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Practical Considerations for Assessing the Waste Assimilative Capacity of Illinois Streams

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PRACTICAL CONSIDERATIONS FOR ASSESSING THE WASTE ASSIMILATIVE CAPACITY OF ILLINOIS STREAMS

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INTRODUCTION

This document has been prepared for the practitioner who has the responsibility for developing or approving the design of waste treatment facilities with discharges to Illinois streams. It sets forth in practical terms the factors that must be considered for determining the capability of stream waters to receive waste. A stream's capacity to assimilate waste as used here is that capacity available for use within the stream without depressing the dissolved oxygen (DO) levels below that required by Water Pollution Regulations in Illinois.[1]

Although the concept of waste assimilation is not new, its implementation for stream pollution abatement has been intermittent and its full potential neglected. Unfortunately it has been considered a dilution phenomenon. It is based upon two principles of reason: 1) a stream is not a sterile entity and 2) waste receiving stream waters are a natural extension of a biological treatment process. Recognition of these principles is essential if stream pollution abatement is to be approached on the basis of cost effectiveness.

An investigation leading to the assessment of the ability of stream waters to accept, ingest, and dispose of liquid waste is a scientific endeavor, though not an exact one. Its purpose is to define the mechanisms inherent in the self-purification capabilities of a stream which are in turn dependent upon interrelated physical, chemical, and biological activity. The basic laws that have been developed in an effort to correlate these interrelations are not absolute. They can be weakened by gross assumptions and distorted by generalized data. The proper selection of reaction rates, biodegradable loads, and stream flows is fundamental; the use of textbook factors or the translation of observations from one stream to another are acceptable only in terms of broad policy decisions.

The techniques suggested herein are founded on theory, logic, and experience. Some effort has been made to evolve traditional theoretical considerations, but more extensive guidance on these can be obtained from the references cited.

Plan of Report

This report has been prepared with two basic situations in mind. These have been designated as Case I and Case II. Case I represents the conceptual approach whereby the necessity arises to evaluate assimilative capacities on the basis of simulation models. The procedures developed for this case are applicable when it becomes necessary to estimate the assimilative capacity of a stream prior to the actual discharge of waste effluents to it. Case II, on the other hand, represents a pragmatic approach based upon observed DO or biochemical oxygen demand (BOD) concentrations at several points along a stream reach. The procedures outlined in this situation are particularly useful where the stream is currently receiving waste water discharges and the effects of the effluents can be evaluated by field investigations.

Considerable reliance has been placed upon the use of examples in demonstrating the application of formulas, and where appropriate, actual field data have been used. Procedures for estimating parametric values, which are based upon the experience of field investigations in Illinois, have also been included. Notations for symbols used throughout the report are given in the back.

Acknowledgments

This report was prepared under the general supervision of Ralph L. Evans, Head of the Water Quality Section, and Dr. William C. Ackermann, Chief, Illinois State Water Survey. The computer program was developed by Robert Sinclair, System Analyst. Katherine Shemas typed the original manuscript; Mrs. J. Loreena Ivens edited the final report; and William Motherway, Jr., prepared the final illustrations. Extensive cooperation and assistance was extended to the authors by all members of the Water Quality Section staff, and notably by Donald H. Schnepper.

THEORY

The theory and procedures developed herein for oxygen balance in streams are based on the premise that the overall dissolved oxygen-biochemical oxygen demand (DO-BOD) relationship in streams is a linear process. Under this assumption, effects of all the component processes affecting the DO-BOD relationship in streams are additive, even though they are in themselves nonlinear.

The analysis of streams to determine their capacity to assimilate organic pollution has been based on the classical theory of Streeter and Phelps.[2] Since their pioneering efforts, several modifications (Camp,[3] Dobbins,[4] O'Connor[5]) have been proposed. Streeter and Phelps considered only two mechanisms affecting the DO budget in streams, namely, the biochemical oxidation of organic matter and the atmospheric reaeration of river water. Dobbins[4] postulated that there are several other mechanisms operative in a river system which affect the DO-BOD relationship. These are:

1) Removal of BOD by sedimentation and adsorption.
2) Addition of BOD along a river stretch by scour of bottom deposits or by diffusion of partly decomposed organic products from the benthal layer into the waters above.
3) Addition of BOD by local runoff.
4) Removal of oxygen from water by diffusion into the benthal layer to satisfy oxygen demand in the aerobic zone of this layer.
5) Removal of oxygen from water by purging action of gases rising from benthal layers.
6) Addition of oxygen by photosynthetic action of plankton and fixed plants.
7) Removal of oxygen by respiration of plankton and rooted plants.

8) Continuous redistribution of BOD and DO by the effect of longitudinal dispersion.

In addition to these eight processes listed by Dobbins,[4] the importance of nitrogen demand as part of the BOD is being increasingly recognized (Butts et al.,[6] O'Connor and DiToro,[7] Gannon[8]). Zanoni,[9] conducting extensive research on the long term deoxygenation behavior of a conventional activated sludge waste water treatment plant effluent, came to the conclusion that the ultimate demands due to carbonaceous and nitrogenous materials are nearly equal. O'Connor and DiToro[7] emphasized the importance of relative magnitudes of ultimate demands of carbonaceous and nitrogenous components, and stated that the effluents from biological treatment plants, particularly the high rate type, should be evaluated for their nitrogenous demands.

In view of the stringent effluent quality standard with respect to suspended solids, it is likely that the effects of solids sedimentation, benthal demand, and BOD addition due to scour on oxygen balance will be minimal and consequently negligible. Gannon[8] in his intensive study of the Clinton River below Pontiac, Michigan, found no extensive sludge deposits and concluded that this factor had a minimal influence. The major waste source in this study consisted of the effluent from the Pontiac waste treatment plant, which gave biological treatment to the waste, either by means of trickling filter or activated sludge units that operated in parallel. O'Connor and DiToro[7] in their analysis of oxygen distribution in the Grand River below Lansing, Michigan, considered settling of organic matter to be minimal in the river downstream of an activated sludge treatment plant. In view of the experiences of these investigators, solids deposition and other related mechanisms, namely items 1, 2, 4, and 5 in the Dobbins list, will have very little influence on oxygen balance studies in most streams.

Gannon[8] found valid the assumption that the addition of oxygen by photosynthetic activity of algae and other plants was balanced by the oxygen consumed by the respiration activity of these algae and rooted plants. Austin and Sollo[10] concluded, on the basis of their 2-year study on the Salt Fork of the Vermilion River, that phytoplankton did not have a great effect on the oxygen balance of the stream. The major waste source into that stream is the effluent from a waste treatment plant managed by the Urbana-Champaign Sanitary District. Most of the field work was carried out during the periods of critical flow when the treatment plant effluent made up 50 percent or more of the stream flow. Camp[3] is of the opinion that, in oxygen balance studies to estimate allowable pollution loads, oxygen furnished through photosynthesis should not be relied upon because there is no assurance that it will be available during critical low flow periods. Consequently, photosynthesis and respiration of aquatic plants will not be considered here in the mathematical formulation for estimating assimilative capacities of streams.

Reliable documentations of the extent of BOD additions along a river stretch by local runoff are not available at this time. The effects of dispersion, though significant in estuarial analysis, are considered insignificant in river pollution analysis. Therefore only three major factors will be considered in the evaluation of the mechanisms of self purification within stream waters, namely, the dissolved oxygen available to the stream system, the biological oxygen demands due to carbonaceous loads, and similar demands due to nitrogenous loads.

HYDROLOGIC AND HYDRAULIC DATA

Flow

Regardless of the procedures selected for assessing the waste assimilative capacity of a stream, certain fundamental data must be available or developed for the hydrology of the watershed and the geometry of the stream system. Basic among these is the need to ascertain the 7-day, once-in-10-year low flow of the receiving water, which is the design flow established by the Water Pollution Regulations of Illinois.[1]

A special hydrologic study is currently (1972) being made under the supervision of J. B. Stall of the Illinois State Water Survey to determine the 7-day, 10-year low flow for all streams in the state. Upon the completion of the study a publication will be issued setting forth the design flows to be incorporated in any water pollution abatement plans.

Certain features of the hydraulic geometry of the stream system are essential for estimating the time-of-travel (velocity) in stream waters and the system's capability for absorbing oxygen from the atmosphere (reaeration). However, such estimates from generalized formulas cannot be relied upon when dealing with low stream flows. For example, the excellent work of Stall and Fok[11] and Stall and Hiestand[12] is not applicable to low flow conditions. These investigators have found that general hydraulic geometry equations are best applied during relatively high flows, i.e., those flows equaled or exceeded about 10 percent of the days each year. For most Illinois streams the 7-day, 10-year low flow will usually be exceeded 90 percent of the time and often 98 percent of the time. During these periods of low flow the effective slope of the water surface is greatly different from the general slope of the stream bed. Pools, riffles, and other impediments consequently cause travel time through the stream reach to be much longer than that computed by generalized equations.

Cross Sections

In dealing with low flow conditions in a stream there is no substitute for cross section data developed from field observations. They are essential for defining the width, depth, and velocity of flow along the course of a stream at a constant discharge. Unfortunately such data are sparse in Illinois. Sources include the U. S. Corps of Engineers, Illinois Division of Waterways, the Illinois State Water Survey, and some local drainage districts.

For major streams, cross sections at points of significant changes in the stream bed are adequate. For small streams, cross sections may be needed at much closer intervals. Gannon and Downs[13] used cross sections at 500-foot intervals on moderately small Michigan streams during extensive waste assimilative studies. The Illinois State Water Survey has found that cross sections on small streams should be taken to define riffle and pool areas in free flowing reaches, but are needed at less frequent intervals in pooled areas upstream of dams. In dealing with cross sections, stream water stages are essential. In addition to the sources previously mentioned for cross section information the U. S. Geological Survey can furnish water level elevations.

To demonstrate the differences between using adequate cross section data and misapplying generalized stream geometry formulas a reach of the Rock River between Rockford and Rockton will be examined. This reach of 22.4 miles from milepoint (MP) 159.1 to MP 136.7 is located between two channel dams. Pertinent cross sections for the reach, provided by the U. S. Corps of Engineers, are set forth in figure 1. River flow and stage data were obtained from the U. S. Geological Survey and Division of Waterways. River flows were considerably in excess of the 7~day, 10-year low flow. The lower 13 miles of the reach is pooled, whereas the upper 9 miles is relatively free flowing. This is typical of many reaches of Illinois streams.

For this example the time-of-travel will be estimated by a computer-oriented volume displacement procedure and compared with generalized formulas,[11,12] and the reaeration rate K_2 will be developed from the findings. The volume displacement procedure can be formulized as:

$$\text{time-of-travel} = \frac{\text{average area} \times \text{length}}{\text{average flow}} \qquad (1)$$

The Illinois State Water Survey has developed a computer program using cross section coordinates to produce a variety of hydrologic and hydraulic data based upon this simple formula. The format and computations (made with a desk calculator) used for this example are given in table 1, and the results of these computations compared with empirical formulas are depicted in figure 2. Reasonable estimates for

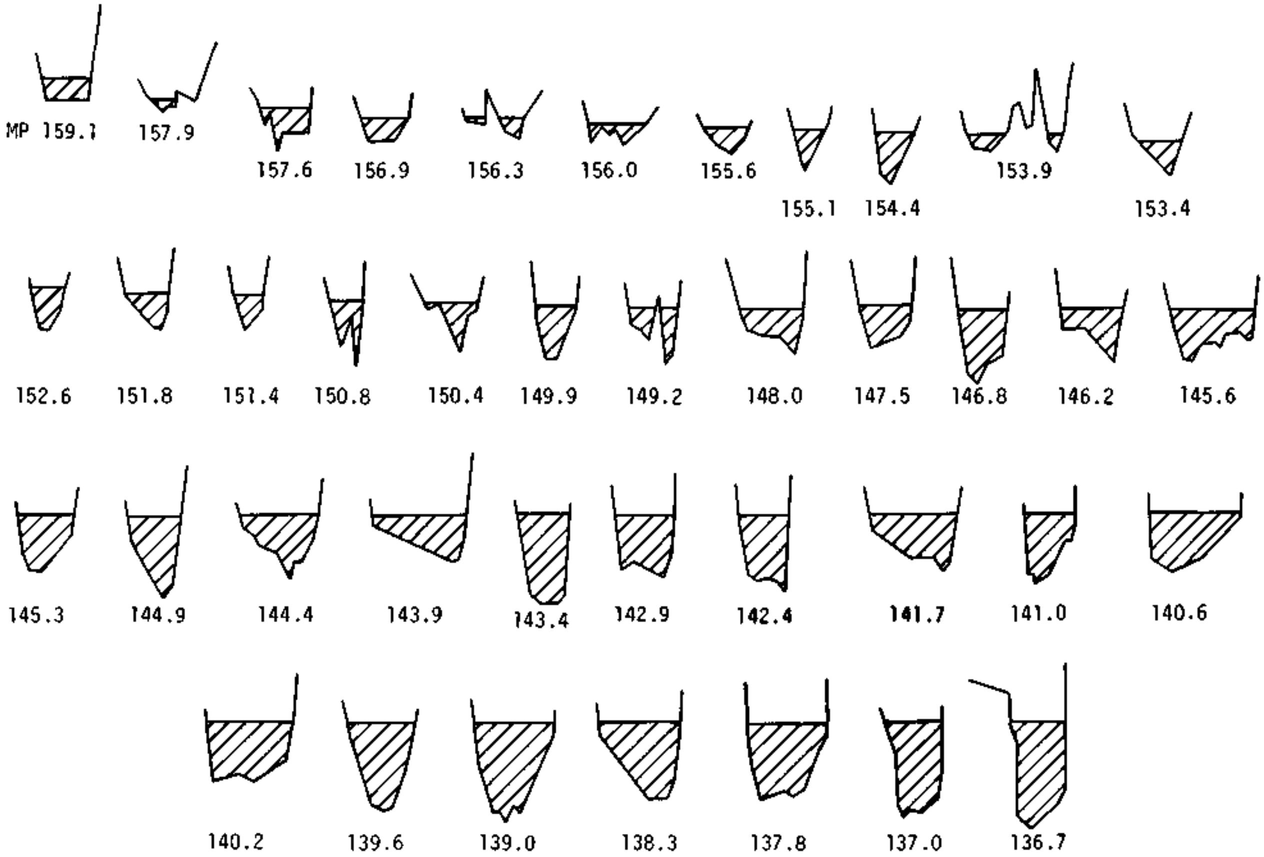


Figure 1. Rook River cross sections from milepoint 159.1 to 136.7

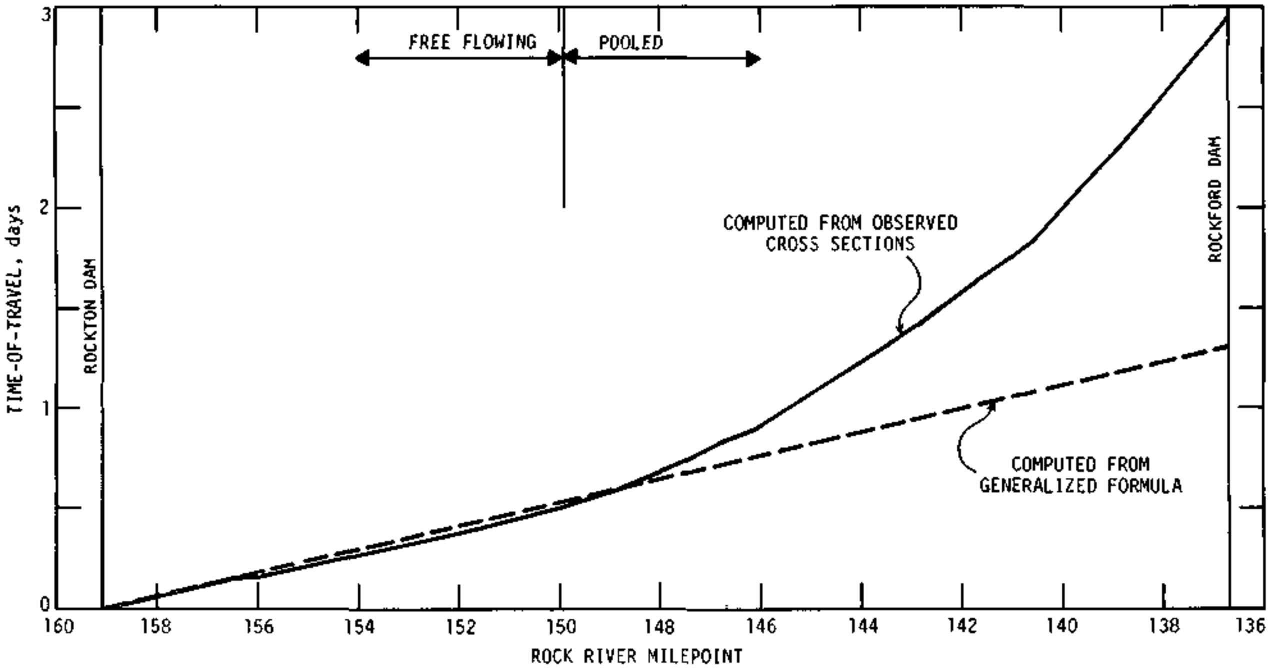


Figure 2. Comparison of time-of-travel by volume displacement and generalized formulas

Table 1. Hydraulic and Hydrologic Computations, 22.4-Mile Reach of Rock River

Mile-point	Area (ft^2)	Distance* (ft)	Volume* ($10^6 ft^3$)	Width (ft)	Avg depth (ft)	Flow (cfs)	Velocity (fps)	t* ($days$)	Σt ($days$)	K_2 ($1/day$)
159.1	830			360	2.31	732	.88			2.20
157.9	319	6336	3.643	200	1.60	734	2.30	.058	.058	9.35
157.6	1106	1584	1.129	380	2.91	735	.66	.018	.076	1.21
156.9	936	3696	3.773	370	2.53	989	1.05	.051	.127	2.32
156.3	479	3168	2.243	350	1.37	990	2.06	.026	.153	10.28
156.0	755	1584	.977	410	1.84	991	1.31	.011	.164	4.42
155.6	723	2112	1.561	320	2.26	992	1.37	.018	.182	3.52
155.1	809	2640	2.022	250	3.24	993	1.23	.024	.206	1.95
154.4	1181	3696	3.678	300	3.94	994	.84	.043	.249	1.03
153.9	638	2640	2.402	400	1.59	994	1.56	.028	.277	6.41
153.4	798	2640	1.896	330	2.42	995	1.25	.022	.299	2.93
152.6	1096	4224	4.000	250	4.38	997	.91	.046	.345	.97
151.8	1021	4224	4.473	320	3.19	998	.98	.052	.397	1.61
151.4	755	2112	1.875	230	3.28	999	1.32	.022	.419	2.07
150.8	1170	3168	3.050	260	4.50	1000	.85	.035	.454	.87
150.4	936	2112	2.224	340	2.75	1001	1.07	.026	.480	2.12
149.9	1404	2640	3.089	300	4.68	1002	.71	.036	.516	.69
149.2	1213	3696	4.838	310	3.91	1003	.82	.056	.572	1.02
148.0	1660	6336	9.105	460	3.61	1005	.60	.105	.677	.82
147.5	1755	2640	4.700	390	4.50	1006	.57	.054	.731	.59
146.8	2628	3696	8.102	370	7.10	1007	.34	.093	.824	.20
146.2	1862	3168	7.112	460	4.05	1009	.54	.082	.906	.64
145.6	2426	3168	6.792	620	3.91	1010	.42	.078	.984	.52
145.3	2372	1584	3.800	420	5.65	1011	.43	.044	1.028	.33
144.9	2713	2112	5.371	400	6.78	1011	.37	.061	1.089	.22
144.4	2787	2640	7.260	580	4.81	1012	.36	.083	1.172	.34
143.9	2128	2640	6.468	680	3.13	1013	.48	.074	1.246	.80
143.4	3319	2640	7.191	380	8.73	1014	.31	.082	1.328	.14
142.9	2713	2640	7.962	430	6.31	1015	.37	.091	1.419	.24
142.4	3074	2640	7.640	360	8.54	1016	.33	.087	1.506	.14
141.7	3298	3696	11.775	640	5.15	1016	.31	.134	1.640	.27
141.0	2426	3696	10.578	380	6.38	1018	.42	.120	1.760	.27
140.6	3723	2112	6.494	670	5.56	1019	.27	.158	1.834	.21
140.2	4766	2112	8.964	530	8.99	1020	.22	.102	1.936	.09
139.6	4010	3168	13.901	480	8.35	1020	.25	.158	2.094	.11
139.0	5500	3168	15.063	590	9.32	1022	.19	.171	2.265	.07
138.3	4383	3696	18.266	600	7.31	1023	.23	.207	2.472	.12
137.8	5532	2640	13.089	600	9.22	1024	.19	.148	2.620	.08
137.0	4596	4224	21.390	410	11.21	1025	.22	.242	2.862	.07
136.7	4649	1584	7.323	400	11.62	1026	.22	.083	2.945	.06

**Between milepoints*

flows considerably in excess of 7-day, 10-year low flows were obtained where free flowing conditions exist. However it is obvious that the generalized formulas cannot be used in reaches where the influence of channel dams exists. Because of the pooled area the average travel time in the reach has been extended to 3 days, whereas under free flowing conditions it would be about 1.3 days. From a waste assimilative standpoint, for a given oxygen demand load and deoxygenation rate, about 2 1/4 times more oxygen would be required within the reach than was estimated by empirical means.

The factors influencing reaeration, i.e., the absorption of oxygen from the atmosphere, will be discussed in detail later in this report. However, for the purpose of emphasizing the importance of cross section information, a generalized and acceptable equation for estimating reaeration rates is one developed by Langbein and Durum:[14]

$$K_2 = 7.63 V H^{-1.33} \tag{2}$$

where

K_2 = reaeration coefficient to the base e, per day

V = average velocity, feet per second (fps)

H = average depth, feet

This equation was applied to the velocities and depths for the Rock River reach, computed first,by the use of empirical formulas and secondly by the use of cross sections, to determine the K_2 values for points along the reach, as shown in figure 3. From the generalized formulas the computed depths and velocities remain constant throughout the reach and thus the K_2 value is constant, i.e., 2.92 per day. From observed cross section data (neglecting the influence of bridge abutments), K_2 values varied from 0.87 to 6.41 per day in the free flowing subreach. Although bridge locations had a significant influence in the free flowing subreach, bridges in the pooled area did not. Within the pooled area, K_2 values ranged from 0.06 to 1.02 per day. The weighted average K_2 values for the free flowing and the pooled areas, respectively, were 3.21 and 0.37 per day. Use of the K_2 value developed by the generalized stream geometry equations would have produced a significant error in computing a dissolved oxygen profile for the 22.4-mile reach.

Textbook values for K_2 pose similar pitfalls. Babbitt and Baumann[15] values of K_2 for the free flowing section of the reach would range from 0.50 to 0.70 per day, while those for the sluggish pool would range from 0.20 to 0.35 per day.

The purpose of this example is to emphasize the essentiality of cross section data to reasonable estimates of parametric values independent of the approach, conceptual or pragmatic, for assessing the waste assimilative capacities of streams.

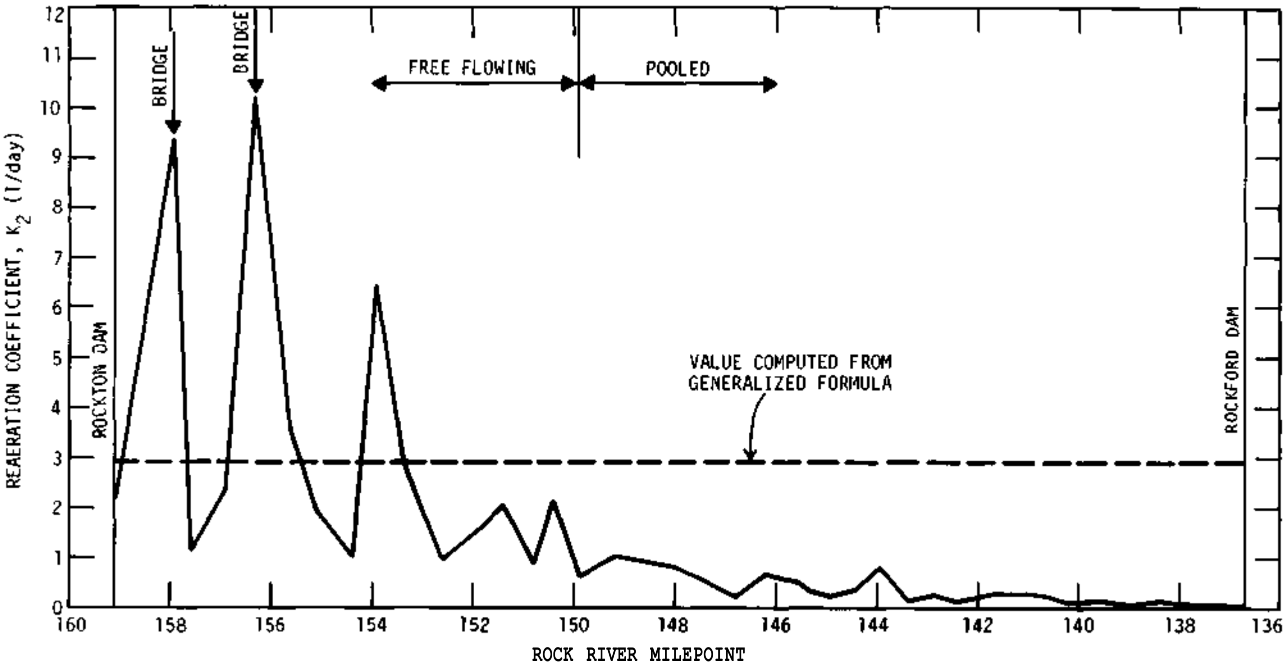


Figure 3. Reaeration coefficients at various locations along upper Rock River

CASE I: CONCEPTUAL APPROACH

As mentioned earlier the procedures that are developed for the conceptual approach are principally applicable to locating a waste treatment plant outlet on a stream that heretofore has not been receiving waste effluents. This is not to suggest that certain field work and laboratory investigations are not required. Certainly the hydrologic characteristics and geometry of the stream system must be determined, and estimates based upon laboratory observations of BOD progressions are in order. Procedures for the BOD evaluations are outlined in a later section of this report.

The predictive equations for BOD and DO profiles along a stream are based upon the following assumptions:

1) Stream flow is steady and uniform.
2) BOD and DO are uniformly distributed over each cross section, so that the equation can be written in one-dimensional form.
3) The process for any river reach as a whole is a steady state process, the conditions at every cross section remaining unchanged with time.
4) Bacterial oxidation of carbonaceous and nitrogenous matter and the absorption of oxygen from the atmosphere are first order reactions.
5) The rate of BOD removal is proportional to the amount of BOD present, and the rate of reaeration is proportional to the oxygen deficit.

If oxygen balance equations are applied to relatively short reaches of the receiving stream, errors introduced by the above assumptions will be of minor importance because the rate coefficients will change very little.

Oxygen Balance Equations

The rate of removal of BOD in a stream, on the basis of first order kinetics, is given by

$$\partial L/\partial t \;=\; -K_C\,L_C - K_N\,L_N \tag{3}$$

where

L = total BOD remaining at any time t, mg/l
L_C = carbonaceous BOD remaining at any time t, mg/l
L_N = nitrogenous BOD remaining at any time t, mg/l
K_C = carbonaceous deoxygenation coefficient to the base e, per day
K_N = nitrogenous deoxygenation coefficient to the base e, per day

Integration of the carbonaceous terms of this equation yields

$$L_C \;=\; L_{aC}\,e^{-K_C t} \tag{4}$$

where L_{aC} is the ultimate carbonaceous demand.
aC

Integration of the nitrogenous terms of equation 3 yields

$$L_N \;=\; L_{aN}\,e^{-K_N t} \tag{5}$$

where L_{aN} is the ultimate nitrogenous demand. If $L = L_C + L_N$ then

$$L \;=\; L_{aC}\,e^{-K_C t} \;+\; L_{aN}\,e^{-K_N t} \tag{6}$$

If there is a time lag, t_o, for the onset of nitrogenous demand from the carbonaceous demand, which invariably is the case, then equation 6 takes the form

$$L \;=\; L_{aC}\,e^{-K_C t} \;+\; L_{aN}\;e^{-K_N(t-t_o)} \tag{7}$$

The useful application of these equations has been discouraged by the fact that L_C, L_N, and the rate coefficients K_C and K_N are usually unknown. Methods for developing reasonable estimates of these factors will be outlined later in this report.

The reaeration process in a stream due to absorption from the atmosphere is also considered a first order reaction. The rate of the process is dependent upon the DO deficit, D, which is defined as the difference between the DO saturation concentration, C_s, at the stream water temperature and the observed DO concentration, C, in the stream. The process can be expressed as

$$\partial C/\partial t \;=\; K_2(C_s - C) \;=\; K_2 D \;=\; -\,\partial D/\partial t \tag{8}$$

where K_2 is the reaeration coefficient to the base e, per day.

Since L, the BOD remaining to be satisfied, is measured in terms of oxygen consumed, and the rate of oxygen consumption equals the rate at which the BOD is satisfied, then

$$\partial D/\partial t = \partial L/\partial t \tag{9}$$

Combining the rates of these two reactions, equations 3 and 8, and writing the resulting equation in terms of the DO deficit gives the response of receiving waters to a single waste source. The differential equation is

$$\partial D/\partial t = K_C L_C + K_N L_N - K_2 D \tag{10}$$

This equation simply states that the rate of change of the DO deficit with time-of-travel downstream is a function of the rate of oxygen utilization by demand loads and the rate of oxygen replenishment from the atmosphere.

The solution of this differential equation yields a modification of the classical dissolved oxygen sag formula postulated by Streeter and Phelps,[2] as follows:

$$\begin{aligned} D = {} & [K_C L_{aC}/(K_2-K_C)]\ (e^{-K_C t} - e^{-K_2 t}) \\ & + [K_N L_{aN}/(K_2-K_N)]\ (e^{-K_N(t-t_o)} - e^{-K_2(t-t_o)}) + D_a\, e^{-K_2 t} \end{aligned} \tag{11}$$

where D_a is the initial dissolved oxygen deficit, t is the time-of-travel or incubation time in days, and all other terms are as previously noted.

Although equation 11 appears complex, with the exception of time (t and t_o), values for only 6 terms are required for a solution. Proper application of the equation will permit estimates of DO deficits along the course of the receiving stream's travel. If nitrification is not likely to occur in the stream, then the term pertaining to nitrogenous demand can be omitted. The determination of time, t, is dependent upon stream flows, in combination with effluent flows, and the stream geometry of the water course system.

As mentioned earlier, methods for estimating rates of oxygen utilization will be covered later. Nevertheless the order of magnitude for K_C and K_N found by other investigators should be of interest. Zanoni,[9] on the basis of his extensive studies of the long term deoxygenation behavior of the effluent from the activated sludge treatment plant in Milwaukee, Wisconsin, found that the values for K_C ranged from 0.07 to 0.17 per day with an average value of 0.12 per day. Values for K_N were found to vary from 0.25 to 0.37 per day with an average of 0.29 per day. He also found that the time lag for the onset of nitrogenous demand varied from 1 to 11 days. Several investigators (Butts et al.,[6] Gannon,[8] O'Connor and DiToro[7]) found that the rate of removal of BOD in-stream was higher than the equivalent BOD removal under standard laboratory conditions. Sometimes the BOD removal may be due to biological abstractions rather than oxidation (Gannon[8]), and at other times BOD removal from the flowing waters may be due to sedimentation. In the latter case, it may be added back as a result of scour, or there may be an exertion of oxygen demand due to the products of putrefaction of the settled solids. It is obviously quite difficult to generalize and assign numerical values for these factors in a conceptual simulation model.

Reaeration Rates

Extensive research has been conducted for evaluating the stream reaeration rate coefficient. Several empirical and semiempirical formulations for predicting K_2 values have been proposed. Austin and Sollo[10] investigated the validity of a few of these formulas by carrying out field tests to evaluate K_2 values for a stream and then comparing them with the computed K_2 values obtained from equations developed by four different groups of investigators. They concluded that the prediction equation of Churchill et al.,[16] as shown below, gave the best fit to their observed K_2 values at 20 C:

$$K_2 = 11.57\, V^{0.969}\, H^{-1.673} \tag{12}$$

The next best fit was obtained with the equation developed by O'Connor and Dobbins:[17]

$$K_2 = 13.0\, V^{1/2}\, H^{-3/2} \tag{13}$$

In these two equations, V is the average velocity of flow in feet per second, and H is the average depth in feet. These two equations have been used extensively in river oxygen balance studies. The equation developed by Churchill et al.[16] is empirical in nature and was developed for rivers in the TVA system with flow depths varying from 2.83 to 11.41 feet and with velocities of flow in the range of 1.85 to 5.00 fps. Caution should be exercised not to use equation 12 for ranges of velocity and depth of flow not contained in the original formulation.

It is imperative that the mean velocity of flow in the stream for the critical low flow be assessed with utmost care. Velocity is a governing factor in the estimation of K_2 values and is also used in computing the time-of-travel between river locations of interest.

Rate—Temperature Relationships

The determinations of BOD, L, and the rates of deoxygenation, K_C and K_N, as well as aeration, K_2, are generally made at a standard temperature of 20 C. To apply these values to stream waters at temperatures other than 20 C, corrections have to be made. In the case of rates the Arrhenius model is generally the basis for applying temperature corrections. Its form is

$$K_T = K_{20}\,\theta^{T-20} \qquad (14)$$

where

K_T = the rate coefficient at temperature T, °C

K_{20} = the rate coefficient at 20 C

T = the observed temperature, °C

θ = a constant

The following expressions are used for temperature correction purposes for the parameters that have thus far been discussed:

$$K_{C(T)} = K_{C(20)} \times 1.047^{T-20} \qquad (15)$$

$$K_{N(T)} = K_{N(20)} \times 1.097^{T-20} \quad \text{Range 10-22 C} \qquad (16)^{9}$$

$$K_{N(T)} = K_{N(20)} \times 1.203 \times 0.877^{T-22} \quad \text{Range 22-30 C} \qquad (17)^{9}$$

$$K_{2(T)} = K_{2(20)} \times 1.024^{T-20} \qquad (18)$$

$$L_{aC(T)} = L_{aC(20)} \times (0.02T + 0.6) \qquad (19)$$

Dissolved Oxygen Saturation

Several investigators have reported on the saturation of dissolved oxygen in fresh water with slightly varying results. Probably the most comprehensive and authoritative is the study undertaken by the American Society of Civil Engineers.[18] The solubility of DO in fresh water was found to be best described by

$$\text{DO} = 14.652 - 0.41022T + 0.0079910T^2 - 0.000077774T^3 \qquad (20)$$

where

DO = the dissolved oxygen, mg/l

T = water temperature, °C

The dissolved oxygen saturation values developed from this expression are included in table 2 for temperatures ranging from 0 to 30 C.

Design Problem

To demonstrate the use of the mathematical expressions developed thus far for a conceptual approach to assessing the assimilative capacity of a stream, a typical design problem will be analyzed.

A treated waste load from a community of 50,000 persons is to be discharged into a stream which is not receiving any other significant waste discharge. The flow of the waste effluent will average 100 gallons per capita per day (gcd) and the 7"day, 10-year low flow, Q, of the stream is 7.0 cfs. The treatment process will be designed to produce an average 5-day BOD (BOD_5) of 4.1 mg/l and an

Table 2. Dissolved Oxygen Saturation Values (Ref. 18)

(Dissolved oxygen in milligrams per liter)

Temperature °C	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	14.65	14.61	14.57	14.53	14.49	14.45	14.41	14.37	14.33	14.29
1	14.25	14.21	14.17	14.13	14.09	14.05	14.02	13.98	13.94	13.90
2	13.86	13.82	13.79	13.75	13.71	13.68	13.64	13.60	13.56	13.53
3	13.49	13.46	13.42	13.38	13.35	13.31	13.28	13.24	13.20	13.17
4	13.13	13.10	13.06	13.03	13.00	12.96	12.93	12.89	12.86	12.82
5	12.79	12.76	12.72	12.69	12.66	12.62	12.59	12.56	12.53	12.49
6	12.46	12.43	12.40	12.36	12.33	12.30	12.27	12.24	12.21	12.18
7	12.14	12.11	12.08	12.05	12.02	11.99	11.96	11.93	11.90	11.87
8	11.84	11.81	11.78	11.75	11.72	11.70	11.67	11.64	11.61	11.58
9	11.55	11.52	11.49	11.47	11.44	11.41	11.38	11.35	11.33	11.30
10	11.27	11.24	11.22	11.19	11.16	11.14	11.11	11.08	11.06	11.03
11	11.00	10.98	10.95	10.93	10.90	10.87	10.85	10.82	10.80	10.77
12	10.75	10.72	10.70	10.67	10.65	10.62	10.60	10.57	10.55	10.52
13	10.50	10.48	10.45	10.43	10.40	10.38	10.36	10.33	10.31	10.28
14	10.26	10.24	10.22	10.19	10.17	10.15	10.12	10.10	10.08	10.06
15	10.03	10.01	9.99	9.97	9.95	9.92	9.90	9.88	9.86	9.84
16	9.82	9.79	9.77	9.75	9.73	9.71	9.69	9.67	9.65	9.63
17	9.61	9.58	9.56	9.54	9.52	9.50	9.48	9.46	9.44	9.42
18	9.40	9.38	9.36	9.34	9.32	9.30	9.29	9.27	9.25	9.23
19	9.21	9.19	9.17	9.15	9.13	9.12	9.10	9.08	9.06	9.04
20	9.02	9.00	8.98	8.97	8.95	8.93	8.91	8.90	8.88	8.86
21	8.84	8.82	8.81	8.79	8.77	8.75	8.74	8.72	8.70	8.68
22	8.67	8.65	8.63	8.62	8.60	8.58	8.56	8.55	8.53	8.52
23	8.50	8.48	8.46	8.45	8.43	8.42	8.40	8.38	8.37	8.35
24	8.33	8.32	8.30	8.29	8.27	8.25	8.24	8.22	8.21	8.19
25	8.18	8.16	8.14	8.13	8.11	8.10	8.08	8.07	8.05	8.04
26	8.02	8.01	7.99	7.98	7.96	7.95	7.93	7.92	7.90	7.89
27	7.87	7.86	7.84	7.83	7.81	7.80	7.78	7.77	7.75	7.74
28	7.72	7.71	7.69	7.68	7.66	7.65	7.64	7.62	7.61	7.59
29	7.58	7.56	7.55	7.54	7.52	7.51	7.49	7.48	7.47	7.45
30	7.44	7.42	7.41	7.40	7.38	7.37	7.35	7.34	7.32	7.31

ammonia-N concentration of 2.5 mg/l. After the mixing of the effluent with the stream waters, the estimated carbonaceous deoxygenation coefficient is 0.12 per day at 20 C; similarly, the nitrogenous deoxygenation coefficient is estimated to be 0.29 per day at 20 C. The time lag, t_o, is estimated to be 1.0 day. The width and depth of flow in the stream bed has been found to be reasonably constant at 15.0 and 1.5 feet, respectively. The highest temperature of the stream waters during critical low flow is 25 C. Average DO concentration in the stream, upstream of the waste discharge, is 7.0 mg/l and the anticipated DO concentration in the effluent is 2.0 mg/l. It is the purpose of this investigation to determine whether or not the Illinois stream water quality standards for DO levels will be maintained under the conditions described.

Known:

Effluent Q = 7.74 cfs $(50{,}000 \times 100 \times 1.54/10^{6})$
[the factor 1.54 converts mgd to cfs]
Upstream Q = 7.00 cfs
Downstream Q = 14.74 cfs
Effluent BOD_5 = 4.1 mg/l (carbonaceous)
Effluent NH_3 = 2.5 mg/l (as N)
Effluent DO = 2.0 mg/l
Stream DO = 7.0 mg/l
Stream width = 15.0 ft
Stream depth H = 1.5 ft
$K_C(20)$ = 0.12 per day
$K_N(20)$ = 0.29 per day
T = 25C

The ultimate carbonaceous demand L_{aC} is determined from equation 4 thusly:

$$L_{aC} = BOD_5(\text{carbonaceous})/(1 - e^{-K_C \times 5})$$

and substituting the values gives

$$L_{aC} = 4.1/(1 - e^{-0.60})$$
$$L_{aC} = 9.10 \text{ mg/l}$$

In determining the ultimate nitrogenous demand L_{aN} the oxygen consumption due to nitrification can be considered to occur at the ratio of 4.57 mg/l of DO per mg/l of NH_3-N. Therefore the ultimate nitrogenous demand is

$$L_{aN} = 2.5 \times 4.57 = 11.4 \text{ mg/l}$$

The reaeration coefficient K_2 can be determined by several means, but here equation 13 will be used. Since the cross section area A, the depth H, and the flow Q of the stream are known, then

$$V = Q/A = 14.74/(15.0 \times 1.5) = 0.66 \text{ fps}$$

and, from equation 13

$$K_2 = 13.0 \times 0.66^{1/2} \times 1.5^{-3/2}$$
$$K_2 = 5.7 \text{ per day}$$

Applying temperature correction factors for the temperature dependent parameters in accordance with equations 15, 17, 18, and 19 yields

$$K_{C(25)} = 0.12 \times 1.047^{5} = 0.15 \text{ per day}$$
$$K_{N(25)} = 0.29 \times 1.203 \times 0.877^{3} = 0.24 \text{ per day}$$
$$K_{2(25)} = 5.7 \times 1.024^{5} = 6.4 \text{ per day}$$
$$L_{aC(25)} = 9.10\ (0.02 \times 25 + 0.6) = 10.0 \text{ mg/l}$$

The initial DO in the stream has been given as 7.0 mg/l, and the DO immediately downstream of the waste effluent is computed as

$$\frac{(7.74 \times 2) + (7.0 \times 7.0)}{(7.74 + 7.0)} = 4.37 \text{ mg/l}$$

From table 2 the DO saturation concentration at 25C is 8.18 mg/l. Therefore the initial DO deficit D_a is

$$D_a = 8.18 - 4.37 = 3.81 \text{ mg/l}$$

The parametric values for determining the DO concentration along the course of the stream have now been determined. There remains some judgment in assigning realistic time-of-travel, t, values along the length of the stream. Although flow is steady it is not necessarily constant. This requires a method of flow-routing between reaches or stream flow gaging stations. Where flow data are meager, a simple unit flow addition per mile of stream or per square mile of drainage area, taking into account tributary flows, will usually be sufficient for 7-day, once-in-10-year low flow conditions.

The values that have been computed for use in equation 11 are:

K_C = 0.15 per day
K_2 = 6.4 per day
L_{aC} = 10.0 mg/l
K_N = 0.24 per day
L_{aN} = 11.4 mg/l
D_a = 3.81 mg/l
t_o = 1 day

The results of using these values and selected t values in equation 11 are tabulated in table 3. Although desirable, it is not possible to give a neat expression for the critical time, i.e., when the DO deficit is maximum, when both nitrogenous and carbonaceous demands are being exerted. Hence it is necessary to trace the DO concentrations, using equation 11, for a sufficient length of time to determine the critical DO point. From table 3 it is seen that the minimum DO, 4.37 mg/l, occurs at the point of discharge. A stream stretch of about 1.0 mile below the outfall is required before a recovery of 6.0 mg/l of DO is attained. The stream recovers rapidly up to MP 10.8 below the outfall. Then, because of the exertion of the nitrogenous demand, a second minimum is reached near MP 16.2. Thereafter the DO levels begin to increase, reaching stability at some point downstream.

Table 3. Dissolved Oxygen Concentration in the Stream at Various Reaches of the Stream

t (*days*)	Miles below outfall	DO deficit (*mg/l*)	DO (*mg/l*)
0.0	0.0	3.81	4.37
0.1	1.08	2.12	6.06
0.2	2.16	1.23	6.95
0.3	3.24	0.75	7.43
0.4	4.32	0.50	7.68
0.5	5.40	0.37	7.81
1.0	10.8	0.21	7.97
1.1	11.9	0.40	7.78
1.2	13.0	0.50	7.68
1.3	14.0	0.55	7.63
1.4	15.1	0.56	7.62
1.5	16.2	0.57	7.61
2.0	21.6	0.53	7.65

It should be pointed out here that the nitrogenous demand term in equation 11 should be omitted in any computations for values of $t < t$. Also it has been assumed that the waste characteristics, stream geometry, and other factors in this example are constant. If there are any significant changes in the stream system, for example, a dam, a tributary, riffles, etc., the parametric values used in equation 11 must be reassessed. In situations of this nature the stream should be divided into reaches for which all the parameters affecting waste assimilative capacities can be taken as reasonably stable. The values determined for D, L_{aC}, and L_{aN} at the end of any reach can then be used as the initial conditions for the next reach in assessing its capacity to assimilate waste.

Summary

The steps involved in estimating the waste assimilative capacity of a stream on the basis of a conceptual approach can be summarized as follows:

1) Develop full understanding of the stream length, its channel geometry, water stage and flow patterns, and the general hydrologic features of the watershed.

2) Determine the 7-day, once-in-10-year low flow of the receiving stream and the design flow of the waste treatment plant.

3) Obtain reasonable estimates of the parameters defining the characteristics of the in-stream treated waste, namely K_C, K_N, L_{aC}, L_{aN}, and t_o.

4) Compute the stream aeration coefficients consistent with channel geometry and flows for the conditions indicated in step 2.

5) Select the highest mean water temperature anticipated for the flow conditions and apply correction factors to all temperature dependent parameters,

6) Determine the mean DO concentration of the stream waters upstream of the waste discharge point. Knowledge of the DO concentration expected in the treated effluent and the DO levels upstream of that point allows computation of the initial DO concentration in the stream after discharge by a mass balance expression.

7) Select the DO saturation concentration from table 2 consistent with the water temperature selected in step 5. The difference between the saturation DO values and the initial DO concentration computed in step 6 represents the initial DO deficit, D_a.

8) Using equation 11 determine the DO deficits and hence the DO concentrations along the stream course for a sufficient length, and at predetermined intervals, to define the DO sag.

9) Re-evaluate the parametric values used in equation 11 whenever warranted by significant physical changes in the stream system.

CASE II: PRAGMATIC APPROACH

This section of the report outlines the procedures applicable to a waste assimilative investigation of a stream where the DO levels in the stream are being influenced by an existing waste discharge, or where detectable BOD changes are occurring in decreasing order with the downstream movement of the stream. Either occurrence, if judiciously measured and properly evaluated, will provide the bases for reasonable assessments for the mechanisms of self-purification within the stream. The procedures outlined will give realistic parametric values for a stream subjected to the discharge of untreated or partially treated waste effluents. However, some accuracy is lost when values under one condition are used to predict another condition.

Oxygen Balance Equation

The dissolved oxygen balance within a stream can be expressed in various ways, but for the pragmatic approach it can be formulated as follows:

$$DO_n = DO_a - DO_u + DO_r + DO_x \tag{21}$$

where

DO_n = net dissolved oxygen at the end of a stream reach

DO_a = initial dissolved oxygen of the reach

DO_u = dissolved oxygen consumed biologically within a reach

DO_r = dissolved oxygen derived from natural aeration within a reach

DO_x = dissolved oxygen derived from channel dams, tributaries, etc.

Basically equation 21 states that the DO used and the DO added within a stream reach can be evaluated separately and added algebraically. This equation uses the same basic concepts as employed in the application of the Streeter-Phelps[2] expression.

The difference lies in the fact that the Streeter-Phelps expression or any modification of it, as demonstrated in the conceptual approach, specifies all the influential components in a single differential form. The following discussion describes the procedures involved in determining the values that should be assigned to the terms in equation 21.

Reaeration

In most streams in Illinois three significant influences affecting the dissolved oxygen levels in various reaches must be considered. These include 1) the stream geometry, 2) dams, and 3) tributaries. For the purpose of this report the oxygen absorbed from the atmosphere because of the physical characteristics of the stream's channel and flow shall be considered natural stream aeration, DO_r . The influence of dams and tributaries will be included in the Do_x term.

Influence of Stream Geometry. The best procedure for estimating natural aeration in streams for use in equation 21 is that proposed originally by Black and Phelps[19,20] and later modified and improved by Velz[21] and Gannon.[22] The Velz modification of the Black and Phelps methods has been used extensively by Butts et al.[6] in estimating the reaeration characteristics of the Illinois River. The reaeration coefficient K_2 , the factor normally associated with stream reaeration, does not directly enter into the computations here, but an equivalent K_2 is developed by the Velz methodology.

Natural stream aeration is based on the premise that the rate at which water is aerated under constant temperature and turbulent mixing conditions is directly proportional to the existing oxygen saturation deficit. Also, aeration is directly proportional to velocity and inversely proportional to depth of a stream reach. In the Velz modification of the Black and Phelps method, a mix or exposure time is used, which is analogous to the velocity factor found in most reaeration formulas. The mix time is the time required to expose an element of water to the air-water interface. It follows that the mix time is highly dependent upon depth. A modified but equivalent form of the original Black and Phelps reaeration equation given by Gannon[22] is

$$R = 100 - [(1 - \frac{B_O}{100})(81.06)(e^{-K} + \frac{e^{-9K}}{9} + \frac{e^{-25K}}{25} + \cdots \frac{e^{-(2n-1)^2K}}{(2n-1)^2})] \qquad (22)$$

where

R = percent of saturation of DO absorbed per mix

B_o = initial DO in percent of saturation

$K = \pi^2 am/4H_1^2$ in which m is the mix or exposure time in hours, H_1 is the average depth in centimeters, and a is a diffusion coefficient used by Velz.

The diffusion coefficient a was found by Velz[23] to vary with temperature according to the expression $a_T = a_{20}(1.1^{T-20})$ in which a_T and a_{20} are the coefficients at T°C and 20C, respectively. When the depth is expressed in feet a_{20} equals 0.00153; therefore, for most engineering investigations

$$a_T = 0.00153\ (1.1^{T-20}) \tag{23}$$

Using average depths in feet and mix time in minutes and reducing factors yields

$$K = (6.2918 \times 10^{-5}/H^2)\ (1.1^{T-20})M \tag{24}$$

where M is the mix time in minutes and H is the average depth in feet.

M can be estimated by either of two equations experimentally developed by Gannon:[22]

$$\text{For depths} >3 \text{ feet, } M = 13.94\ (\log_e H) - 7.45 \tag{25}$$

$$\text{For depths} <3 \text{ feet, } M = 0.721H + 2.279 \tag{26}$$

Equations 22, 2k, and 25 or 26 are required to determine the natural stream aeration term DO_r included in equation 21. These expressions are easily programmed on a digital computer or on an advanced programmable desk type calculator. The series expansion in equation 22 is continued until the last additive term change is no greater than 1 in the tenth decimal place. Butts and Schnepper[24] have developed a nomographical solution for equations 22, 2k, and 25 for streams having average depths *greater than* 5 feet. This nomograph is presented as figure k. For depths *less than* 5 feet, equation 2k can be solved using M computed by either equation 25 or 26; the computed value for K is then used to select R_o from the modified Velz curve shown as figure 5.

To illustrate the use of these expressions in determining the quantity of oxygen absorbed from the atmosphere, in a reach of stream without tributary sources of oxygen and where channel dams do not exist, two examples are given. The first one is for a depth of 12.1 feet.

Assume:

Depth H = 12.1 ft
Upstream T = 24.9C
Downstream T = 25.1C
Average T = 25C
Upstream Q = 1240 cfs
Downstream Q = 1280 cfs
Average Q = 1260 cfs

Average V = 1.8 fps
Length of reach = 2.8 miles
DO_a = 3.5 mg/l

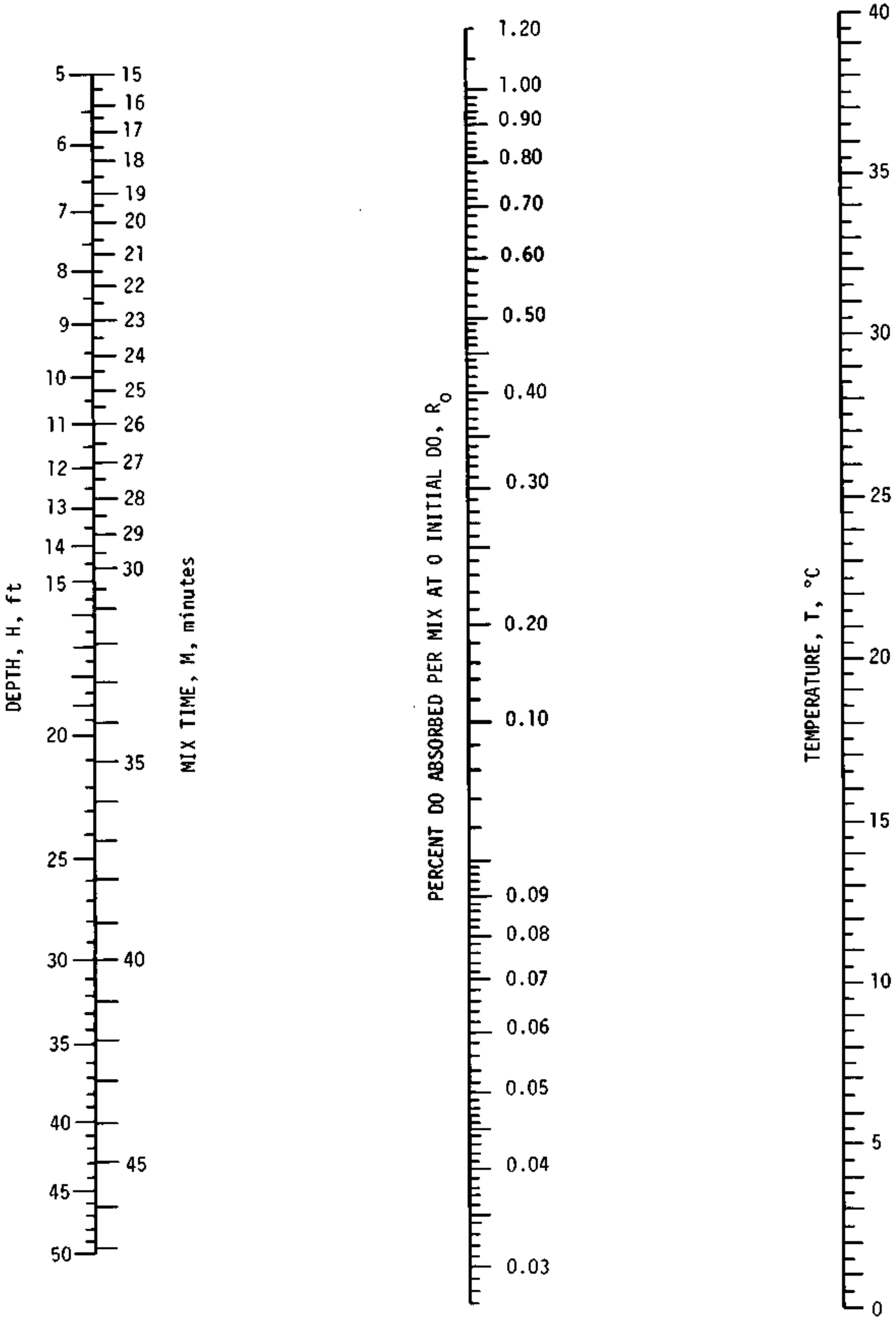


Figure 4. Oxygen absorption nomograph

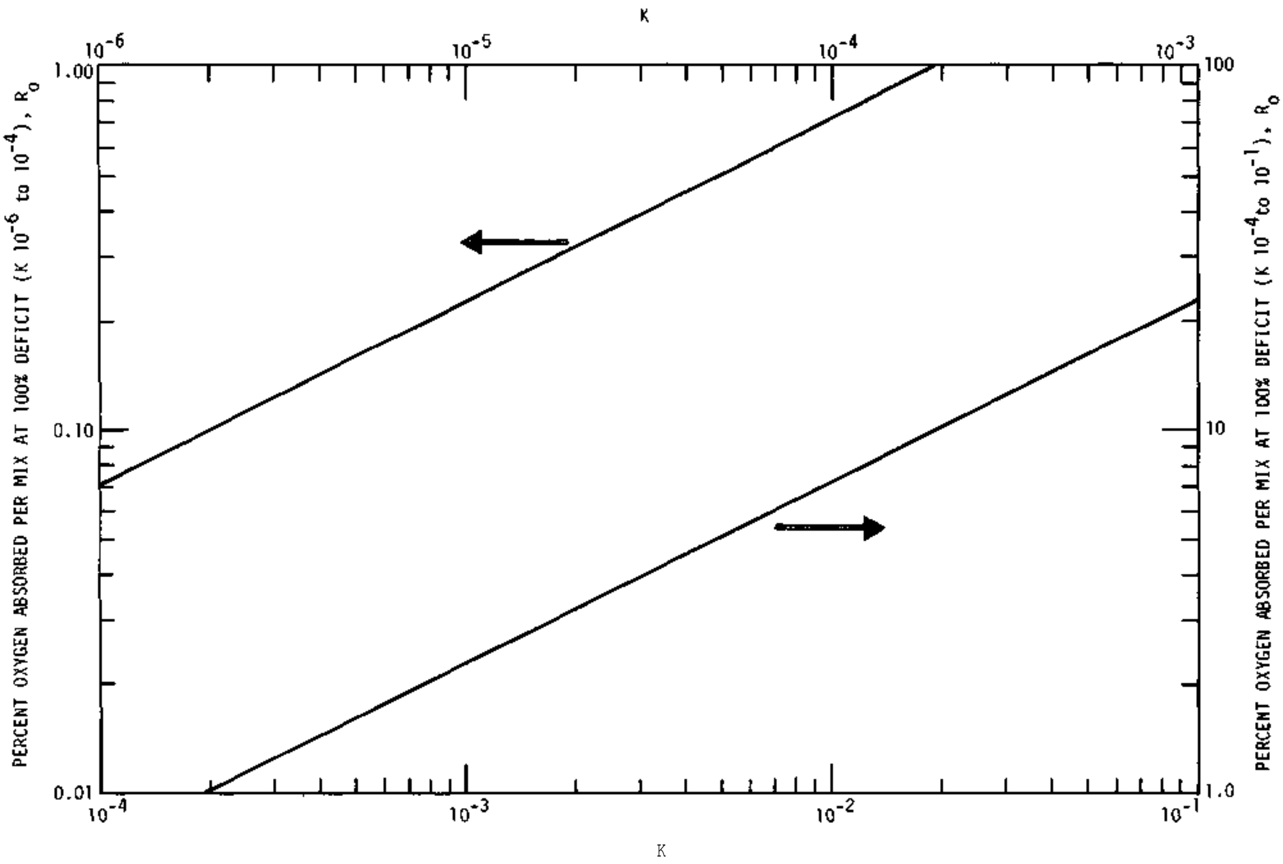


Figure 5. Modified Velz curve

Find:

Mix time, M; the percent DO absorbed per mix at zero initial DO, R_o: and the quantity of atmospheric oxygen absorbed in the stream reach, DO_r

Equations 22, 24, and 25 can be used for a solution, but the nomograph (figure 4) will be used here. On figure *k*, by a straightedge connect the depth scale H at 12.1 with 25C (average T) on the temperature scale; read 27 minutes on the mix time M scale and 0.31 on the R_o scale. Therefore:

M = **27 minutes**

R_o = **0.31 percent**

R is the percent of DO saturation absorbed per mix; R_o is that percent absorbed per mix at an initial DO of zero, i.e., 100 percent deficit. From table 2 the DO saturation in the stream is found to be about 43 percent (i.e., 100 x 3.5/8.18). Since, as mentioned earlier, the rate of reaeration is proportional to the DO deficit, the oxygen absorbed during an initial DO of zero would be greater than when the initial DO is 3.5 mg/l. Therefore to determine R, corrections must be made for R_o thusly: :

$$R = 0.31\ (1-3.5/8.18) = 0.18 \text{ percent}$$

At a saturation level of 8.18 mg/l, the pounds per day of oxygen, in the stream at average stream flow, is found to be 55,657 (i.e., $1260 \times 5.4 \times 8.18$). The value 5.k is a conversion factor for converting flow in cfs to million pounds. Therefore the quantity of oxygen absorbed is

$$(0.18/100)\ 55{,}657 = 100 \text{ lb/day/mix}$$

What is the number of mixes within the reach? The flow time in minutes, t', in the reach is

$$t' = (5280 \times 2.8)/(1.8 \times 60) = 137 \text{ minutes}$$

Since the mix time M from figure 4 is 27 minutes, the number of mixes within the reach is 5.1 (i.e., 137/27) and the quantity of dissolved oxygen absorbed is

$$DO_r = 5.1 \times 100 = 510 \text{ lb/day}$$

These procedures and computations can be summarized by the following expression:

$$DO_r = [1 - (DO_a/DO_s)]\ (R_o/100)\ (t'/M)\ (5.4\ Q \times DO_s) \tag{27}$$

where

DO_r = dissolved oxygen absorbed, lb/day

DO_a = initial dissolved oxygen concentration, mg/l

DO_s = dissolved oxygen at saturation, mg/l

R_o = percent dissolved oxygen absorbed at 100 percent DO deficit

t' = flow time of the stream within the reach, minutes

M = mix time, minutes

Q = average stream flow, cfs

In the second example, which follows, all assumptions are the same except the depth H is 2.8 feet. Here the solution requires the use of equations 2k and 26 in combination with figure 5. Substituting 2.8 feet into equation 26 gives

$$M = (0.721)\ (2.8) + 2.279$$

$$M = 4.28 \text{ minutes}$$

Substituting 4.28 (M) into equation 24 then gives the value of K as 3.89×10^{-5}, and from figure 5 the percent of oxygen absorbed per mix at 100 percent deficit is

$$R_o = 0.45 \text{ percent}$$

Substituting in equation 27 the values for R_o and M shows the quantity of dissolved oxygen absorbed DO_r to be 4460 lb/day. The significance of stream depth to reaeration is amply demonstrated by the two examples.

These examples have been used to illustrate only the mechanics of computing reaeration quantities as set forth by equation 27. In a situation where the reaeration is significant within a reach, an adjustment should be made in DO_a to take into account the change in dissolved oxygen. This adjustment is made by re-computing DO_r using for DO_a the average of the beginning and ending dissolved oxygen concentrations,

Influence of Dams. Low head channel dams and large navigation dams are very important factors when assessing the oxygen balance in a stream. There are certain disadvantages from a water quality standpoint associated with such structures but their reaeration potential is significant during overflow, particularly if a low DO concentration exists in the upstream pooled waters. This source of DO replenishment must be considered.

Procedures for estimating reaeration at channel dams and weirs have been developed by researchers in England.[25-28] The methods are easily applied and give satisfactory results. Little has been done in developing similar procedures for large navigation and power dams. The investigator is referred to the work of Preul and Holler[29] for these larger structures.

Often in small sluggish streams more oxygen may be absorbed by water overflowing a channel dam than in a long reach between dams. However, if the same reach were free flowing, this might not be the case, i.e., if the dams were absent, the reaeration in the same stretch of river could conceivably be greater than that provided by overflow at a dam. If water is saturated with oxygen, no uptake occurs at the dam overflow. If it is supersaturated, oxygen will be lost during dam overflow. Water, at a given percent supersaturation, will lose oxygen at the same rate that water at the same percent deficit will gain oxygen.

The basic channel dam reaeration formula takes the general form

$$r = 1 + 0.11\, qb\, (1 + 0.046T)h \tag{28}$$

where

r = dissolved oxygen deficit ratio at temperature T

q = water quality correction factor

b = weir correction factor

T = water temperature, °C

h = height through which the water falls, ft

The deficit dissolved oxygen ratio is defined by the expression

$$r = (C_s - C_A)/(C_s - C_B) \tag{29}$$

where

C_A = dissolved oxygen concentration upstream of the dam, mg/l

C_B = dissolved oxygen concentration downstream of the dam, mg/l

C_s = dissolved oxygen saturation concentration, mg/l

Although equations 28 and 29 are rather simplistic and do not include all potential parameters which could affect the reaeration of water overflowing a channel dam, they have been found to be quite reliable in predicting the change in oxygen content of water passing over a dam or weir.[27] The degree of accuracy in using the equations is dependent upon the estimate of factors q and b.

For assigning values for q, three generalized classifications of water have been developed from field observations. They are: clean or slightly polluted water $q = 1.25$; moderately polluted water $q = 1.0$; and grossly polluted water $q = 0.8$. A slightly polluted water is one in which no noticeable deterioration of water quality exists from sewage discharges; a moderately polluted stream is one which receives a significant quantity of sewage effluent; and a grossly polluted stream is one in which noxious conditions exist.

For estimating the value of b, the geometrical shape of the dam is taken into consideration. This factor is a function of the ratio of weir coefficients, W, of various geometrical designs to that of a free weir where

$$W = (r-1)/h$$

Weir coefficients have been established for a number of spillway types, and Gameson[25] in his original work has suggested assigned b values as follows:

Spillway type	b
Free	1.0
Step	1.3
Slope (ogee)	0.58
Sloping channel	0.17

For special situations engineering judgment is required. For example, a number of channel dams in Illinois are fitted with flashboards during the summer. In effect, this creates a free fall in combination with some other configuration. A value of b, say 0.75, could be used for a flashboard installation on top of an ogee spillway. This combination would certainly justify a value less than 1.0, that for a free weir, because the energy dissipation of the water flowing over the flashboards onto the curved ogee surface would not be as great as that for a flat surface such as usually exists below free and step weirs.

Aeration at a spillway takes place in three phases: 1) during the fall, 2) at the apron from splashing, and 3) from the diffusion of oxygen due to entrained air bubbles. Gameson[25] has found that little aeration occurs in the fall. Most DO uptake occurs at the apron; consequently, the greater the energy dissipation at the apron the greater the aeration. Therefore, if an ogee spillway is designed with an energy dissipator, such as a hydraulic jump, the value of b should be increased accordingly.

The following example illustrates the use of the expressions developed when considering the reaeration potential of a dam.

Assume:

Shape of dam = sloping channel type with a hydraulic jump at the bottom

C_A = 1.9 mg/l

T = 28.9C

h = 6.0 ft

q = 0.8

b = 0.17+; because of hydraulic jump use 0.30

Find:

DO concentration immediately below the dam, C_B.

Use of equation 28 with substitution of values for q, b, T, and h gives a dissolved oxygen deficit ratio r equal to 1.37. Use of equation 29 with a value of C_s equal to 7.59 mg/l (from table 2) determines C_B to be 3.44 mg/l.

Influence of -Tributaries. Tributary sources of DO are often an important consideration in deriving a DO balance in stream waters. These sources may be tributary streams or outfall sewers. The downstream effect of any DO input is determined by mass balance computations. In terms of pounds per day the tributary load can simply be added to the mainstream load occurring above the confluence. Two examples will be used to demonstrate the influence of tributary sources of DO. The first one involves a tributary stream.

Assume:

Tributary flow Q_1 = 230 cfs

Tributary DO = 5.5 mg/l

Mainstream Q_2 = 560 cfs

Mainstream DO = 4.0 mg/l

Find:

DO load (lb/day) and concentration downstream of the confluence.

$$\text{Tributary load} = (Q_1)(DO)(5.4) = (230)(5.5)(5.4) = 6831 \text{ lb/day}$$

Similarly

Mainstream load = 12,096 lb/day

The load downstream of the confluence is therefore 18,927 lb/day (i.e., 6831 + 12,096). The DO concentration downstream of the confluence is thus

$$18{,}927/(Q_1 + Q_2)(5.4) = 4.4 \text{ mg/l}$$

The second example involves the design of an outfall structure to achieve a minimum DO at the point of discharge. From preliminary design data it appears that a 5-foot difference in elevation is possible between the outfall crest and the 7-day, 10-year low flow river stage. Utilizing this head difference, determine if a free falling two-step weir at the outfall will insure a minimum DO of 5.0 mg/l at the point of discharge.

Assume:

Sewage Q = 5 mgd = 7.73 cfs
Stream Q = 4 mgd = 6.18 cfs
Stream DO = 6.0 mg/l
Temperature of effluent = 20C
Final tank effluent DO = 2.0 mg/l
Quality factor q = 0.8
Weir fall h = 5.0 ft

Find:

DO in the stream at the point of discharge and the oxygen mass balance.

From the proposed spillway type, b equals 1.3. Use of equations 28 and 29, where the final tank effluent DO of 2.0 mg/l is equivalent to C_A, gives values for r and C_B of 2.1 and 5.67 mg/l, respectively. In this case C_B is the effluent DO as it reaches the stream. The resultant DO at the point of discharge is

$$\frac{(7.73)(5.67) + (6.18)(6.0)}{(7.73 + 6.18)} = 5.88 \text{ mg/l}$$

The quantity of oxygen in the upstream sector is

6.18 x 6.0 x 5.4 = 200 lb/day

That added by the effluent is

7.73 x 5.67 x 5.4 = 237 lb/day

Oxygen Used

The DO_u term in equation 21 represents the oxygen consumed biologically within a stream reach. This term can be evaluated by three methods: 1) observed dissolved oxygen concentrations in conjunction with reaeration estimates, 2) bottle BOD and deoxygenation rate determinations of river water samples, and 3) long term bottle BOD progression evaluation of a waste effluent. The methods are listed in order of preference; however, the one used will probably be dictated by the existing data or the resources available for collecting usable data.

Method 1 is basically a computational procedure involving observed field DOs adjusted for DO from natural aeration and DO inputs from other sources as previously discussed. Rearrangement of equation 21 yields

$$DO_u = DO_a - DO_n + DO_r + DO_x \tag{30}$$

Note that the mathematical difference between the first two terms, DO_a and DO_n, is simply the observed reduction in DO concentration during stream travel from the beginning of the reach to the end of the reach. The DO_u is this reduction plus any DO inputs between each station for which DO concentrations have been observed. For a series of stream reaches, each incremental DO_u value is added and the accumulated sums, with the corresponding time-of-travel in the stream, are fitted to the first order exponential expression

$$Y = L_a\left(1 - e^{-K_d t}\right) \tag{31}$$

where

Y = oxygen demand exerted (DO_u)
L_a = ultimate oxygen demand, including carbonaceous and nitrogenous
K_d = in-stream deoxygenation coefficient to the base e, per day
t = time-of-travel, days

Note: The coefficient K_d is comparable to the composite of the terms K_C and K_N previously defined in the discussion regarding the conceptual approach to waste assimilative analysis.

Observed data cannot be fitted directly to equation 31. It must be done by an iterative, trial and error process. This can be accomplished by a linearization process such as developed by Reed and Theriault[30] or least squares, or by a statistical nonlinearization process such as the method of steepest descent. These methods require use of either digital computer facilities or large capacity programmable calculators. The flow diagram and the computer program used by the Illinois State Water Survey for solving the problem by the Reed-Theriault method are given in appendixes A and B. There are other methods for solving equation 31 which will be discussed later.

The use of field DOs in estimating the waste degradation characteristics of a stream has certain advantages. The need for laboratory BOD tests is eliminated which saves time and money. The reliability of the results should be better since the measurement of dissolved oxygen is far more precise and accurate than the BOD test. Also, stream measured DOs take into account the *in situ* oxygen demand in the stream. This would include both dissolved and benthic demand, whereas laboratory BOD results generally reflect only the oxygen demand exerted by dissolved matter.

Table 4. Computed Values of DO Used and Time-of-Travel

Station	ΣDO_u (lb/day)	Σt (days)	Comments
1	0	0	
2	14,947	0.625	
3	25,701	1.500	Above a dam
4	25,701	1.500	Below a dam
5	39,474	1.750	
6	52,760	2.250	
7	61,246	3.250	
8	67,429	4.375	
9	70,253	5.175	Above tributary
10	70,253	5.175	Below tributary
11	72,890	6.000	

The main disadvantage of the method is that estimates have to be made of the reaeration inputs. However, this disadvantage, relative to the bottle BOD methods, is not as great as it might seem since reaeration estimates need to be made anyway in the final waste assimilative analysis of the stream.

Estimating K_d *and* L_a. Thus far, all of the expressions for use in undertaking a pragmatic approach to waste assimilative analysis have been defined and their application demonstrated except the terms L_a and K_d in equation 31. These will now be examined. The assumption here is that a summation of DO_u values and corresponding time-of-travel, t, have been computed along a stream (for a water temperature of 22C) as shown in table *k*.

In dealing with data of this nature for determining L_a and K_d the use of a computer or programmable calculator simplifies the solution, but here use will be made of the procedures developed by Thomas,[31] often called the Thomas slope method. The factor $(\Sigma t/\Sigma DO_u)^{1/3}$ is plotted on arithmetic graph paper against Σt values as shown in figure 6. A line of best fit is then drawn, often neglecting the first 2 points. The ordinate intercept y is noted as 0.031; the slope of the line, S, is computed as 0.00207 [i.e., (0.04347 − 0.03100)/6]. $K_{d(22)}$ is found to equal 0.4 per day from the expression

$$K_d = (6S)/y \qquad (32)$$

and $L_{a(22)}$ is found to equal 83,784 lb/day by the expression

$$L_a = 1/(K_d y^3) \qquad (33)$$

A curve fit of the DO_u data versus time-of-travel, using the values of K_d and L_a found by the Thomas slope method, is shown in figure 7. It can be seen that the value of I (computed DO_u in contrast to the observed DO_u) is determined by $Y = 83,784(1 - e^{-0.40t})$ A similar curve fit for the same data with a computer-oriented method is shown in figure 8. At this point it is well to recall that since the values computed are not based upon an observed temperature of 20C they must be recomputed for that base temperature by equations 15 and 19 for K_d and L_a, respectively.

Design Problem

The use of the mathematical expressions that have been developed for a pragmatic approach to assessing the

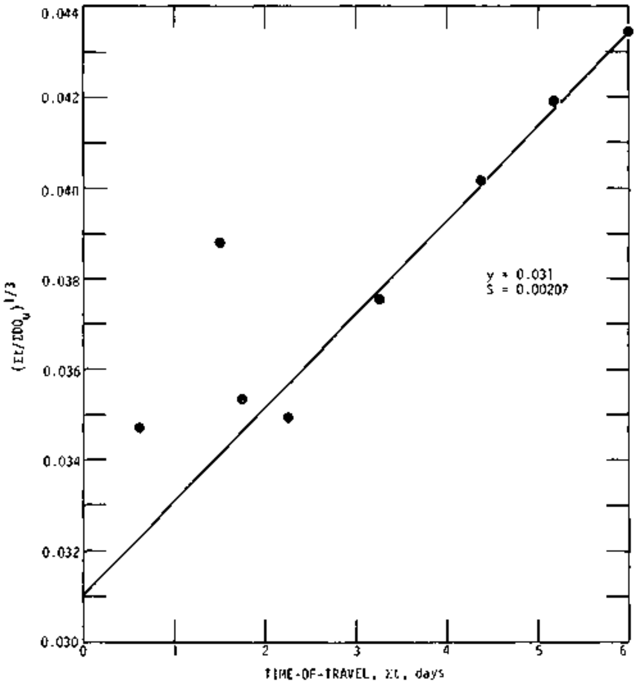


Figure 6. Thomas slope data transformation plot

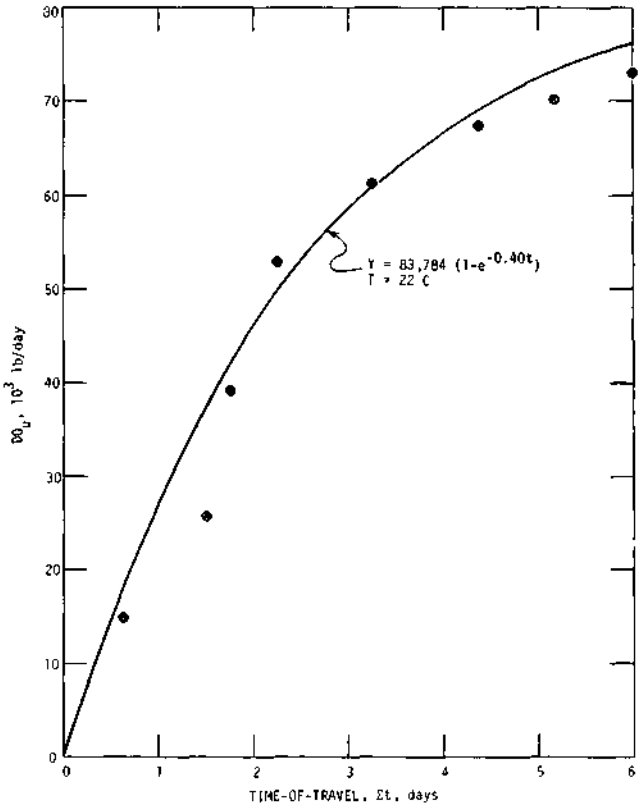


Figure 7. Thomas slope method curve fit

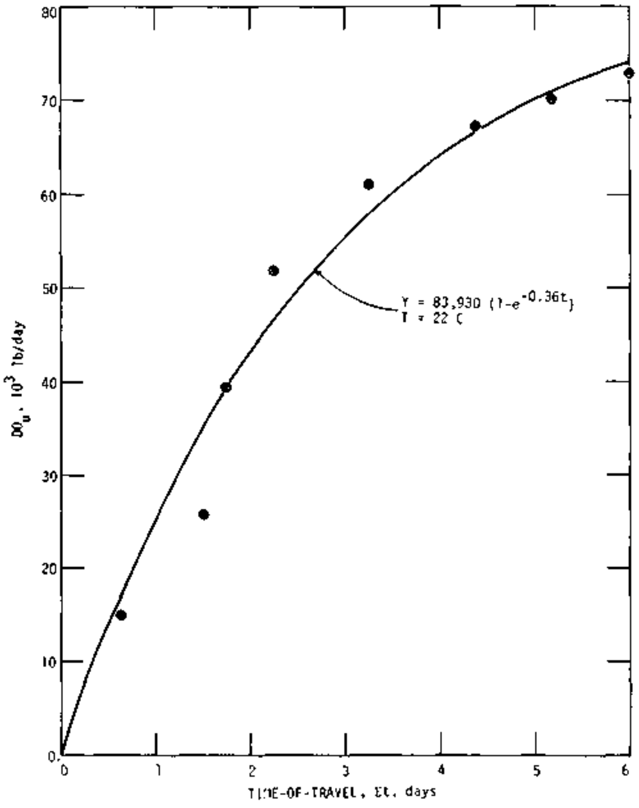


Figure 8. Computer calculated curve fit by steepest descent method

assimilative capacity of a stream will be demonstrated by a detailed analysis of a problem.

A long reach of a moderately sized river is subjected to pollution from an overloaded secondary treatment plant presently operating at a 65 percent BOD removal efficiency. Within the reach under investigation are an ogee type channel dam and inflow from a small, relatively clean tributary stream. Hydraulic and hydrologic analyses have been completed with the use of cross sections and flow data furnished by others. Some DO and BOD data were available but a preliminary review showed it to be inadequate. The 7-day, 10-year low flow of the receiving stream was found to be 560 cfs and that of the tributary 160 cfs. A design temperature of 28 C has been selected. The problem is to determine the configuration of DO concentrations along the reach of the stream at design flow and temperature if additional capacity were provided to raise the level of treatment to that expected in an efficient secondary sewage treatment plant.

The problem will be approached in two phases. First the deoxygenation coefficient K_d will be determined for existing conditions. Secondly a predictive profile of DO concentrations will be developed for design conditions.

A field survey for DO and temperature determinations at 11 stations on the stream was performed during summer low flow conditions. Field observations and subsequent computations are set forth in table 5.

Table 5. Field Observations and DO Computations

(1) Station	(2) Flow at sta (cfs)	(3)* Avg flow (cfs)	(4) Temperature (°C)	(5)* H (ft)	(6)* t (days)	(7) Obs (mg/l)	(8) DO_a (lb/day)	(9)* DO_{avg} (mg/l)	(10)* (lb/day)	(11) DO_a-DO_n (lb/day)	Comments
1	700		21.8			6.00	22,649				
2	710	705	21.9	5.1	0.625	3.15	12,058	4.58	17,370	10,591	
3	726	713	22.0	5.8	0.875	2.20	8,611	2.68	10,302	3,447	Above dam
4	726		22.0			4.10	16,048				Below dam
5	730	728	22.0	2.8	0.250	4.10	16,137	4.10	16,092	-89	
6	739	735	22.0	3.9	0.500	3.95	15,738	4.03	15,970	399	
7	756	747	22.1	5.0	1.000	3.90	15,896	3.93	15,787	-158	
8	775	765	22.1	5.9	1.125	4.15	17,340	4.03	16,621	-1,444	
9	788	782	22.2	6.5	0.800	4.50	19,118	4.33	18,256	-1,778	Above trib
10	988		22.0			4.80	25,568	4.65		-6,450	Below trib
11	1000	994	22.0	7.1	0.825	5.10	27,496	4.95	26,527	-1,928	

*Value per reach

Col 8 = Col 2 × Col 7 × 5.39136

Col 10 = Col 3 × Col 9 × 5.39136

Note: In Cols 7 and 8 the DO_a at the end of a reach is the DO_n of that reach

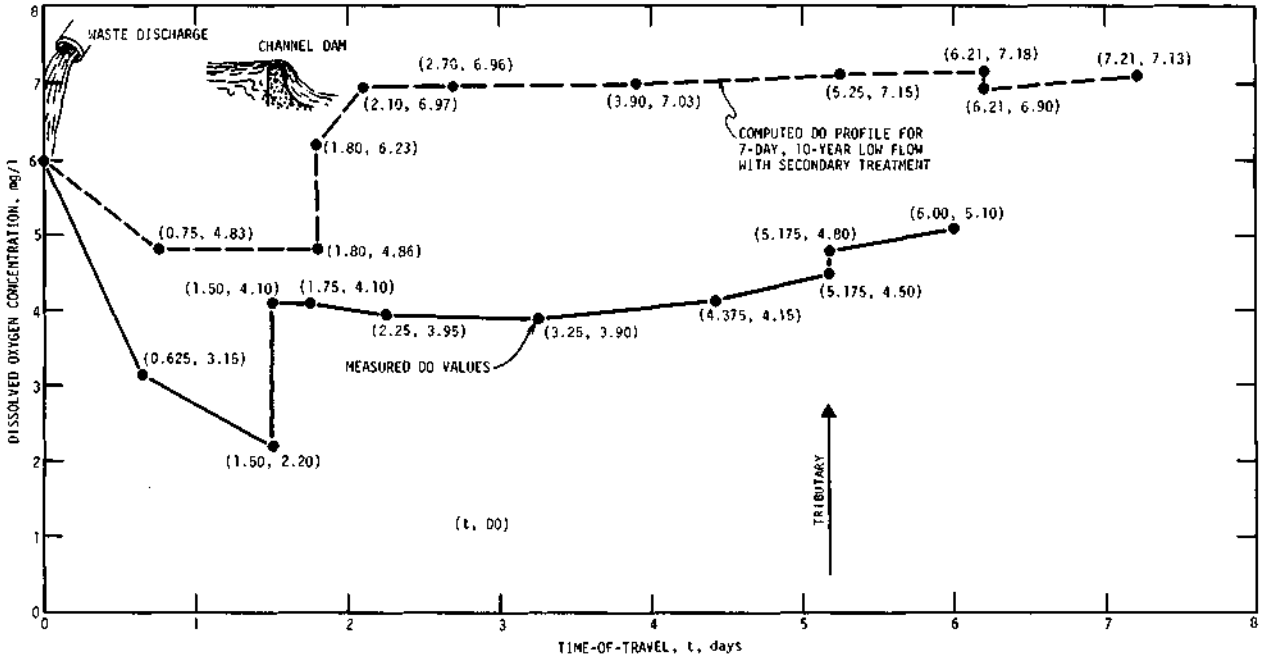


Figure 9. Observed and estimated DO profiles

During the time of field work the flow of the receiving stream in the vicinity of the waste discharge was 700 cfs and the tributary flow was 200 cfs. From the DO measurements (Col. 7) and the time-of-travel (Col. 6) it is found that a pronounced DO sag exists in the stream below the principal source of waste discharge. This condition is depicted by the lower curve in figure 9. Other computations in table 5 lead to a solution for the term $DO_a - DO_n$ (Col. 11) in pounds per day which will be used in equation 30.

The natural reaeration, DO_r, aspects of the stream are assessed by equation 27. The factors for use in this procedure are included in table 6. Values for DO_s at 22 C are obtained from table 2; M and R_o are developed from figures 4 and 5 on the basis of the physical dimensions of the stream established in table 5.

Next the DO_u is computed from equation 30 (note that DO_x does not yet apply), and the results are tabulated in table 7. A computational format which the State Water Survey has found useful with a computer for tabulating the results in tables 5, 6, and 7 is included in appendix C.

At this point $K_{d(22)}$ and $L_{a(22)}$ are found, by the methods previously described, to be 0.4 per day and 83,784 lb/day, respectively. From equations 15 and 19, the values for $K_{d(28)}$ and $L_{a(28)}$ are 0.527 per day and 89,424 lb/day, respectively.

Since the present overloaded secondary treatment plant removes 65 percent of the incoming BOD load, then the incoming load is 89,424/0.35 or 255,497 lb/day. The expanded secondary sewage treatment plant may be expected to remove 90 percent

Table 6. Reaeration Computations

(1) Station	(2) DO_s (mg/l)	(3) Avg DO_s (mg/l)	(4) $5.4Q \times DO_s$ (lb/day)	(5) M (min)	(6) $\frac{R_Q}{100}$	(7) $1-\frac{DO_a}{DO_s}$	(8) $\frac{t'}{M}$	(9) DO_r (lb/day)
1	8.70							
2	8.68	8.69	33,030	15.2	.0047	.474	59.2	4,356
3	8.67	8.68	33,367	17.1	.0043	.691	73.7	7,307
4	8.67	8.67						
5	8.67	8.67	34,029	4.3	.0092	.527	83.7	13,862
6	8.67	8.67	34,357	11.5	.0112	.535	62.6	12,887
7	8.65	8.66	34,877	15.0	.0047	.547	96.0	8,664
8	8.65	8.65	35,676	17.4	.0043	.534	93.1	7,627
9	8.63	8.64	36,427	18.6	.0041	.499	61.9	4,602
10	8.67							
11	8.67	8.67	46,463	19.9	.0039	.429	59.7	4,665

Note: Col 9 = Col 7 × Col 6 × Col 8 × Col 4

Table 7. DO Used Computations

(1) Station	(2) t $(days)$	(3) DO_a-DO_n (lb/day)	(4) DO_r (lb/day)	(5) DO_u (lb/day)	(6) $\Sigma\ DO_u$ (lb/day)	(7) Σt $(days)$
1						
2	0.625	10,591	4,356	14,947	14,947	0.625
3	0.875	3,447	7,307	10,754	25,701	1.500
4					25,701	1.500
5	0.250	-89	13,862	13,773	39,474	1.750
6	0.500	399	12,887	13,286	52,760	2.250
7	1.000	-158	8,644	8,486	61,246	3.250
8	1.125	-1,444	7,627	6,183	67,429	4.375
9	0.800	-1,778	4,602	2,824	70,253	5.175
10		-6,450			70,253	5.175
11	0.825	-1,928	4,665	2,637	72,890	6.000

Note: Col 5 = Col 3 + Col 4

of the BOD load. Therefore the load L_a to the stream will be 0.10 × 255,497 or about 25,550 lb/day. For use in developing the predictive DO profile of the receiving stream after the new secondary treatment is functioning, the following design factors are now known:

Stream temperature = 28 C
Main stem Q = 560 cfs
Tributary Q = 160 cfs
Tributary DO = 6 mg/l
At the dam, q = 1.0
At the dam, b = 0.58
At the dam, h = 6.3 ft
$L_a(28)$ = 25,550 lb/day
$K_d(28)$ = 0.527 per day
H and t available from cross section data

The results of computations for solving equation 21 are tabulated in table 8, and the DO values in column 20 of table 8 were used to plot the predictive DO profile shown in the upper portion of figure 3.

Table 8. DO Profile Computations at 28C and a 7-Day, 10-Year Low Flow

L_a = 25,550 lb/day; K_d = 0.527 per day; D_a = 6.00 mg/l

(1) Station	(2) Flow at sta (cfs)	(3) Avg flow (cfs)	(4) H (ft)	(5) M (min)	(6) t (days)	(7) Σt (days)	(8) $1-e^{-K_d \Sigma t}$	(9) Total DO_u (lb/day)	(10) Subreach DO_u (lb/day)	(11) DO_a-DO_u (lb/day)
1	560									
2	568	564	4.6	13.8	.750	.750	.327	8,355	8,355	9,756
3	580	570	5.2	15.5	1.050	1.800	.613	15,662	7,307	7,485
4	580									
5	584	582	2.5	4.1	.300	2.100	.669	17,093	1,431	18,046
6	591	588	3.5	10.0	.600	2.700	.759	19,392	2,299	19,653
7	605	598	4.5	13.5	1.200	3.900	.872	22,280	2,888	19,270
8	620	613	5.3	15.7	1.350	5.250	.937	23,940	1,660	21,275
9	630	625	5.9	17.3	.960	6.210	.962	24,579	639	23,240
10	790									
11	800	795	6.4	18.3	1.000	7.210	.978	24,988	409	29,149

Station	(12) Adj DO_a	(13) DO at saturation (mg/l)	(14) DO at saturation (lb/day)	(15) $\frac{R_O}{100}$	(16) $\frac{1440t}{M}$	(17) $1-\frac{DO_a}{DO_s}$	(18) DO_r (lb/day)	(19) DO_n (lb/day)	(20) DO_n (mg/l)	Comments
1								18,111	6.00	
2	13.934	7.72	23,470	.00675	78.3	.406	5,036	14,792	4.83	
3	11,139	7.72	23,719	.00630	97.5	.530	7,722	15,207	4.86	Above dam
4								19,477	6.23	Below dam
5	18,760	7.72	24,219	.00680	105.4	.225	3,906	21,952	6.97	
6	20,802	7.72	24,468	.00790	86.4	.150	2,505	22,158	6.96	
7	20,714	7.72	24,884	.00685	128.0	.168	3,665	22,935	7.03	
8	22,105	7.72	25,508	.00620	123.8	.133	2,604	23,879	7.15	
9	23,560	7.72	26,008	.00585	79.9	.094	1,143	24,383	7.18	Above trib
10								29,558	6.94	Below trib
11	29,353	7.72	33,082	.00550	78.7	.113	1,618	30,767	7.13	

Note: Col 9 = L_a × Col 8
Col 10 = Col 9_{n+1} - Col 9_n
Col 11 = Col 19 - Col 10
Col 12 = (Col 11 + Col 19) ÷ 2
Col 17 = 1 - (Col 12 ÷ Col 14)
Col 18 = (Col 14 × Col 15 × Col 16 × Col 17)
Col 19 = initial value is 560 × 5.39 × 6.0 and thereafter Col 11 + Col 18

Dam: $b = 0.58$; $q = 1.0$; $T = 28C$; $h = 6.3$; $C_s = 7.72$
$r = 1 + (.11)(1.0)(.58)(6.3)[1 + .046(28)] = 1.91961$
$C_B = (4.86 - 7.72)/ 1.91961 + 7.72 = 6.23$

Tributary: 160 × 5.39 × 6.00 = 5175 lb/day

Summary

The steps involved in estimating the waste assimilative capacity of a stream based upon a pragmatic approach can be summarized as follows:

1) Develop a full understanding of the stream length, its channel geometry, water stage and flow patterns, and the general hydrologic features of the watershed.

2) Determine the 7-day, 10-year low flow of the stream and select a design water temperature.

3) Define the location of all dams and their physical features; define also the location of all tributary flows and relevant data regarding them.

4) Divide the stream into reaches consistent with significant changes in cross sections and determine the volumes and average depth in each reach.

5) At the beginning and end of each reach, during low flow conditions and summer temperatures, undertake a series of field determinations for at least water temperature and dissolved oxygen concentrations; and if desired collect water samples for BOD determinations.

6) Compute the time-of-travel within each reach at stream flows observed during the time of sampling as well as that during 7-day, 10-year low flow.

7) From the observed DOs, flow, and time-of-travel compute $DO_a - DO_n$ as demonstrated in table 5.

8) Select DO saturation values from table 2 for observed stream temperature conditions and compute the natural reaeration for each reach using figures 4 and 5 in conjunction with appropriate equations for finding the mix time, M, and the percent absorption at 100 percent deficit, R_o. Keep in mind the need to make adjustments in accordance with weir and mass balance formulas where dams and tributaries are encountered.

9) Calculate, by summation, the DO_u for each reach as demonstrated in table 7.

10) From an array of the DO_u vs t data, determine L_a an K_d, eferably by the methods of Reed-Theriault, steepest descent, or least squares. For a graphical solution, the Thomas slope method is satisfactory. Adjust the values for L_a and K_d for the selected design water temperature by the use of equations 15 and 19.

11) Apply the removal efficiency anticipated to the computed ultimate oxygen demand L_a and develop the required expression $DO_u = L_a(1 - e^{-Kt})$.

12) From the values developed in step 6 for time-of-travel and depth at 7-day, 10-year low flow, use the observed DO just upstream of the discharge point (D_a at beginning of reach) as a starting point, and follow the computation outlined in table 8.

13) Note whether or not the removal efficiency selected will permit predicted DO concentrations in the stream compatible with water quality standards.

The steps that have been outlined can be readily arranged for computer management. A format for the DO_u putations is included in appendix C as previously mentioned, and the format shown in table 8 can be used for the DO profile computations. These steps have been based principally upon DO observations in contrast to BOD determinations. Several examples will follow to demonstrate the use of BOD data to estimate the parametric values required to solve for the DO used.

Estimating Parametric Values

Procedures have been developed for using the data obtained from field work as well as laboratory work for estimating deoxygenation coeffi K_d, K_N, K_C, nd ultimate oxygen dem L_a, L_{aC}, L_{aN}. e DO_u in a stream or a bottle is often characterized by a first order reaction, i.e., $Y = L_a(1 - e^{-Kt})$; and, after ascertaining values for Y as a function of t, there are numerous methods for estimating L_a and K. Among these are the Thomas slope[31] which has been previously demonstrated, method of moments,[32] rapid ratio,[33] daily difference,[34] Reed-Theriault,[30] and least squares. State Water Survey investigators have favored the Reed-Theriault method.

There will be times when the data collected will not, upon analysis, resemble a first order reaction. The purpose here is to review some of the situations an investigator is likely to encounter in estimating the parametric values from field and laboratory work.

Field Work. Generally the DO_u as a function of time, takes one of the general shapes shown in figure 10.

Figure 10a shows two first order reactions; one commences immediately, the other is delayed and starts at some later time designated t_o. The best procedure for analyzing this type of data is to fit two first order curves to each segment of the plot. The first curve is derived from the data up to t_o, the inflection point. The origins of the ordinate and abscissa are then transposed to the inflection point, and a new curve is derived. Thus two values are computed from each curve segment.

Certain conditions may cause stream DO_u data to plot similarly to that depicted in figure 10a. These are: 1) dams, either channel or navigation types, and 2) instream nitrification, i.e., conversion of ammonia to nitrate. The effect of dams is clearly demonstrated by a plot of actual observed conditions for the Upper Illinois

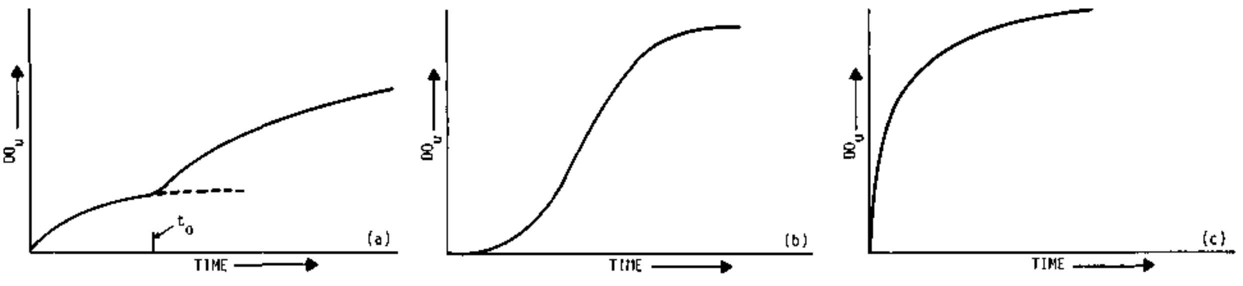


Figure 10. Various DO used progression curves

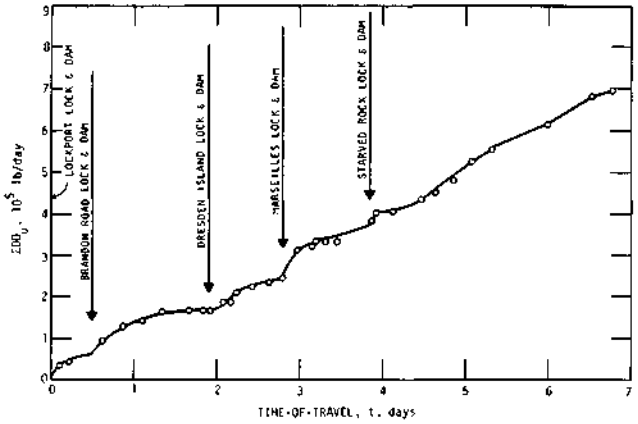


Figure 11. DO used vs time-of'-travel, Upper Illinois Waterway, showing influence of navigation dams and pools

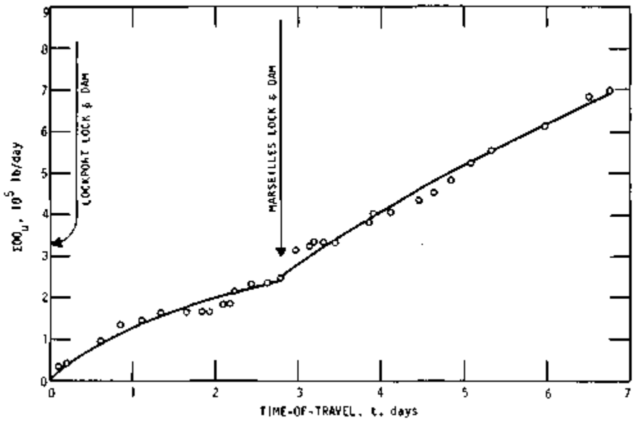


Figure 12. DO used vs time-of-travel, Upper Illinois Waterway, indicating first and second stage BOD curves

Waterway shown in figure 11. Note that each pool appears to contain its own individual first order DO_u curve. A first order curve could be force-fitted through all the points; however, a better simulation of the observed DOs would be achieved by developing L_a and K_d for each individual pool.

For the condition where the break in the first stage DO_u is caused by nitrification, separate L_a and K_d coefficients should be developed for each distinct curve. The data points of figure 11 are replotted in figure 12 to show that a break in the overall data occurs as a result of nitrification commencing near Marseilles. Without a thorough in-field investigation, the complex phenomenon depicted by figures 11 and 12 would not have been taken into consideration, and any generalized assumptions would have inaccurately portrayed the real conditions.

The DO_u progression curve shown as figure 10b is indicative of an organic load that is not biodegraded immediately. Time is required to build up a viable bacterial population capable of oxidizing the waste. A curve similar to this could be expected in a stream receiving a high ammonia load with respect to a carbonaceous load This reaction involves a time lag also, and the curve is similar to a Gaussian distribution; consequently, a good fit for DO_u can be achieved by the expression

$$DO_u = L_a \{1 - e^{-[K_d(t-t_o)]^2}\} \tag{34}$$

The solution to equation 34 necessitates the use of a digital computer. Squaring of the exponential power implies that K_d is no longer a constant but is a variable in relation to time.

The curve shown as figure 10c is indicative of a reaction that proceeds immediately and rapidly. For such a reaction to occur, the bacterial population must be in the log-growth phase. Such an array of data may be represented by fitting the data points to a second order curve. One model that has gained some acceptance is a second order reaction curve in the form of a transposed, rotated hyperbola. This model assumes that the reaction rate is not constant but changes with time. Woodward[35] presented this model mathematically as

$$DO_u = t/(p+zt) \tag{35}$$

where, in the terminology of this report

$$p = 1/(K_o L_a^2) \tag{36}$$

$$z = 1/L_a \tag{37}$$

K_o = time varying deoxygenation coefficient

L_a = ultimate or maximum DO_u

The reciprocal of p represents the initial (maximum) reaction rate while the reciprocal *z* represents the ultimate DO_u. A least squares analytical solution to equation 35 can be achieved by linearizing the observed data with the use of the reciprocals of the DO_u and the reciprocals of the corresponding time.

Woodward[35] showed that the second order reaction formula can produce a good fit to observed data. However, he stressed that a definite statement cannot be made as to what actual mathematical relationship a given set of data follows. Young and Clark[36] compared the results achieved by the use of the first and second order reaction equations for sewage, glucose-gluatamic acid, and Metrecal BODs. In all three cases the second order equation was found to fit with the same degree of precision as the first order equation. Dougal and Baumann[37] compared both techniques in use with raw and treated sewage. They concluded that BOD progression is not monomolecular and that the second degree equation is more representative of biochemical reactions. On the other hand, Butts and Kothandaraman[38] found that for most samples of Illinois River water (La Grange Pool) the second order curve did not fit as well as the basic first order curves derived from the Reed-Theriault curve fitting technique.

In some instances though, the second order equation does show an extremely good fit to computed DO_u values. One such example is illustrated by figure 13, a plot of DO_u versus time-of-travel for the Dresden Island Pool of the Upper Illinois Waterway.

The numerical values of p and z in equation 35 can be computed by

$$p = [n\Sigma\, u\, v - (\Sigma u)(\Sigma v)]/[n\, \Sigma u^2 - (\Sigma u)^2] \tag{38}$$

$$z = [\Sigma v\, (\Sigma u^2) - (\Sigma u)(\Sigma u\, v)]/[n\Sigma u^2 - (\Sigma u)^2] \tag{39}$$

where

n = number of observations

u = reciprocal of t

v = reciprocal of DO_u

Figure 13 will be used to demonstrate the analysis of a second order reaction for use with equation 21. From figure 13, p equals 0.145 and z equals 0.718. Substituting these values into equation 37 yields $L_a = 139{,}200$ lb/day; and from equation 36 $K_o = 3.563$. If a 75 percent reduction in stream BOD is required, L_a would then equal 34,800 lb/day. Substituting this value and $K_o = 3.563$ into equations 36 and 37 gives values for p and z of 2.318 and 2.874 respectively. The DO_u for use in equation 21 is therefore

$$DO_u = t/(2.318 + 2.874\,t)$$

The methods that have been discussed for estimating in-stream deoxygenation coefficients and ultimate demands have been based solely on observed DOs and reaeration estimates. Under some circumstances it may be advantageous to use stream BOD data for estimating K_d and L_a. O'Connor[39] has proposed a procedure for such use and the technique used here is a modification of his work.

The method consists generally of plotting on semi-log paper the stream BOD values determined by incubation in the laboratory against time-of-travel representative of sampling points along the stream. The example used here was developed from a sector of the Illinois River where nitrogenous demand was the basic sink for dissolved oxygen. River water samples were incubated for 9 days and the BOD_9 values were plotted against time as shown in figure 14.

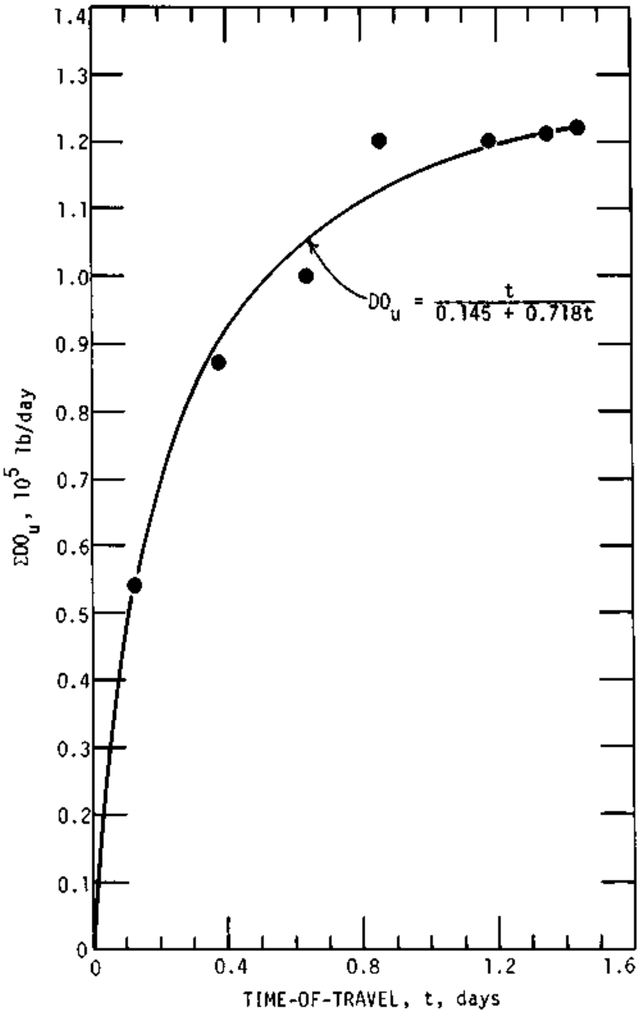


Figure 13. Second order DO used curve fit, Dresden Island Pool of Upper Illinois Waterway

The general form of the expression for figure 14 is

$$Y_{N\tau} = L_{aN\tau} e^{-K_{dN} t} \qquad (40)$$

where $Y_{N\tau}$ = nitrogenous demand at τ days incubation time. $L_{aN\tau}$ and K_{dN} can be calculated from the following:

$$\log L_{aN\tau} = (\Sigma t^2)(\Sigma \log Y_{N\tau}) - \Sigma t \ (\Sigma t \log Y_{N\tau}) \qquad (41)$$

$$K_{dN} = [n(\Sigma t \log Y_{N\tau}) - \Sigma t \ (\Sigma \log Y_{N\tau})]/\{0.4343 \ [n \ (\Sigma t^2) - (\Sigma t)^2]\} \qquad (42)$$

From these expressions $L_{aN\tau}$ was found to equal 475,000lb/day and K_{dN} was found to equal -0.14 per day. Letting $L_{aN\tau}$ equal I and K_{dN} equal K_d for a time t of 9 days and solving for L_a in equation 31 results in a DO_u expression to be used in equation 21 of $DO_u = 1{,}832{,}000 \ e^{-0.15t}$.

Laboratory Work. The oxygen demand of liquid wastes or substances subject to biochemical degradation in water is generally characterized by the 5-day biochemical oxygen demand (BOD_5) test.[40] Usually the results are interpreted as the demand for oxygen exerted by the substances being tested or more often a measure of the biodegradable matter available for oxidation by microbes. Just as important but less considered is the fact that the test reflects the activity and population of the microbes in the tested media.

Whether or not there is a carbonaceous or nitrogenous demand, assuming unlimited access to food, depends on the presence of cooperating bacteria. In the case of carbonaceous oxidation the heterotrophs are capable of using a carbon source for energy and producing as a by-product carbon dioxide. For nitrogenous oxidation the autotroph *Nitrosomonas* utilizes ammonia as an energy source converting it to nitrite whereas the autotroph *Nitrobacter* utilizes nitrite as an energy source converting it to nitrate.

There is an important difference in the oxygen requirements for these fundamental oxidation processes. Each part of carbonaceous demand, since it is expressed in terms of oxygen, requires one part of oxygen; whereas for the nitrogenous demand, about 4.57 parts of oxygen are required for one part ammonia-N oxidized. In effect 1 mg/l NH_3-N has a potential oxygen demand equivalent of 4.57 mg/l.

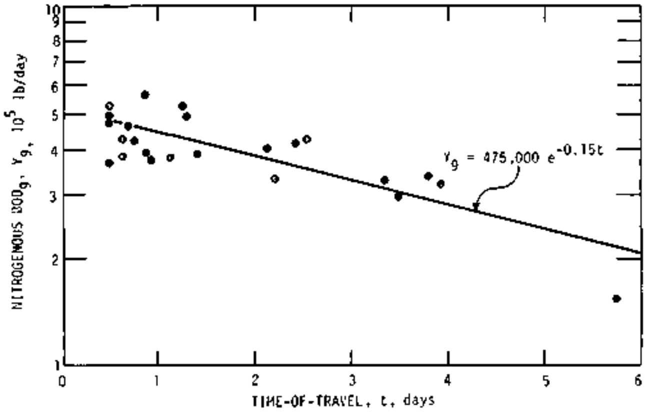


Figure 14. River BOD plot, upper end of Peoria Pool of Illinois Waterway

Another aspect of the BOD_5 test, and a limiting one, is that frequently 5 days is not sufficient time to develop a dynamic nitrifying bacteria population. Thus a BOD_5 result may not include the effects of nitrogenous oxidation.

With these basic concepts in mind it is also important to realize that the ultimate oxygen demand, not the BOD_5, is the important consideration for stream waters. The ultimate demand L_a is dependent on the deoxygenation coefficient K, and the basis for determining these values is either BODs performed at varying periods of time, or determinations of actual dissolved oxygen used between selected points on a stream.

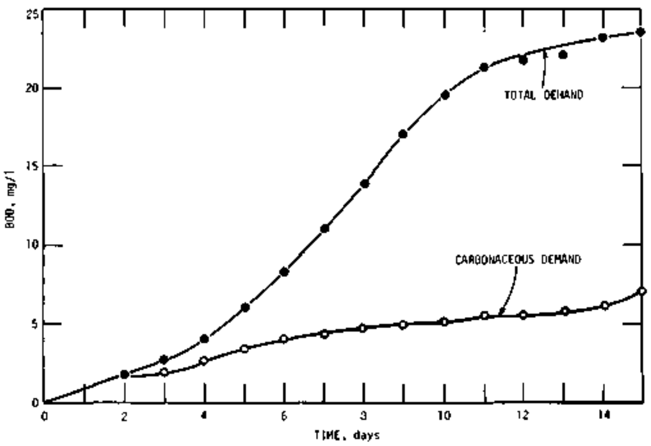


Figure 15. BOD progression curves, Illinois River at Morris, June 15, 1972

For the laboratory determinations of stream BODs at 20 C the technique devised by Elmore[1*1] is the State Water Survey's method of choice. The procedure eliminates the necessity for dilution water. The results from diluted stream waters have been found by several researchers[6,22,42,43] to be incompatible with stream-like conditions

In differentiating between nitrogenous and carbonaceous demand, several techniques have been used by the Survey. Acidification by Hurwitz et al.,[44] N-Serve by Young,[1*5] and ATU by Montgomery and Borne[46] have all been used extensively. The Survey's choice is a modification of that suggested by Montgomery and Borne,[1*6] which employs as an inhibiting agent for *Nitrosomonas* activity, the chemical allythiourea (ATU). For long term BODs a concentration of 0.5 mg/l is needed in the BOD bottle at intervals of 5 days to inhibit the oxidation of ammonia. *Nitrobacter* is not affected, thus any nitrite present may be oxidized.

The effect of an inhibitor is shown in figure 15. A portion of a sample of stream water is incubated without an inhibitor. This is represented by the upper curve and is the total oxygen demand. A portion of the sample was treated with ATU. This is represented by the lower curve, and since nitrification is inhibited, it depicts the carbonaceous demand. The difference between the total demand and the carbonaceous demand is the nitrogenous demand. An analysis of the data supporting these demands will permit evaluations for L_a and K_d; L_{aC} and K_C; and L_{aN} and K_N. Several methods for determining the ultimate oxygen demand and deoxygenation coefficients have been either described or cited earlier, and a computer program utilizing the Reed-Thierault method is included in the appendix.

Where a treated discharge is proposed for a stream, these parameters should be estimated from a test sample containing a comparable effluent and the stream water in a mixture having a dilution ratio equivalent to that for the design flow of the proposed treatment works and the 7-day, 10-year low flow of the stream. If the stream's 7-day, 10-year low flow is zero, i.e., no flow, the BOD procedures should be applied to an undiluted sample of an effluent comparable in quality to that anticipated from the treated discharge.

REFERENCES

1 *Water pollution regulations of Illinois.* 1972. Illinois Environmental Protection Agency, Springfield, 36 p.

2 Streeter, H. W., and E. B. Phelps. 1925. *A study of the pollution and natural purification of the Ohio Eiver.* U. S. Public Health Service Bulletin 146 (1958 reprint).

3 Camp, T. R. 1963. *Water and its impurities.* Reinhold Publishing Corporation, New York, p. 287-310.

4 Dobbins, W. E. 1964. *BOD and oxygen relationships in streams.* American Society of Civil Engineers Journal of the Sanitary Engineering Division v. 90(6): 53-78.

5 O'Connor, D. J. 1967. *The temporal spatial distribution of dissolved oxygen in streams.* Water Resources Research v. 3(0):65¯79.

6 Butts, T. A., D. H. Schnepper, and R. L. Evans. 1970. *Dissolved oxygen resources and waste assimilative capacity of the La Grange Pool, Illinois River.* Illinois State Water Survey Report of Investigation *6k.*

7 O'Connor, D. J., and D. M. DiToro. 1966. *An analysis of the dissolved oxygen variation in a flowing stream.* Paper presented at the University of Texas, April 4, 11 p.

8 Gannon, J. J. 1966. *River and laboratory BOD rate considerations.* American Society of Civil Engineers Journal of the Sanitary Engineering Division v. 92(2):135-161.

9 Zanoni, A. E. 1967. *Effluent deoxygenation at different temperatures.* Marquette University, Milwaukee, Wisconsin, Civil Engineering Department Report 100-SA, 78 p.

10 Austin, J. H., and F. W. Sollo. 1969. *Oxygen relationships in small streams.* University of Illinois Department of Civil Engineering, Sanitary Engineering Series 52, 192 p.

11 Stall, J. B., and Yu-Si Fok. 1968. *Hydraulic geometry of Illinois streams.* University of Illinois Water Resources Center, Urbana, Research Report No. 15, 45 p.

12 Stall, J. B., and D. W. Hiestand. 1969. *Provisional time-of-travel for Illinois streams.* Illinois State Water Survey Report of Investigation 63, 31 p.

13 Gannon, J. J., and T. D. Downs. 1963. *Programming river DO calculations.* Water and Sewage Works v. 110(4):157-159.

14 Langbein, W. B., and H. W. Durum. 1967. *Aeration capacity of streams.* U. S. Geological Survey Circular 542, 6 p.

15 Babbitt, H. E., and E. R. Baumann. 1958. *Sewage and sewage treatment.* John Wiley 6 Sons, Inc., 8th Ed., New York, 379 p.

16 Churchill, M. A., R. L. Elmore, and R. A. Buckingham. 1962. *The prediction of stream reaeration rates.* American Society of Civil Engineers Journal of the Sanitary Engineering Division v. 88(7):l-46.

17 O'Connor, D. J., and W. E. Dobbins. 1956. *The mechanisms of reaeration in natural streams.* American Society of Civil Engineers Journal of the Sanitary Engineering Division v. 82(12):1115-1130.

18 Committee on Sanitary Engineering Research. 1960. *Solubility of atmospheric oxygen in water.* American Society of Civil Engineers Journal of the Sanitary Engineering Division v. 86(7):41-53.

19 Black, W. M., and E. B. Phelps. 1911. *Location of sewer outlets and discharge of sewage in New York harbor.* New York City Board of Estimate, March 23.

20 Phelps, E. B. 1944. *Stream sanitation.* John Wiley & Sons, Inc., New York, p. 153-155.

21 Velz, C. J. 1939. *Deoxygenation and reoxygenation.* Transactions of the American Society of Civil Engineers v. 104:560-572.

22 Gannon, J. J. 1963. *River BOD abnormalities.* University of Michigan School of Public Health, Ann Arbor, Final Report USPHS Grants RG-6905, WP-187(C1), and WP-187(C2), November, p. 172.

23 Velz, C. J. 1947. *Factors influencing self-purification and their relation to pollution abatement.* Sewage Works Journal v. 19(7):629-644.

24 Butts, T. A., and D. H. Schnepper. 1967. *Oxygen absorption in streams.* Water and Sewage Works v. 114(10):385-386.

25 Gameson, A. L. H. 1957. *Weirs and the aeration of rivers.* Journal of the Institution of Water Engineers (British) v. 11(6):477, October.

26 Gameson, A. L. H., K. G. Vandyke, and C. G. Ogden. 1958. *The effect of temperature on aeration at weirs.* Water and Water Engineering (British) v. 62: 489, November.

27 Barrett, M. J., A. L. H. Gameson, and C. G. Ogden. 1960. *Aeration studies at four weir systems.* Water and Water Engineering (British) v. 64:407, September.

28 Grindrod, J. 1962. *British research on aeration at weirs.* Water and Sewage Works v. 109(10) :395, October.

29 Preul , H. C, and A. G. Holler. 1969. *Reaeration through low days in the Ohio River.* Proceedings of the 24th Purdue University Industrial Waste Conference, Part two, p. 1249-1270.

30 Reed, L. J., and E. J. Theriault. 1931. *The statistical treatment of reaeration velocity data, II.* Journal of Physical Chemistry v. 35(4):950~971.

31 Thomas, H. A., Jr. 1950. *Graphical determination of BOD curve constants.* Water and Sewage Works v 97(3) : 123-124.

32 Moore, E. W., H. A. Thomas, and W. B. Snow. 1950. *Simplified method for analysis of BOD data.* Sewage and Industrial Wastes v. 22(10) : 1343.

33 Sheehy, J. P. 1960. *Rapid methods for solving monomolecular equations.* Journal of Water Pollution Control Federation v. 32(6):646-652.

34 Tsivoglou, E. C. 1958. *Oxygen relationships in streams.* Robert A. Taft Sanitary Engineering Center, Cincinnati, Ohio, Technical Bulletin W-58-2.

35 Woodward, R. L. 1953. *Deoxygenation of sewage, a discussion.* Sewage and Industrial Wastes v. 25(8):918.

36 Young, J. C, and J. W. Clark. 1965. *Second order equation for BOD.* American Society of Civil Engineers Journal of the Sanitary Engineering Division v. 9KSA1) :43.

37 Dougal, M. D., and R. E. Baumann. 1967. *Mathematical models for expressing the biochemical oxygen demand in water quality studies.* Proceedings of the Third Annual Conference, American Water Resources Association, Urbana, Illinois.

38 Butts, T. A., and V. Kothandaraman. 1970. *Fitting first and second order BOD reaeration equation to stream data.* Water and Sewage Works v. 117(8):276-278.

39 O'Connor, D. J. 1966. *Stream and estuarine analysis.* Summer Institute in Water Pollution Control, Training Manual, Manhattan College, New York.

40 American Public Health Association, American Water Works Association, and Water Pollution Control Federation. 1971. *Standard methods for the examination of wastewater.* American Public Health Association, Inc., 12th Ed., New York, 874 p.

41 Elmore, H. L. 1955. *Determinations of BOD by a reaeration technique.* Sewage and Industrial Wastes v. 27(9):993-1002.

42 Wisely, W. H., and C. W. Klassen. 1938. *The pollution and natural purification of Illinois River below Peoria.* Sewage Works Journal v. 10(3):569¯595.

43 Gaudy, A. F. 1972. *Water pollution microbiology.* John Wiley S Sons, Inc., New York, p. 305-331.

44 Hurwitz, E., G. R. Barrett, R. E. Beaudian, and H. P. Kramer. 1947. *Nitrification and BOD.* Sewage Works Journal v. 19(6) :995-999.

Young, J. C. 1969. *Chemical methods for nitrification control.* Proceedings of the 24th Industrial Waste Conference, Part 2, Purdue University, Lafayette, Indiana, p. 1090-1102.

Montgomery, H. A. C., and B. J. Borne. 1966. ***The inhibition of nitrification in the BOD test.*** Journal and Proceedings of the Institute of Sewage Purification, Water Pollution Research Laboratory, Stevenage, England, Part 4, p. 357-368.

NOTATIONS

A = area of cross section, square feet
a = diffusion coefficient used by Velz in Black and Phelps reaeration formula
a_T = Velz diffusion coefficient at any temperature T
B_o = initial dissolved oxygen in percent saturation
b = weir correction factor
BOD = biochemical oxygen demand without specifying time
C = dissolved oxygen observed, mg/l
C_A = dissolved oxygen upstream of a dam, mg/l
C_B = dissolved oxygen downstream of a dam, mg/l
C_s = dissolved oxygen at saturation, mg/l
D = dissolved oxygen deficit, mg/l or lb/day
D_a = initial dissolved oxygen deficit, mg/l or lb/day
DO = general expression for dissolved oxygen
DO_a = initial dissolved oxygen in a reach, mg/l or lb/day
DO_{avg} = average dissolved oxygen, mg/l or lb/day
DO_n = dissolved oxygen at the end of a reach, mg/l or lb/day
DO_r = dissolved oxygen derived from natural reaeration, mg/l or lb/day
DO_s = dissolved oxygen at saturation, mg/l or lb/day
DO_u = dissolved oxygen consumed biologically, mg/l or lb/day
DO_x = dissolved oxygen derived from dams, tributaries, etc., lb/day
H = average water depth, feet
H_1 = average depth, centimeters
h = height through which water falls, feet
K = exponent in the Black and Phelps reaeration formula
K_2 = reaeration coefficient to base e, per day
K_c = carbonaceous deoxygenation coefficient to the base e, per day

K_d = in-stream deoxygenation coefficient to the base e, per day

K_N = nitrogenous deoxygenation coefficient to the base e, per day

K_o = time varying deoxygenation coefficient

K_T = rate coefficient to the base e at temperature T, per day

L = total BOD remaining at any time t, mg/l or lb/day

L_a = ultimate oxygen demand including nitrogenous and carbonaceous, mg/l or lb/day

L_{aC} = ultimate carbonaceous demand, mg/l or lb/day

L_{aN} = ultimate nitrogenous demand, mg/l or lb/day

L_C = carbonaceous demand remaining at any time t, mg/l or lb/day

L_N = nitrogenous demand remaining at any time t, mg/l or lb/day

M = mixture or exposure time, minutes

m = mixture or exposure time, hours

n = number of observations

p = factor for second order reaction

Q = flow, cfs

q = water quality correction factor at dams

R = percent of saturation of dissolved oxygen absorbed per mix

R_o = percent of saturation of dissolved oxygen absorbed per mix at 100 percent deficit

r = dissolved oxygen deficit ratio at temperature T, °C

S = slope of the best fit in Thomas slope method

T = observed temperature, °C

t = time-of-travel or incubation period, days

t_o = lag time for nitrogenous demand, days

t' = time, minutes

τ = period of incubation at 20 C, days

θ = a constant used in temperature corrections

u = reciprocal of t

V = average velocity, fps

v = reciprocal of DO_u

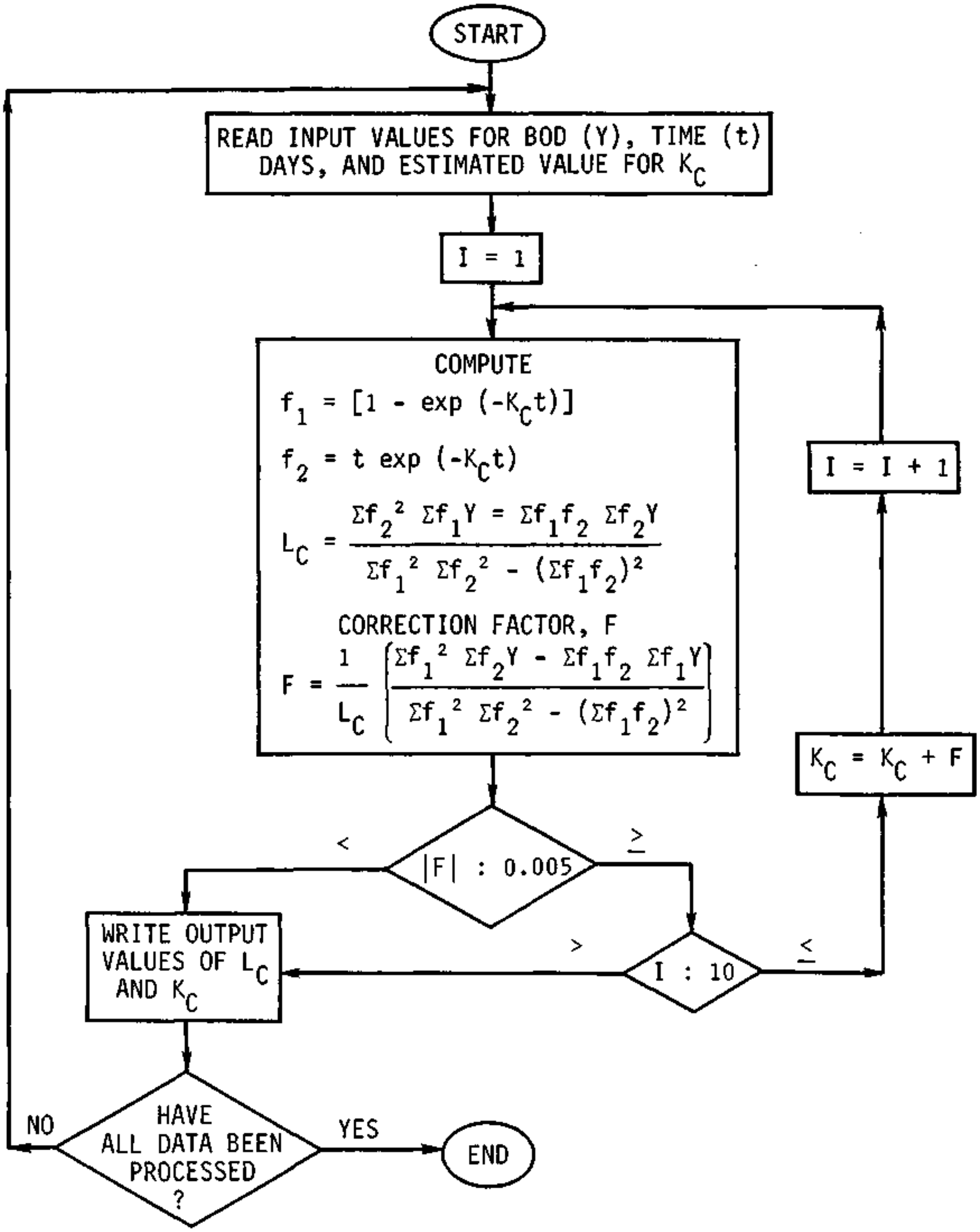
W = ratio of weir coefficients

Y = oxygen demand exerted, mg/l or lb/day

y = intercept on the ordinate

z = factor for second order reaction

Appendix A. Flow Diagram for Computing K_c and L_c Values by Reed-Theriault Method



Appendix B. Program Listing for Evaluating K_c and L_c by Reed-Theriault Method

STATEMENT OF THE PROBLEM

```
C     CALCULATION OF KC VALUE (THE DEOXYGENATION COEFFICIENT) AND THE
C     ULTIMATE FIRST STAGE DEMAND
C     Y = BOD SATISFIED AT TIME T  IN MG PER LITER
C     T = TIME OF OBSERVATION  IN DAYS
C     EK = ESTIMATED VALUE OF KC SUPPLIED BY THE USER
C     F3 = EXP(-KC*T)
      DIMENSION Y(50),T(50)
   12 READ (5,1)N,EK
    1 FORMAT(I10,F10.4)
      READ (5,2)(T(J),J=1,N)
      READ (5,2)(Y(J),J=1,N)
    2 FORMAT(8F10.3)
      I = 1
C     SUMF1Y = SUM OF THE TERMS F1*Y
C     SUMF2Y = SUM OF THE TERMS F2*Y
C     SF1F1 = SUM OF THE TERMS F1*F1
C     SF1F2 = SUM OF THE TERMS F1*F2
C     SF2F2 = SUM OF THE TERMS F2*F2
    7 SUMF1Y = 0.0
      SUMF2Y = 0.0
      SF1F1 = 0.0
      SF1F2 = 0.0
      SF2F2 = 0.0
      DO 3 J= 1,N
      F3 = EXP(-EK*T(J))
      F1 = 1.0-F3
      F2 = T(J)*F3
      SUMF1Y= SUMF1Y+F1*Y(J)
      SUMF2Y = SUMF2Y + F2*Y(J)
      SF1F1 = SF1F1+F1*F1
      SF1F2 = SF1F2+F1*F2
    3 SF2F2 = SF2F2 + F2*F2
      DENOM = SF1F1*SF2F2-SF1F2*SF1F2
      AL = (SF2F2*SUMF1Y-SF1F2*SUMF2Y)/DEN0M
      B= (SF1F1*SUMF2Y-SF1F2*SUMF1Y)/DEN0M
      F = B/AL
      IF(ABSF(F)-0.005)4,5,5
    5 IF(I-10)6,4,4
    6 EK = EK + F
      I = I + 1
      GO TO 7
    4 WRITE (6,8)
    8 FORMAT(1H0,15X,13H TIME IN DAYS, 6X,14H BOD SATISFIED  )
      DO 9 J = 1,N
    9 WRITE (6,10) T(J), Y(J)
   10 FORMAT(15X,F10.3,10X,F10.3)
      WRITE (6,11) EK, AL, F, I
   11 FORMAT(5H K1 =,F6.3,3X,15H ULTIMATE BOD =,E15.5,3X,4H F =,F7.3,3X,
     I20H NO. OF ITERATIONS =,I2)
      GO TO 12
      END
```

Appendix C. Format for DO Used Computations

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Station number	Flow at station (*cfs*)	Average flow in reach (*cfs*)	Temp (°*C*)	H (*ft*)	t (*days*)	M (*min*)

(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
Obs DO_a		Obs avg DO		Percent sat	DO_a-DO_n	DO at saturation			
						At station		Average	
(*mg/l*)	(*lb/day*)	(*mg/l*)	(*lb/day*)		(*lb/day*)	(*mg/l*)	(*lb/day*)	(*mg/l*)	(*lb/day*)

(18)	(19)	(20)	(21)	(22)	(23)	(24)
Percent abs R_o	$1-\frac{\text{Percent sat}}{100}$	$\frac{1440t}{M}$	DO_r (*lb/day*)	DO_u (*lb/day*)	$\Sigma\ DO_u$ (*lb/day*)	Σt (*days*)

Col 12 = Col 10 ÷ Col 16 × 100
Col 13 = Col 9_{n+1} — Col 9_n
Col 19 = 1-(Col 12 ÷ 100)
Col 20 = 1440 × Col 6 ÷ Col 7
Col 21 = (Col 17 × Col 18 × Col 19 × Col 20) ÷ 100
Col 22 = Col 13 + Col 21