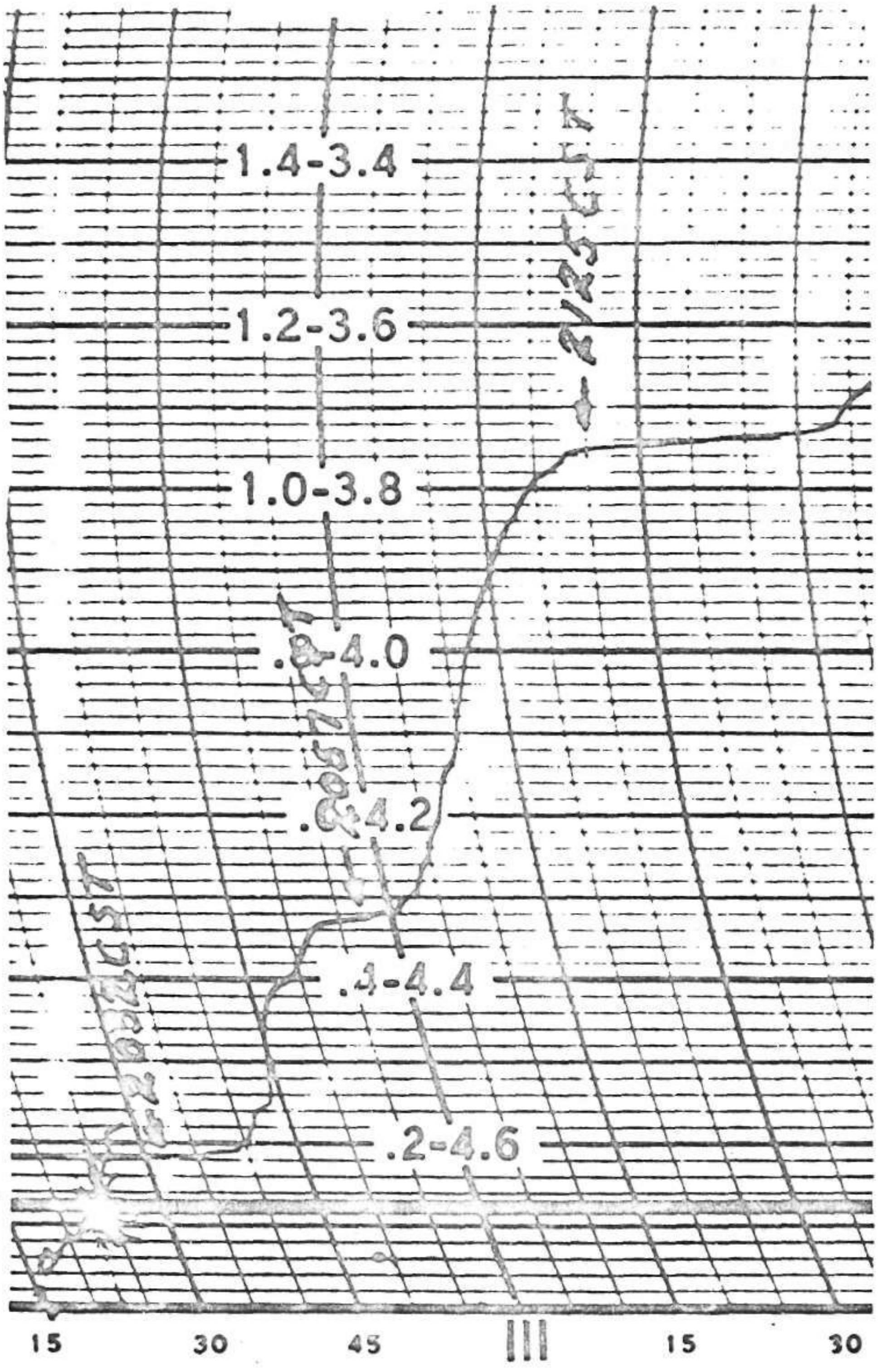


FIG. 5 RAIMGAGE TRACE
10 OCTOBER 1954

Table II summarizes the Z-R analysis of this study and the results of other investigators. Shown in Table II is the Z-R relationship for the 1211 ungrouped observations presented as Z = Z (R) and R = R (Z) with the correlation coefficient and the number of observations. Also shown in Table II are the results of an attempt to improve the accuracy of estimating the rainfall rate from radar reflectivity by classifying the data under three rainfall types: Thundershowers, rainshowers, and continuous rains. In the three classifications, the relationships of Z = Z (R)'and R = R (Z) are given, along with the linear correlation coefficients relating the logarithms of the two variables, and the number of observations entering into each equation.

It will be noted that there are obvious differences between the Z-R relationships of the three rainfall types. This leads to the conclusion that the drop size-distribution of each rainfall type is unique. The difference amounts to 2.9 db. at a rate of 25 mm per hour for rainshowers as compared to thundershowers (Figure 6). This is particularly unfortunate since the PPI presentations of thundershowers and rainshowers are quite similar whereas a continuous rain is easily distinguished from the other two.

There is a decided difference between the coefficient and exponent of R from this study as compared with the results obtained by other investigators. This is partially explained by the choice of R = R (Z) instead of Z = Z (R) as used by the other investigators and given in Table II. The difference not explained by the choice of dependent variable has not been accounted for.

Second-Order Logarithmic Equation

When the points used for obtaining the Z-R relationship for all storms were plotted on a log R - log Z diagram, it was noted that there was

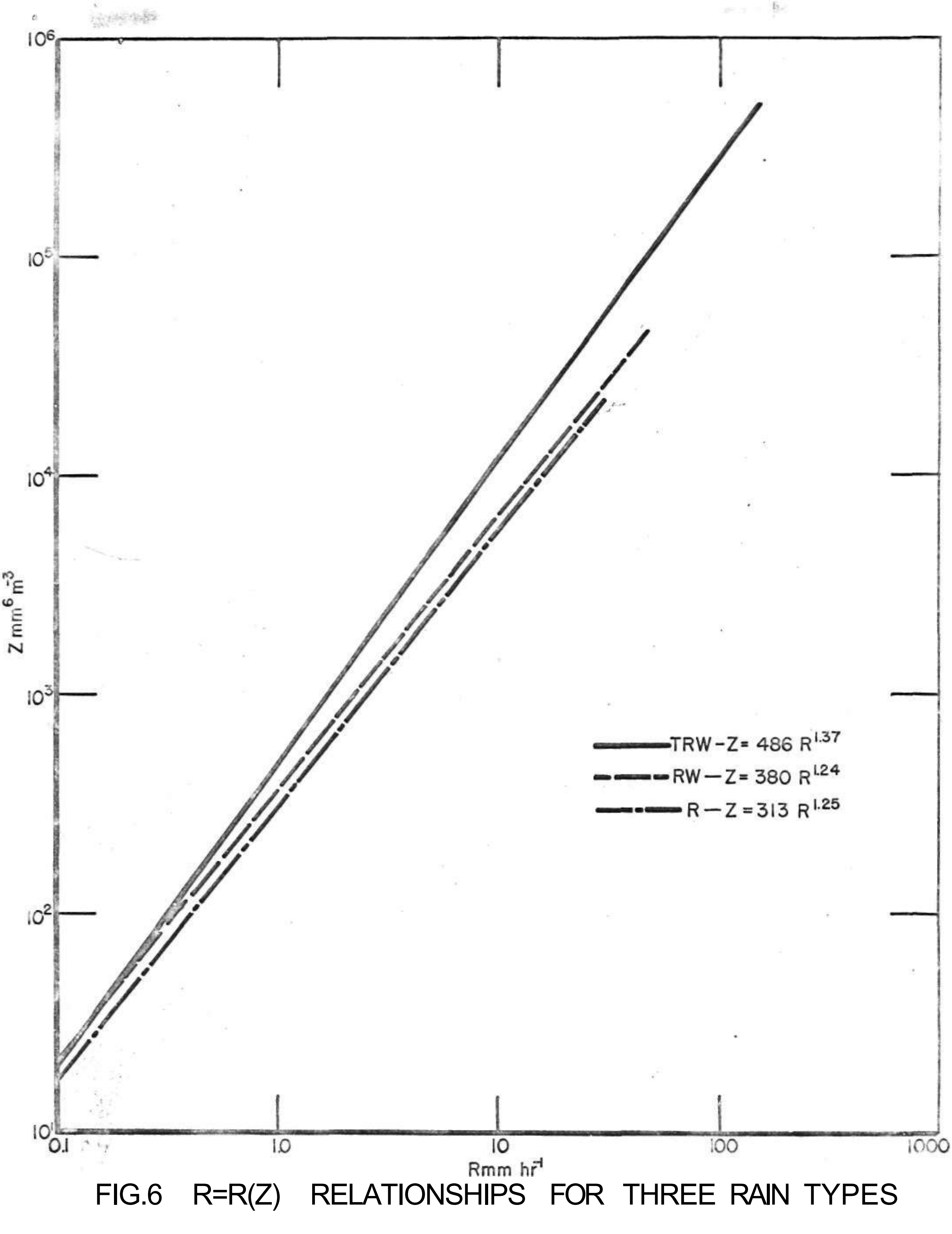


FIG.6 R=R(Z) RELATIONSHIPS FOR THREE RAIN TYPES

TABLE II

COMPARISON OF Z-R RELATIONSHIPS

| Investigator | Z = Z (R) | Z = Z(R) ~~R = R (Z)~~ | n* | r** |
|---|---|---|---|---|
| Anderson (1946) Hilo, Hawaii | $Z = 208R^{1.53}$ | | 49 | |
| Wexler (1947) from smoothed data of Laws and Parsons DCA | $Z = 320R^{1.44}$ | | 49 | |
| Wexler (1948) from original data of Laws and Parsons DCA | $Z = 214R^{1.58}$ | | 98 | |
| Marshall and Palmer (1948) Toronto | $Z = 220R^{1.60}$ | | 250 | |
| Best (1950) Ynslas | $Z = 257R^{1.45}$ | | 299 | |
| Shoeburyness | $Z = 505R^{1.44}$ | | 90 | |
| East Hill | $Z = 436R^{1.64}$ | | 149 | |
| Hood (1950) | $Z = 295R^{1.61}$ | | 270 | |
| Boucher (1951) | $Z = 180R^{1.55}$ | | 63 | |
| Jones (1956) | R = R(Z) | Z = Z(R) | | |
| All Storms | $Z = 372R^{1.47}$ | $Z = 396R^{1.35}$ | 1211 | .96 |
| Thundershowers | $Z = 435R^{1.48}$ | $Z = 486R^{1.37}$ | 515 | .96 |
| Rainshowers | $Z = 370R^{1.31}$ | $Z = 380R^{1.24}$ | 314 | .97 |
| Rain | $Z = 311R^{1.43}$ | $Z = 313R^{1.25}$ | 382 | .93 |

* Number of Observations
** Linear Correlation Coefficient of Log Z and Log R

a decided tendency for log Z to increase non-linearly as log R increased- The 1211 points were fitted to a second order equation relating log R and log Z. The result was

$$\text{Log } Z = 2.569 + 1.317 \ (\log R) + 0.072 \ (\log R)^2 . \qquad (6)$$

Figure 7 is a plot of the first and second order equations of log R - log Z for all types of. storms with the observed points. Table III gives differences between the first and second order equations at several first order rainfall rates.

## DISTRIBUTION CHANGES WITH TIME

It was stated in the INTRODUCTION that insofar as possible one-minute observations were made through complete storms. This observational procedure makes it possible to follow the Z-R relationships minute by minute. Several storms were analyzed for evidence of reflectivity changes with time. The trend was expected to be from large reflectivity values at the beginning of a shower to abnormally low values at the end of the shower. This trend might be expected since large raindrops fall faster than small raindrops. In order to make the results comparable, all of the reflectivity values were reduced to a normalized rainfall rate of 1 mm per hour. Thus, the normalized Z values appeared as deviations about the coefficient of R in the particular Z-R relationship. In most cases, as expected, the normalized reflectivity appeared to be highest at the beginning of a shower, although other high points occurred in the normalized Z curves of the storms. Unexpectedly, a rising tendency was usually observed in the normalized Z value on the trailing end of the storm where the rainfall rate was decreasing. There seems to be a tendency for the reflectivity to

TABLE III

DIFFERENCES IN DB BETWEEN FIRST AND SECOND ORDER EQUATIONS AT SEVERAL DIFFERENT FIRST ORDER RAINFALL RATES

| FIRST ORDER $R$ mm hr$^{-1}$ | SECOND ORDER $R$ mm hr$^{-1}$ | DB DIFFERENCES $10 \log \frac{(\text{First Order})}{(\text{Second Order})}$ |
|---|---|---|
| 1.0 | 1.0 | ± 0.00 |
| 10.0 | 9.7 | - 0.14 |
| 25.0 | 22.5 | - 0.45 |
| 50.0 | 40.9 | - 0.87 |
| 100.0 | 75.0 | - 1.25 |
| 150.0 | 104.8 | - 1.60 |

exceed the normal where the rainfall rate is increasing and for it to be less than normal where the rainfall rate is decreasing. These points are illustrated by Figures 8 and 9 which are graphs of the normalized Z values for the storms of 14 August 1954 and 26 August 1954. The storm of 14 August 1954 vas a heavy thunderstorm while the storm of 26 August 1954 was a heavy rainshower (Table I).

Unfortunately, the age of these storms was not obtainable from radar or raingage network data, since the camera was located at the radar site and the normal ground clutter extended for a 20-mile radius about the radar. It would be desirable to know the observational period during the life of the storm to determine the effect of the age of the storm upon the drop size-distribution.

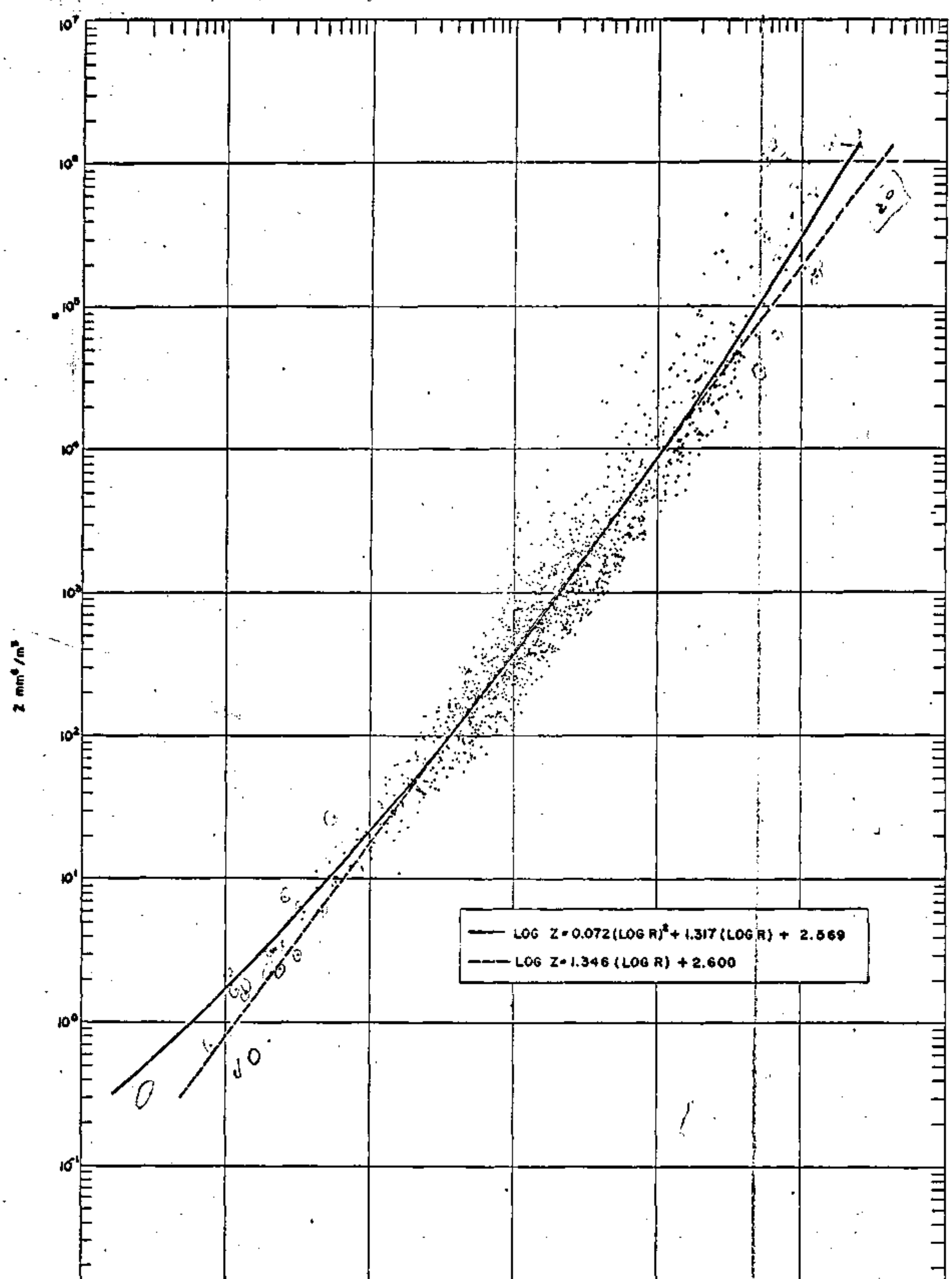


FIG.7. Z-R RELATIONSHIPS FOR UNGROUPED 1953-1954 DATA

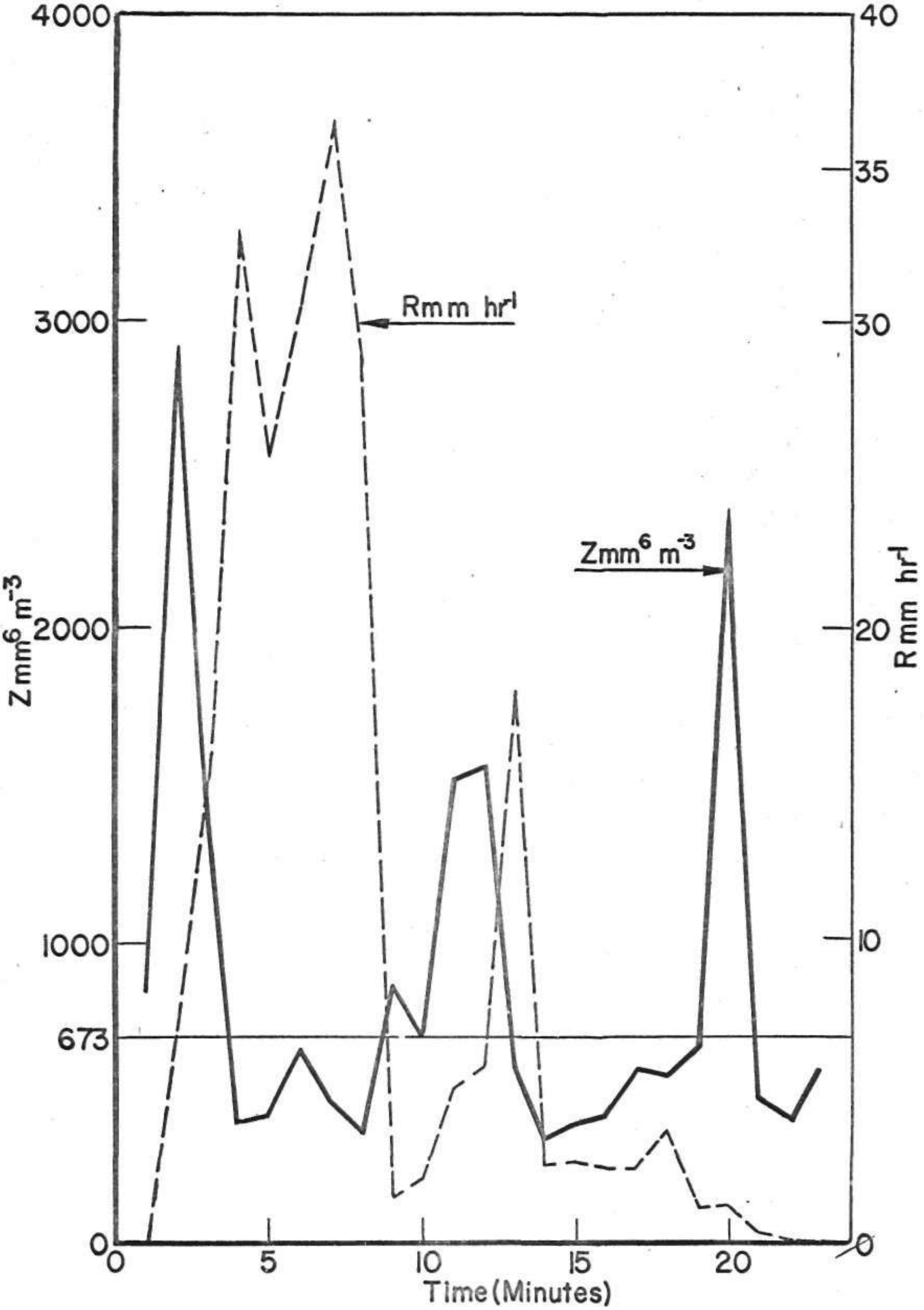


FIG.8 Z NORMALIZED TO I mm hr$^{-1}$
STORM OF 14 AUGUST 1954

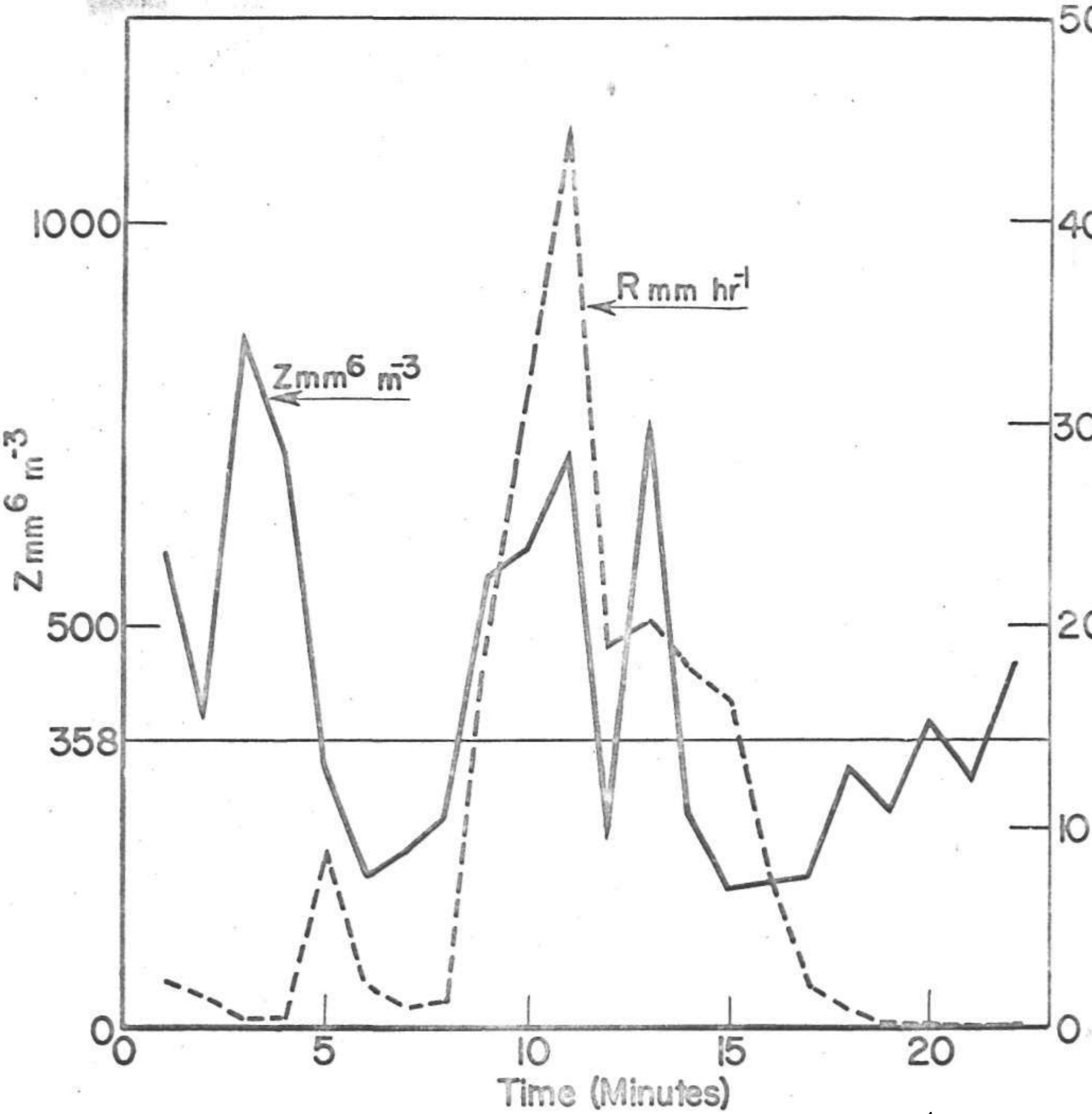


FIG.9 2 NORMALIZED TO I mm hr$^{-1}$
STORM OF 26 AUGUST S954

## BACK SCATTERING - RAINFALL RELATIONSHIPS

When the first photographs of raindrops obtained by the raindrop camera depicted the shape of the larger raindrops, it was realized that an opportunity was presented to relate a more exact back-scattering theory to the observed drop size-di6tributions than, had been possible in the past. Consequently, Mathur and Mueller[10] have determined the validity of Stevenson's[11,12] back-scattering equations for oblate and prolate spheroids and set up the tables necessary for the application of these functions to the observed drop size-distributions. Time has allowed the analysis to be carried out only for 3-cm wavelength radiation for one storm from each of the three storm rainfall types. A steady rain of 8 August 1954 gave the relation

$$\sigma = 0.072R^{1.32} \tag{7}$$

where $\sigma$ is the Stevenson back-scattering function in $mm^2$ and R is the rainfall rate in mm $hr^{-1}$. The relationship has a correlation coefficient of the logarithms of R and $\sigma$ of 0.98 which may he compared with the Z-R correlation coefficient of 0.99 for the same storm. The dependent variable, as before, is assumed to be R such that $R = R(\sigma)$. The 26 August 1954 rainshower gave the relationship

$$\sigma = 0.168R^{1.43} \tag{8}$$

with a correlation coefficient of 0.98 while the corresponding Z-R coefficient is 0.99. For the thunderstorm of 10 October 1954 the relationship was

$$\sigma = 0.198R^{1.56} \tag{9}$$

with a correlation coefficient of 0.90 as compared to 0.93 for-the Z-R relationship. The smaller correlation coefficients of the $\sigma$ - R relationships as compared with the Z-R relationships indicates that there is a greater scatter of points in the $\sigma$ - R relationships. Again it will be noted (Figure 10) that there are significant differences between the three rainfall types. A comparison of the results with the corresponding Z-R

TABLE IV

Z - R AND $\sigma$ - R RELATIONSHIPS FOR 3-CM RADIATION

| Date 1954 | $\sigma$ - R | $Z_\sigma$ - R | Z - R |
|---|---|---|---|
| 8 August | $\sigma = 0.072R^{1.32}$ | $Z_\sigma = 229R^{1.32}$ | $Z = 181R^{1.25}$ |
| 26 August | $\sigma = 0.168R^{1.43}$ | $Z_\sigma = 535R^{1.43}$ | $Z = 358R^{1.36}$ |
| 10 October | $\sigma = 0.198R^{1.56}$ | $Z_\sigma = 630R^{1.56}$ | $Z = 475R^{1.38}$ |

equations is shown in Table IV. The comparison calculations are made by applying the appropriate correction constant which is

$$\Sigma \sigma = \frac{\pi^5 |K|^2 \Sigma d^6}{\lambda^4} \tag{10}$$

or

$$\Sigma \sigma = 3.14 \times 10^{-4} \Sigma d^6 \tag{11}$$

Equation (11) may be written as

$$\sigma = 3.14 \times 10^{-4} Z \tag{12}$$

$$Z_\sigma = \frac{\sigma}{3.14 \times 10^{-4}} \tag{13}$$

where $Z_\sigma$ indicates that the equation is derived from Stevenson's back-scattering functions and expressed in the Rayleigh form. For the 26 August 1954 storm, which was a rainshower with only slightly deformed large drops, the transformed $\sigma$ - R relationship becomes

$$Z_\sigma = 535 R^{1.43}. \tag{14}$$

At 25.4 mm per hour, comparison of the Z and $Z_\sigma$ equations results in a 2.8 db difference, with $Z_\sigma$ indicating the greater power return for this storm.

## LIQUID WATER CONTENT, MEDIAN VOLUME DIAMETER, AND RAINFALL RATE

All samples of the 1211 which were complete (the photographic sample included 36 exposures in 12 seconds) were extracted for a study of liquid water content per cubic meter. For each rainfall rate a drop diameter was determined which divided the liquid water content of the sample such that 50 percent of the water was contributed by the drops smaller in size and 50 percent was contributed by drops larger in size. The complete samples included rates up to 157 mm $hr^{-1}$, but hail was observed to have fallen in some of the storms with the highest rates and these storms were excluded from the study. In spite of the exclusion of the storms which included hail, raindrops in all sizes up to 9.6 mm equivalent spherical diameter were observed. A person standing in the rain at the observation site did not observe hail in the storms during which the extra-large drops were observed. After excluding hailstorms and incomplete observations, only 84 samples remained with rates from 5.0 mm per hr to 107 mm per hr.

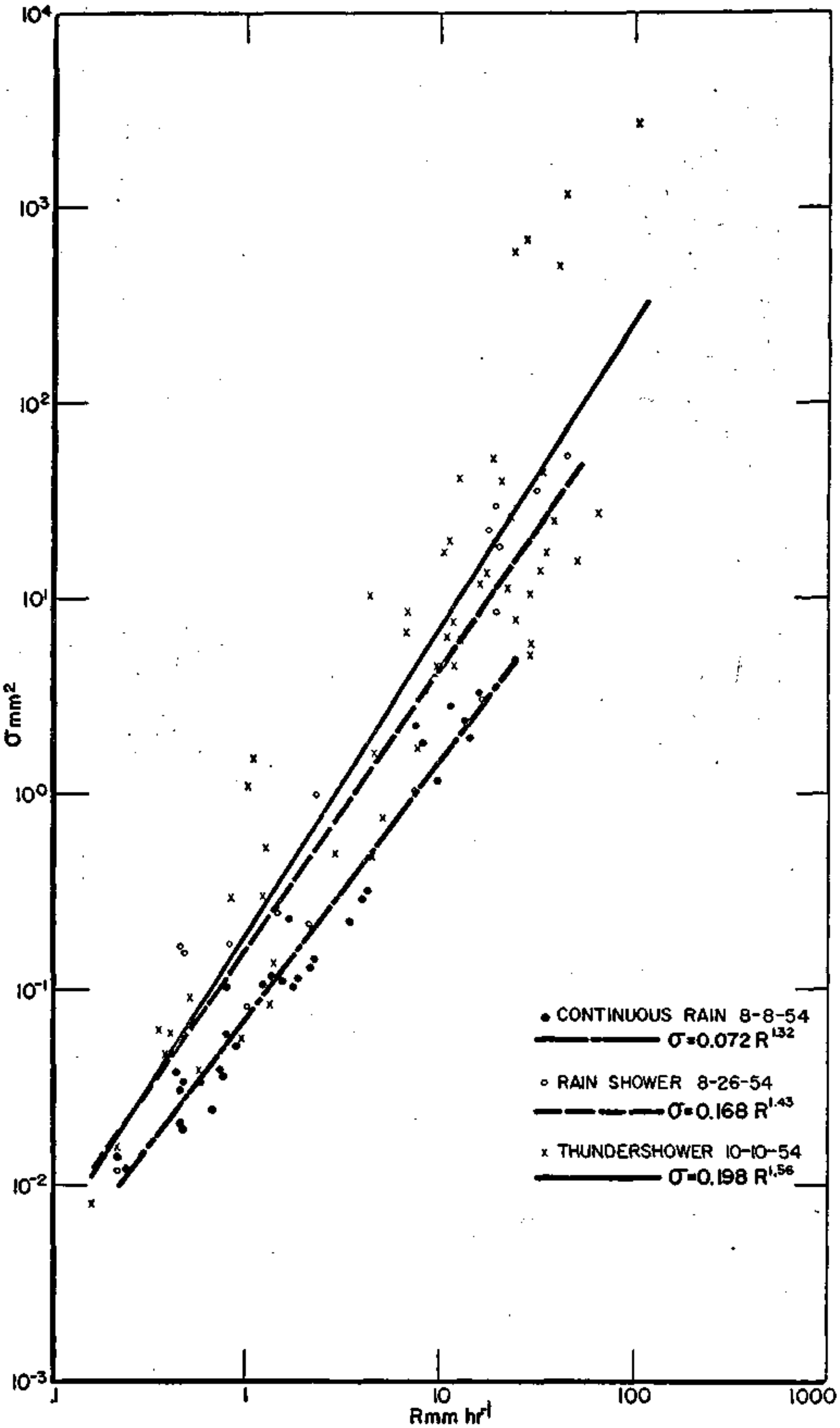


FIG. 10 σ-R RELATIONSHIPS FOR THREE RAINSTORMS

Liquid Water Content

A study of the liquid water content, W in gms $m^{-3}$ for one storm, 26 August 1954, was made. This is summarized in the logarithmic regression equation

$$W = .052 \ R^{.97} \tag{15}$$

which is graphed in Figure 11. The points calculated for each minute of the storm agree reasonably well with the values found by Atlas and Plank[13] who reported the equation,

$$W = .072 \ R^{.88}. \tag{16}$$

Median Volume Diameter

Figure 12 has plotted on it the points from 84 samples without hail. Two regression lines of log $d_{50}$ - log R are shown where $d_{50}$ is the median volume diameter in mm. The dashed curve,

$$d_{50} = 0.97 \ R^{0.26} \tag{17}$$

includes rainfall rates up to 33 mm per hr and is comparable to Atlas and Plank's[13] equation,

$$d_{50} = 0.92 \ R^{0.21} \tag{18}$$

which includes rates up to 3.28 mm per hr. The solid curve,

$$d_{50} = 0.80 \ R^{0.34} \tag{19}$$

includes rates to 107 mm per hr. It will be noted that the scatter of the points increases at rates above 33 mm per hr. On the basis of this data it may be inferred that what is a simple relationship at low rainfall rates becomes a complex relationship at higher rainfall rates because of the greater scatter in $d_{50}$ at the higher rates.

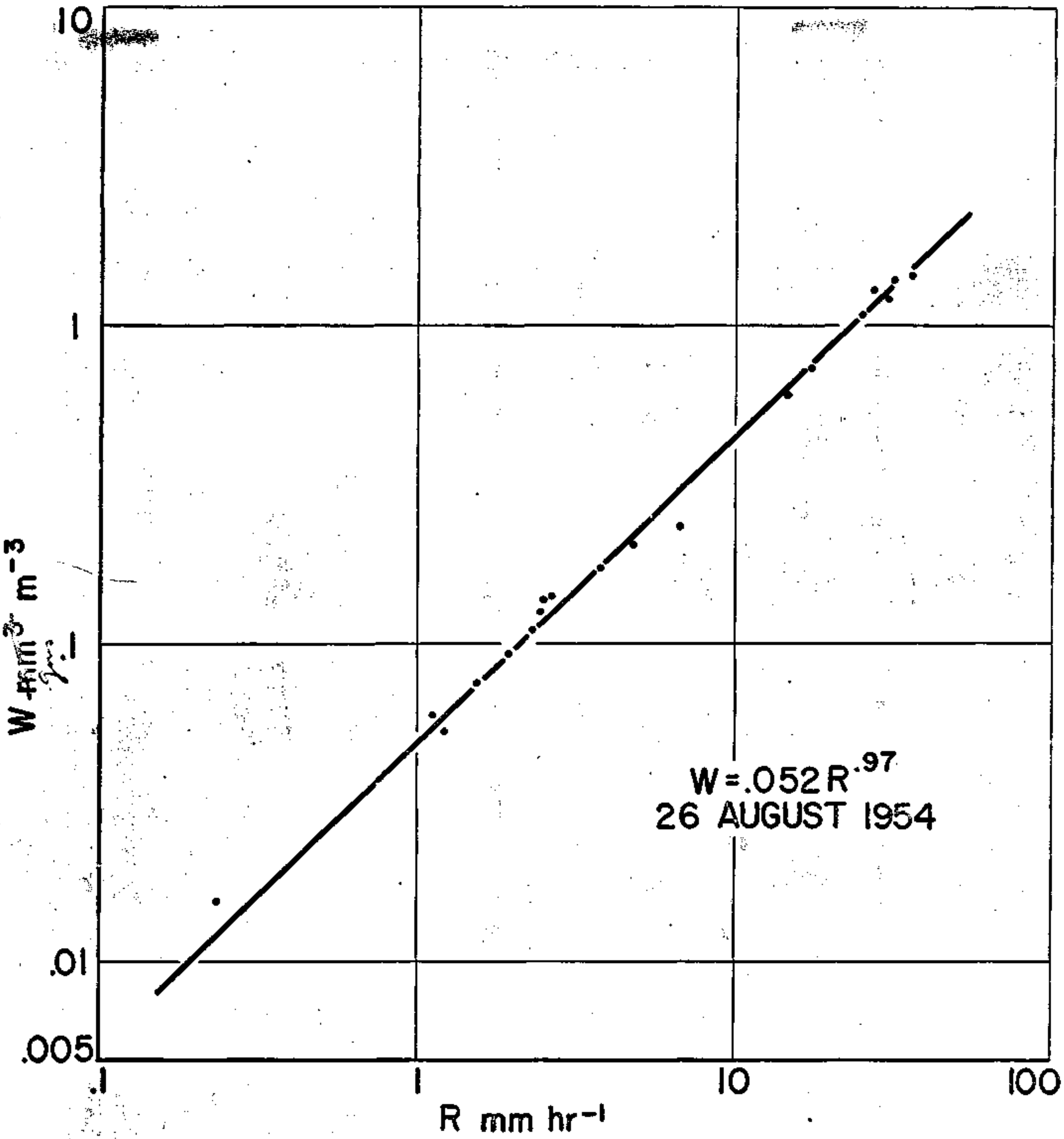


FIG.11 LIQUID WATER CONTENT VS RAIN INTENSITY FOR A HEAVY SHOWER

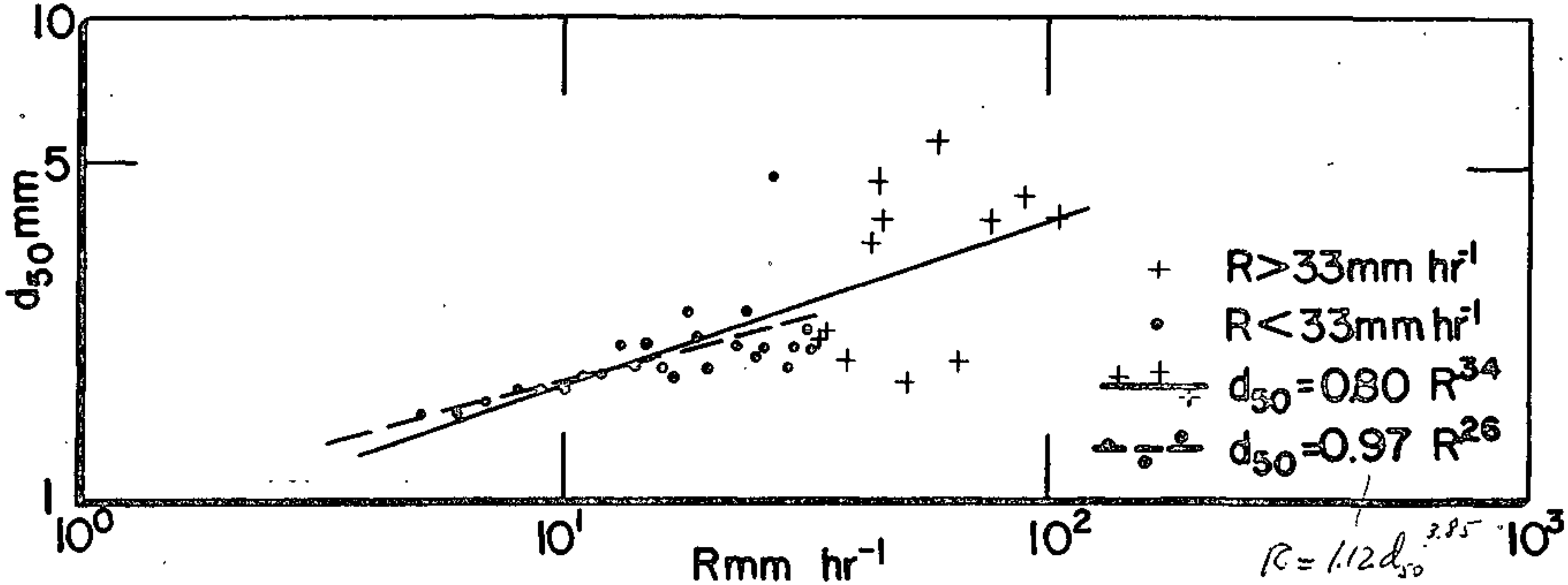


FIG. 12 AVERAGE RELATION BETWEEN RAINFALL RATE AND MEDIAN VOLUME DROP DIAMETER

# CONCLUSIONS

It is believed that sufficient data have been collected and analyzed to define the variability of radar reflectivity and rainfall rate witn some accuracy. It had been shown that there are 13-14 db differences in radar echo power return, Z, for a given rainfall rate between different rainstorms. Some improvement can be obtained if the type of rainfall occurring (thunderstorm, rainshower, or rain) is known and the proper Z-R relationship used. However, a greater variability in back-scattering energy per rainfall rate than in reflectivity per rainfall rate might be expected, since the correlation coefficient relating log $\sigma$ to log R is smaller than the coefficient relating log Z to log R.

A plot of the observations of Z and R on log-log graph paper indicated that a curve of best fit would not be a straight line. Both a first order logarithmic equation and a second order logarithmic equation were fitted to the points by the method of least squares. Visual inspection of the two curves indicated that the second order equation was the better fit.

It has been shown that there is a tendency for larger raindrops to predominate at the beginning of a shower and smaller drops to predominate near the end of a shower.

The liquid water content of one storm was found to agree reasonably well with the values observed by Atlas and Plank.

The median volume diameter was found for each of 84 rainfall rates. It was found that the observed points had little scatter up to R = 33 mm per hour. Above this rainfall rate the scatter of points increased.

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